AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C1 Curve Sketching (Answers)

Name:

MME
Mathsmadeeasy.co.uk

Total Marks: /20

1. Consider the function $f(x) = x^2 + x - 6$:

   (a) $x = -3$ or $x = 2$. [2]

   (b) $f(0) = -6$. [1]

   (c) We simply complete the square to get $f(x) = \left(x + \frac{1}{2}\right)^2 - \frac{25}{4}$ and hence $f(x)$ has a line of symmetry at $x = -\frac{1}{2}$. [2]

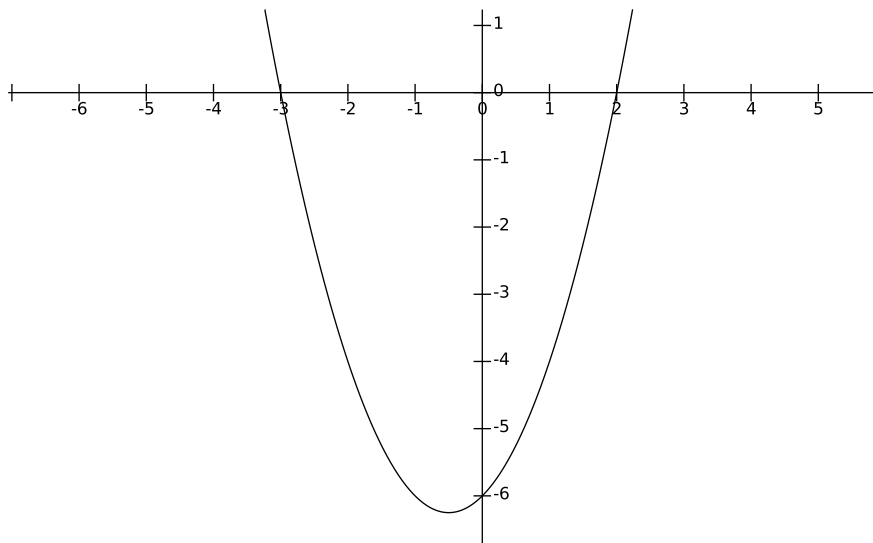   (d) The minimum point is $\left(-\frac{1}{2}, -\frac{25}{4}\right)$. [1]

   (e) [1]

Figure 1: $f(x) = x^2 + x - 6$

   (f) The curve is translated by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, so we know that $g(x) = f(x-3) + 1$. Hence, using part (c) we write $g(x) = \left(x - \frac{5}{2}\right)^2 - \frac{21}{4}$. Expanding gives $g(x) = x^2 - 5x + 1$ as required. [3]

2. Consider the function $f(x) = x^3 + 3x^2 - x - 3$:

   (a) $f(-3) = 0$. [1]

   (b) Since $f(-3) = 0$ we know that $(x+3)$ must be a factor of $f(x)$. We proceed with polynomial division:

$$
\begin{array}{r}
x^2 \qquad\;\; -1 \\
x+3 \overline{)\; x^3 + 3x^2 - x - 3} \\
\underline{-x^3 - 3x^2} \qquad\qquad \\
-x - 3 \\
\underline{x + 3} \\
0
\end{array}
$$

   And so we may write $f(x) = (x+3)(x^2 - 1)$, which we can factorise further to obtain:
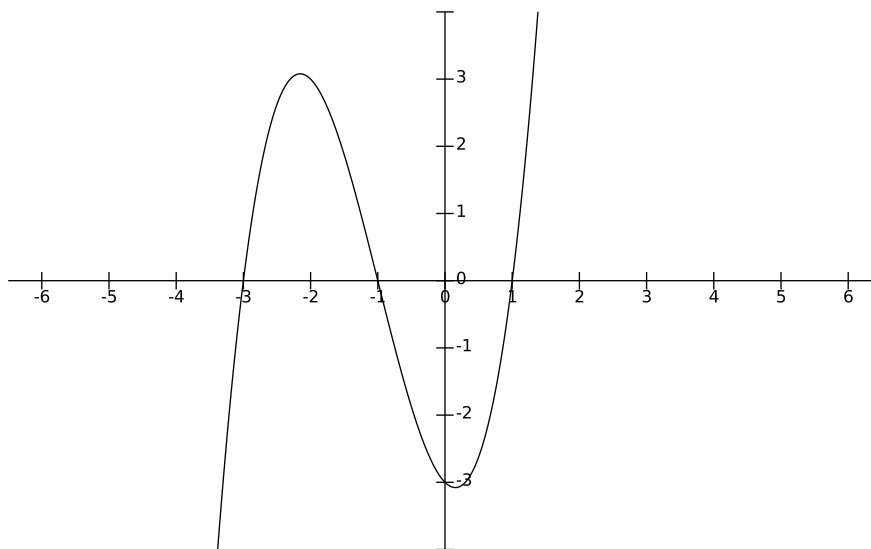
$$f(x) = (x+3)(x+1)(x-1).$$

[4]

   (c) [1]

Figure 2: $f(x) = x^3 + 3x^2 - x - 3$

   (d)

$$
\begin{aligned}
g(x) &= f(x+2) + 1 \\
&= ((x+2) + 3)\,((x+2) + 1)\,((x+2) - 1) \\
&= (x+5)(x+3)(x+1) + 1 \\
&= x^3 + 9x^2 + 23x + 16,
\end{aligned}
$$

   as required. [3]

   (e) Translation by $\binom{-2}{1}$. [1]