AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Proof (Answers)

Name:

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Total Marks: /22

1. This question is easy. Not all integers are even! We use the counter example of 1. The number 1 is an odd integer. [1]

2. Let n be any integer. When we add n to itself we get $n + n = 2n = 2 \times n$ and we know that $2n$ is even as it is $2\times$something. [2]

3. Let three consecutive even numbers be $2n, 2n + 2$ and $2n + 4$. When we multiply these together we get:

$$\begin{aligned} 2n(2n+2)(2n+4) &= 2n \times 2 \times (n+1)(2n+4) \\ &= 4n(n+1)(2n+4) \\ &= 4\left[n(n+1)(2n+4)\right] \\ &= 4 \times \text{ Something.} \end{aligned}$$

   And so it is divisible by 4. [4]

4. False!

   Let n be some arbitrary integer. We square this to get n^2. If we then add this onto twice the original number we get $n^2 + 2n$. Now we may factorise this as follows:

$$n^2 + 2n = n(n + 2).$$

   But this isn't the product of two *consecutive* integers; there is a difference of 2 between the two numbers. If it were the product of two consecutive numbers we would have $n(n + 1)$ which we do not, so the statement is false. This hold for any integer as n is arbitrary.

5. True!

   We factorise as follows:

$$n^3 + n = n(n^2 + 1).$$

   Now consider case 1: when n is even. Then the above is obviously even as we are multiplying $(n^2 + 1)$ by an *even* number, and EVEN $\times$ Anything = EVEN. What about case 2: when n is odd. Then n^2 is odd and so $n^2 + 1$ is even. Then again the above is even as we are multiplying n, an odd number, by $n^2 + 1$, an even number, and EVEN $\times$ Anything = EVEN. So the statement is true. [3]

6. False!

   We adopt proof by exhaustion. When $n = 2$ we get $n^3 + n = 10$, which has a factor of five. When $n = 3$ we get $n^3 + n = 30$, which has a factor of 5. When $n = 4$ we get $n^3 + n = 68$, which does not have a factor of 5. So the statement is false. [3]

7. This one is easy! The answer is False!

   We prove this by a simple counter example. The line $y = 2x$ has a gradient of 2, so the statement is false. [2]

8. True!

   This is a quadratic function. Let's complete the square to find the minimum point. This gives:

$$\begin{aligned} f(x) &= x^2 - 2x - 1 \\ &= (x-1)^2 - 1 - 1 \\ &= (x-1)^2 - 2. \end{aligned}$$

   So the minimum point is $(1, -2)$ and so we know that $f(x) \geq -2$. So the statement is true! [3]