AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Functions (Answers)

Name:

MME
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Total Marks: /61

C3 - Functions (Answers)
MEI, OCR, AQA, Edexcel

1. Consider the functions $f(x) = x + 2$ and $g(x) = x^2$. Find the composite functions:

   (a)

$$\begin{aligned} fg(x) &= f(g(x)) \\ &= f(x^2) \\ &= x^2 + 2. \end{aligned}$$

[2]

   (b)

$$\begin{aligned} gf(x) &= g(f(x)) \\ &= g(x+2) \\ &= (x+2)^2 \\ &= x^2 + 4x + 4. \end{aligned}$$

[2]

2. Consider the function $f(x) = 2x - 1$:

   (a) $f^{-1}(x) = \frac{1}{2}(x+1)$. [3]

   (b)

$$\begin{aligned} f^{-1}f(x) &= f^{-1}(f(x)) \\ &= f^{-1}(2x-1) \\ &= \frac{1}{2}\left((2x-1)+1\right) \\ &= x. \end{aligned}$$

[2]

   (c) Again, $ff^{-1}(x) = x$. [2]

3. A function is invertible if and only if it is a bijection. In other words, the function must be one-to-one and onto. [2]

4. A function $f(x)$ is said to be an *odd* function if $f(-x) = -f(x)$. Similarly, $f(x)$ is said to be an *even* function if $f(-x) = f(x)$:

(a) Let $f(x)$ and $g(x)$ be two odd functions. Denote $s(x)$ to be the function obtained by adding these two functions together. Thus,

$$s(x) = f(x) + g(x).$$

Now we just need to show that $s(x)$ is an odd function: we need to show that $s(-x) = -s(x)$. Let's compute $s(-x)$ then:

$$\begin{aligned} s(-x) &= f(-x) + g(-x) \\ &= -f(x) - g(x) \\ &= -(f(x) + g(x)) \\ &= -s(x). \end{aligned}$$

Hence this proves that the sum of two odd function is odd, as required.

[3]

(b) Let $f(x)$ and $g(x)$ be two odd functions. Let $p(x)$ be their product. Thus

$$p(x) = f(x)g(x).$$

Now let's compute $p(-x)$:

$$\begin{aligned} p(-x) &= f(-x)g(-x) \\ &= [-f(x)] \times [-g(x)] \\ &= f(x)g(x) \\ &= p(x). \end{aligned}$$

Thus, $p(-x) = p(x)$ so we know that $p(x)$ is an even function. Thus, the product of two odd functions gives an even function. [3]

(c)

$$\begin{aligned} h(-x) &= f(-x)g(-x) + g(-x) \\ &= -f(x)g(x) - g(x) \\ &= -[f(x)g(x) + g(x)] \\ &= -h(x). \end{aligned}$$

Thus, $h(x)$ is an *odd* function! [4]

(d) If $f(x)$ is an even function then its graph is symmetrical about the y axis. [2]

(e) If $g(x)$ is an odd function then its graph is antisymmetrical about the y axis. In other words, the graph to the left-hand side of the y axis is a rotation by $180°$ of the graph on the right-hand side of the y axis. [2]

5. Consider the function $f(x) = 2x + 2$:

(a) $f^{-1}(x) = \frac{1}{2}x - 1$. [2]

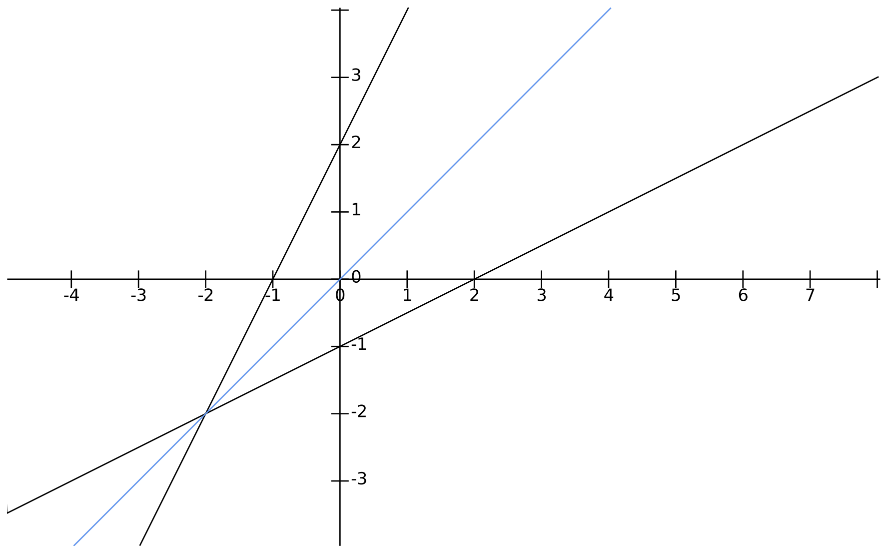

Figure 1: The graphs of $f(x)$, $f^{-1}(x)$ and $g(x) = x$ plotted on the same axes.

(b) The graph of $f^{-1}(x)$ is a *reflection* along the line $y = x$. *(This gives us a very easy way to sketch inverse functions if we already have the graph of the function).* [3]

6. In order for a function to be invertible it needs to be one-to-one and onto. The function $y = \arcsin x$ (or $y = \sin^{-1} x$) is the inverse of the sine function $y = \sin x$. The problem is that the sine function is not one-to-one. In order to find the inverse we must restrict the domain. This can be done by only taking x values in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ as shown in the plot below:

(a) [2]

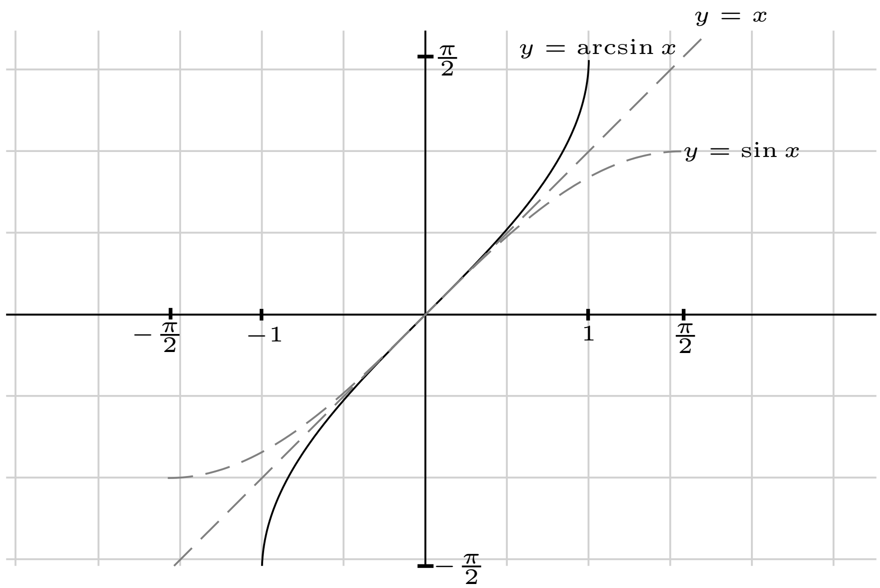

Figure 2: The graphs of $y = x$, $y = \sin x$ and $y = \arcsin x$ plotted on the same axes.

(b) The domain is now $-1 \leq x \leq 1$ and the range is $(-\frac{\pi}{2}, \frac{\pi}{2})$. [2]

7. Sketch the following functions, clearly indicating any points of intersection with the axes and the location of any minimum/maximum points:

(a) [2]

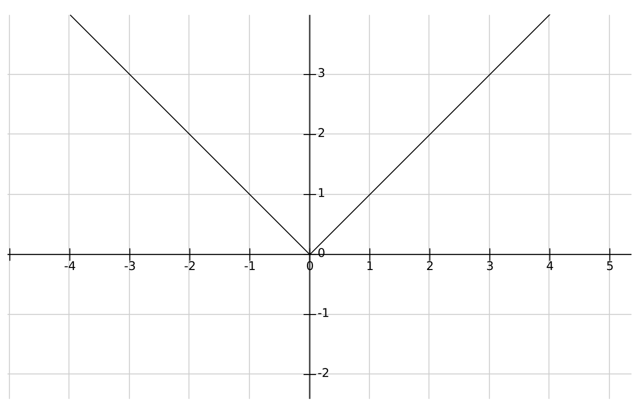

Figure 3: $y = |x|$.

(b) [2]

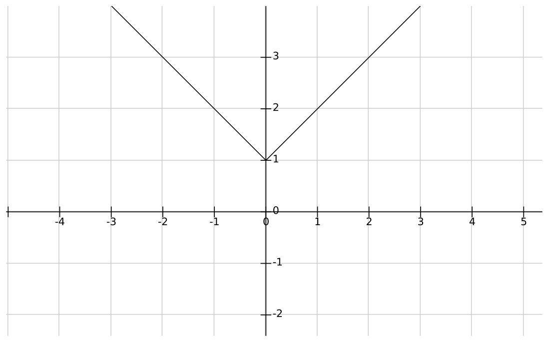

Figure 4: $y = |x| + 1$.

(c) [2]

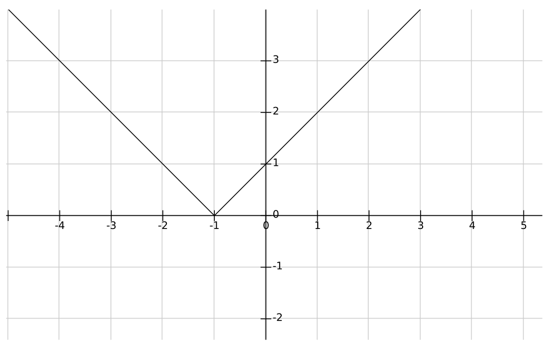

Figure 5: $y = |x + 1|$.

(d) [2]

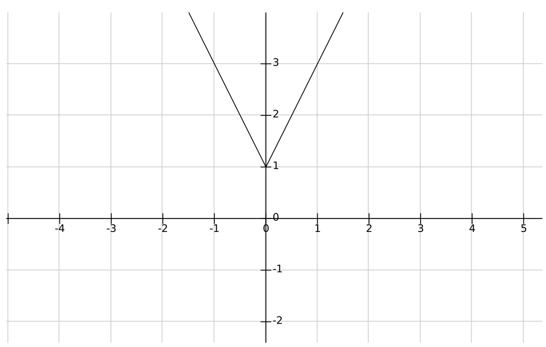

Figure 6: $y = |2x| + 1$.

8. Solve the following equations:

   (a) $x = \pm 1$. [2]

   (b) $x = -1$ and $x = 4$. [2]

9. Solve the following inequalities. *Hint: you may find sketches to be helpful*:

   (a) $-3 < x < 3$. [2]

   (b) $-1 < x < 2$. [2]

   (c) $-2 < x < 2$. [3]

   (d) $-\frac{1}{3} < x < 1$. [3]

   (e) $x < -1$ and $x > \frac{1}{2}$. [3]