AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Algebra (Answers)

Name:

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Total Marks: /34

C4 - Algebra (Answers)
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In order to obtain the solutions to these exercises you will be expected to recall the general binomial formula:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\ldots(n-k+1)}{1\cdot 2\ldots k}x^k + \ldots$$

1. Expand the following expressions. Include only the first three terms:

   (a) $1 - x + x^2 + \cdots$ [2]

   (b) $1 + 2x + 4x^2 + \cdots$ [3]

   (c) $\sqrt{2} + \frac{x}{2\sqrt{2}} - \frac{x^2}{16\sqrt{2}} + \cdots$ [3]

   (d) We can use the answer to part a) to save us a lot of time:

$$\begin{aligned}\left(\frac{1}{3} + \frac{1}{3}x\right)^{-1} &= \left(\frac{1}{3}\right)^{-1}(1+x)^{-1} \\ &= 3(1+x)^{-1} \\ &= 3 - 3x + 3x^2 + \cdots\end{aligned}$$

   [2]

   (e) We can use the answer to part c) to save us a lot of time:

$$\begin{aligned}(32 + 16x)^{\frac{1}{2}} &= (16)^{\frac{1}{2}}(2+x)^{\frac{1}{2}} \\ &= 4(2+x)^{\frac{1}{2}} \\ &= 4\sqrt{2} + \sqrt{2}x - \frac{x^2}{4\sqrt{2}} + \cdots\end{aligned}$$

   [2]

2. Express the following in partial fractions:

   (a) $\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$. [2]

   (b) $\frac{3}{x+3} - \frac{2}{x+2}$. [2]

   (c) $\frac{2}{x+1} - \frac{1}{(x+1)^2} - \frac{2}{x+2}$. [3]

   (d) $\frac{3}{x+1} + \frac{6-3x}{x^2-2}$. [3]

(e) In this example we have a top heavy fraction (cubic is a higher power than the quadratic on the denominator). In order to resolve this issue we may use polynomial division:

$$\begin{array}{r} x-3. \\ x^2+3x+2\overline{)\;x^3} \\ \underline{-x^3-3x^2-2x} \\ -3x^2-2x \\ \underline{3x^2+9x+6} \\ 7x+6 \end{array}$$

Hence we may write the expression as:

$$\frac{x^3}{(x+2)(x+1)} = x-3+\frac{7x+6}{(x+2)(x+1)}.$$

Now we just use standard partial fractions on $\frac{7x+6}{(x+2)(x+1)}$ to get:

$$\frac{7x+6}{(x+2)(x+1)} = -\frac{1}{x+1}+\frac{8}{x+2}.$$

Thus our final answer is:

$$\frac{x^3}{(x+2)(x+1)} = x-3-\frac{1}{x+1}+\frac{8}{x+2}.$$

[4]

3. Consider the expression:

$$\frac{8}{(x-2)(x-1)}.$$

(a) $\frac{8}{x-2}-\frac{8}{x-1}$. [2]

(b)

$$8(x-2)^{-1} = -4-2x-x^2+\cdots$$
$$8(x-1)^{-1} = -8-8x-8x^2+\cdots$$

Thus

$$\frac{8}{(x-2)(x-1)} = 8(x-2)^{-1}-8(x-1)^{-1} = 4+6x+7x^2+\cdots,$$

as required. [6]