AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Functions

Name:

M M E

Mathsmadeeasy.co.uk

Total Marks: /61

C3 - Functions
MEI, OCR, AQA, Edexcel

1. Consider the functions $f(x) = x + 2$ and $g(x) = x^2$. Find the composite functions:

   (a) $fg(x)$ [2]

   (b) $gf(x)$ [2]

2. Consider the function $f(x) = 2x - 1$:

   (a) find $f^{-1}(x)$. [3]

   (b) Compute $f^{-1}f(x)$ [2]

   (c) Compute $ff^{-1}(x)$. [2]

3. What condition(s) are required for a function to be invertible? [2]

4. A function $f(x)$ is said to be an *odd* function if $f(-x) = -f(x)$. Similarly, $f(x)$ is said to be an *even* function if $f(-x) = f(x)$:

   (a) Prove that the sum of two odd functions is also an odd function. [3]

   (b) Prove that the product of two odd functions is an even function. [3]

   (c) Let $f(x)$ be an even function and $g(x)$ be an odd function. Consider the function $h(x) = f(x)g(x) + g(x)$. Is $h(x)$ odd or even (or neither)? [4]

   (d) Let $f(x)$ be an even function. What symmetrical property does this mean that its graph has? *Hint: think about its symmetry about the y axis. If you're still struggling, plot the graph of* $y = \cos x$ *and see what you notice about its reflection in the y axis. The function* $y = \cos x$ *is an even function.* [2]

   (e) Let $g(x)$ be an odd function. What symmetrical property does this mean that its graph has? *Hint: think about its symmetry about the y axis. If you're still struggling, plot the graph of* $y = \sin x$ *and see what you notice about its reflection in the y axis. The function* $y = \sin x$ *is an odd function.* [2]

5. Consider the function $f(x) = 2x + 2$:

   (a) Find $f^{-1}(x)$. [2]

   (b) On the same axis, carefully sketch the lines $f(x)$, $f^{-1}(x)$ and the line $g(x) = x$. What do you notice between the graphs of $f(x)$ and $f^{-1}(x)$? *Hint: think about how they relate to eachother in terms of reflections.* [3]

6. In order for a function to be invertible it needs to be one-to-one and onto. The function $y = \arcsin x$ (or $y = \sin^{-1} x$) is the inverse of the sine function $y = \sin x$. The problem is that the sine function is not one-to-one. In order to find the inverse we must restrict the domain. This can be done by only taking x values in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ as shown in the plot below:

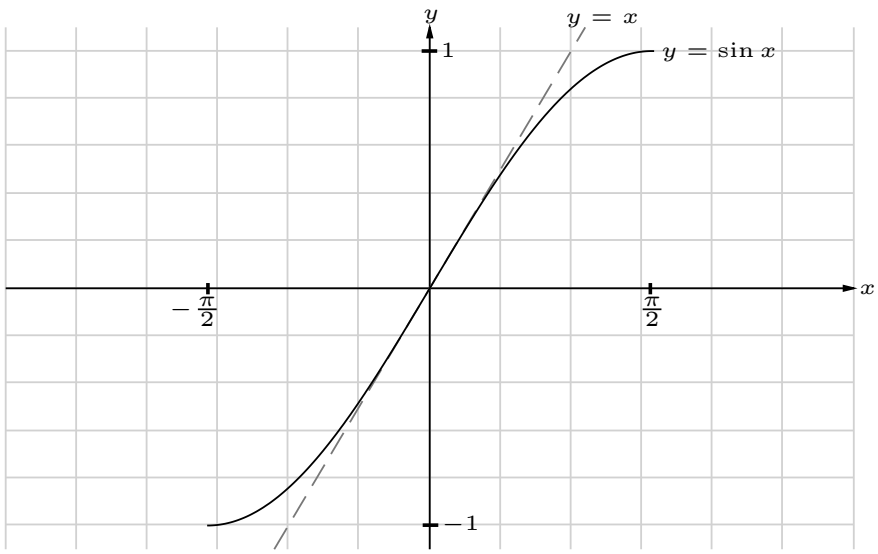

Figure 1: The graphs of $y = x$ and $y = \sin x$ plotted on the same axes.

(a) The graph of an inverse of a one-to-one function $f(x)$ is actually easy to plot once you have the graph of a function: the graph of $f^{-1}(x)$ is simply a reflection of $f(x)$ along the line $g(x) = x$. Using this information, sketch the graph of $y = \arcsin x$. You may wish to draw your sketch on Figure 1 above. [2]

(b) What is the domain and range of the function you sketched in part a)? [2]

7. Sketch the following functions, clearly indicating any points of intersection with the axes and the location of any minimum/maximum points:

(a) $y = |x|$ [2]

(b) $y = |x| + 1$ [2]

(c) $y = |x + 1|$ [2]

(d) $y = |2x| + 1$ [2]

8. Solve the following equations:

(a) $|2x| = 2$. [2]

(b) $|2x - 3| = 5$. [2]

9. Solve the following inequalities. *Hint: you may find sketches to be helpful*:

(a) $|3x| < 9$. [2]

(b) $|4x - 2| < 6$. [2]

(c) $\left|x^2 - 2\right| < 2$. [3]

(d) $|2x| < |x + 1|$. [3]

(e) $|3x| > |x - 2|$. [3]