AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C2 Calculus (Answers)

Name:

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Total Marks: /138

C2 - Calculus (Answers)
MEI, OCR, AQA, Edexcel

1. For each of the following functions calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$:

   (a) $\frac{dy}{dx} = 1, \qquad \frac{d^2y}{dx^2} = 0.$ [2]

   (b) $\frac{dy}{dx} = 3x^{-\frac{2}{3}}, \qquad \frac{d^2y}{dx^2} = -\frac{2}{9}x^{-\frac{5}{3}}.$ [2]

   (c) $\frac{dy}{dx} = 4x^2, \qquad \frac{d^2y}{dx^2} = 8x.$ [2]

   (d) $\frac{dy}{dx} = 20x^3 + 3, \qquad \frac{d^2y}{dx^2} = 60x^2.$ [3]

   (e) $\frac{dy}{dx} = 2x - 1, \qquad \frac{d^2y}{dx^2} = 2.$ [3]

   (f) $\frac{dy}{dx} = -3x, \qquad \frac{d^2y}{dx^2} = -3.$ [3]

   (g) $\frac{dy}{dx} = 6x^2 - 32x + 30, \qquad \frac{d^2y}{dx^2} = 12x - 32.$ [3]

   (h) $\frac{dy}{dx} = 1 - \frac{2}{x^2}, \qquad \frac{d^2y}{dx^2} = \frac{4}{x^3}.$ [3]

   (i) $\frac{dy}{dx} = \frac{1}{4}\left(11x^{\frac{7}{4}} + 2x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right), \qquad \frac{d^2y}{dx^2} = \frac{1}{16}\left(77x^{\frac{3}{4}} - 6x^{-\frac{7}{4}} + 5x^{-\frac{9}{4}}\right).$ [3]

   (j) $\frac{dy}{dx} = 4x, \qquad \frac{d^2y}{dx^2} = 4.$ [4]

2. Find the gradients of the following functions at the speficied points:

   (a) 12. [2]

   (b) $-\frac{2}{3}$. [3]

   (c) 2. [4]

3. Consider the function $f(x) = x^2 - 2x + 4$:

   (a) First we verify that the point $(1, 3)$ lies on the curve which indeed it does. Nwo $f'(x) = 2x - 2$ and $f'(1) = 0$ and so $(1, 3)$ is a stationary point. [5]

   (b) $f''(x) = 2 > 0$ therefore the stationary point is a minimum. [2]

   (c) Completing the square yields $f(x) = (x-1)^2+3$ and so $(1, 3)$ is the minimum point of the quadratic; it is a stationary point. [2]

   (d) $f'(4) = 6$ therefore the gradient is 6. [2]

   (e) When $x = 4$, $f(4) = 12$, so the point $(4, 12)$ lies on the curve. Now we use coordinate geometry on the point $(4, 12)$ using a gradient of 6 to obtain the tangent line $g(x) = 6(x - 2)$ as required. [5]

4. Consider the function $f(x) = \frac{2}{3}x^3 + bx^2 + 2x + 3$, where b is some undetermined coefficient:

(a) $f'(x) = 2x^2 + 2bx + 2$ and $f''(x) = 4x + 2b$. [4]

(b) $f(x)$ has a stationary point at $x = 2$ therefore we know that $f'(2) = 0$, where $f'(2) = 0 = 10 + 4b$. Rearranging gives $b = -\frac{5}{2}$. [3]

(c) We need to solve $f'(x) = 0$. Thus we need to solve the quadratic $2x^2 - 5x + 2 = 0$. The solutions are $x = 2$ and $x = \frac{1}{2}$. And so the other stationary point has coordinates $\left(\frac{1}{2}, \frac{83}{24}\right)$. [2]

(d) $f''(2) = 3 > 0$ and $f''(\frac{1}{2}) = -3 < 0$, therefore the point at $x = 2$ is a minimum and the point at $x = \frac{1}{2}$ is a maximum point. [3]

5. Integrate the following functions. *Remember to include a constant of integration*:

(a) $y = x + c$. [2]

(b) $y = \frac{3}{2}x^{\frac{4}{3}} + c$. [2]

(c) $y = \frac{3}{16}x^4 + x$. [2]

(d) $y = \frac{1}{5}x^5 + \frac{3}{2}x^2 + 8x + c$. [3]

(e) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + c$. [3]

(f) $y = -\frac{5}{6}x^3 + 5x + c$. [3]

(g) $y = \frac{1}{2}x^4 - \frac{16}{3}x^3 + 15x^2 + c$. [3]

6. $f(x) = \frac{1}{2}x^2 + 3x + 1$. [4]

7. $f(x) = 4x^4 + 3x^3 + \frac{1}{2}x - 10$. [5]

8. Consider the second derivative $f''(x) = 6x + 4$ of some cubic function $f(x)$.

(a) $f'(x) = 3x^2 + 4x + c$. [2]

(b) $f(x) = x^3 + 2x^2 + 10$. [4]

(c) We solve $f'(x) = 0 = 3x^2 + 4x$. The solutions to the quadratic are $x = 0$ and $x = -\frac{4}{3}$. The point $(0, 10)$ is a minimum and the point $(-\frac{4}{3}, \frac{302}{27})$ is a maximum. [5]

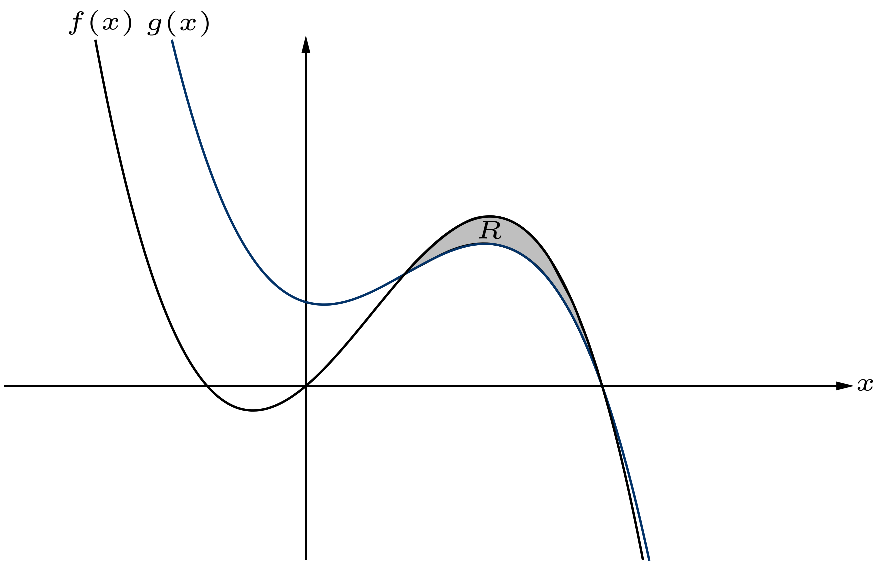
9. Consider the quadratic function $f(x) = 3x^2 + 2x + 4$.

(a) $\int_{-1}^{2} f(x)\, dx = 24$. [4]

(b) The area under the curve $f(x)$ between $x = -1$ and $x = 2$. [2]

10. $y = 2x^2 + \frac{1}{x} + 1$. [4]

11. Consider the functions $f(x) = -x^3 + 2x^2 + 3x$ and $g(x) = -x^3 + 3x^2 - x + 3$ sketched below.



(a) $f'(x) = -3x^2+4x+3$. We solve the quadratic equation $f'(x) = 0$ to obtain the solutions $x = \frac{2}{3} \pm \frac{\sqrt{13}}{3}$ as required. [5]

(b) $(1, 4)$ and $(3, 0)$. [4]

(c) $\frac{28}{3}$. [3]

(d) $\int_{1}^{3} g(x)\, dx = 8$. [3]

(e) Area$= \frac{28}{3} - 8 = \frac{4}{3}$. [2]

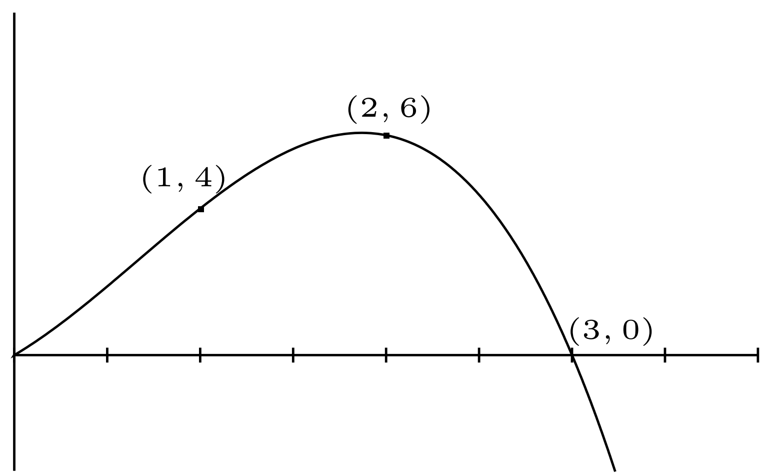
12. Consider the function $f(x) = x^2 + 1$. By differentiating *from first principles* show that $f'(x) = 2$.

Hint: Calculate the following limit:

$$\begin{aligned}\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{2(x+h) + 1 - (2x+1)}{h} \\ &= \lim_{h\to 0} \frac{2x + 2h + 1 - 2x - 1}{h} \\ &= \lim_{h\to 0} \frac{2h}{h} \\ &= \lim_{h\to 0} 2 = 2,\end{aligned}$$

as required. [4]

13. Consider the curve plotted below.



(a) A simple application of the trapezium rule formula: $\int_a^b y\,dx \approx \frac{1}{2}h\left((y_o + y_n) + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right)\right)$, where $h = \frac{b-a}{n}$.

Here $y_0 = 0, y_1 = 4, y_2 = 6$ and $y_3 = 0$.

Substituting these values into the formula yields Area$= 10$. [4]