AQA, Edexcel, OCR

A Level

A Level Mathematics

C3 Trigonometry (Answers)

Name:

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Total Marks: /32

C3 - Trigonometry (Answers)
OCR, AQA, Edexcel

1. Consider the well-known trigonometric identity:

$$\sin^2 x + \cos^2 x = 1.$$

(a) We simply divide both sides of the equation by $\cos^2 x$:

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.$$

Hence:

$$\tan^2 x + 1 = \sec^2 x,$$

as required. [2]

(b) This time we divide both sides by $\sin^2 x$ to get:

$$1 + \cot^2 x = \text{cosec}^2 x.$$

[2]

2. Simplify the following trig expressions:

(a) $\sin(x + y)$. [2]

(b) $\cos(2x)$. [2]

(c) $\sin(2x)$. [2]

(d) $\tan^2 x$. [2]

(e) $4\sin^2(2x)$. [2]

(f)

$$\begin{aligned}\sin x \cos(2x) + 2\sin x \cos^2 x &= \sin x \cos(2x) + 2\sin x \cos x \times \cos x \\ &= \sin x \cos(2x) + \sin(2x)\cos x \\ &= \sin(2x + x) \\ &= \sin(3x).\end{aligned}$$

[3]

(g)

$$\begin{aligned}\cos^4 x - \frac{1}{2}\sin^2(2x) + \sin^4 x &= cos^4 x - \frac{1}{2}\left(2\sin x \cos x\right)^2 + \sin^4 x \\ &= \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x \\ &= \left(\cos^2 x - \sin^2 x\right)^2 \\ &= \left(\cos(2x)\right)^2 \\ &= \cos^2(2x).\end{aligned}$$

[3]

3. Write the following expressions in the form $R\sin(x+\alpha)$:

(a) $4\sin\left(x+\frac{\pi}{3}\right)$. [3]

(b) $3\sin\left(x+\frac{\pi}{4}\right)$. [3]

Turn over

4. The positive double angle formulas for sine and cosine are given by:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B,$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

(a) Using the identities above:

$$\begin{aligned}
\tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} \\
&= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\
&= \frac{\frac{\sin A}{\cos A}\cos B + \frac{\cos A}{\cos A}\sin B}{\frac{\cos A}{\cos A}\cos B - \frac{\sin A}{\cos A}\sin B} \\
&= \frac{\tan A \cos B + \sin B}{\cos B - \tan A \sin B} \\
&= \frac{\tan A \frac{\cos B}{\cos B} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\cos B} - \tan A \frac{\sin B}{\cos B}} \\
&= \frac{\tan A + \tan B}{1 - \tan A \tan B},
\end{aligned}$$

as required. [4]

(b)

$$\tan(2A) = \tan(A+A).$$

All we need to do is replace B with A in the above formula. This yields:

$$\begin{aligned}
\tan(2A) &= \tan(A+A) \\
&= \frac{\tan A + \tan A}{1 - \tan A \tan A} \\
&= \frac{2\tan A}{1 - \tan^2 A}.
\end{aligned}$$

[2]