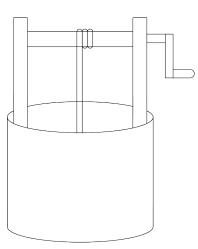


1. To make lifting water from a well much easier, a long handle (50 cm) is attached to the axle around which the rope is wound, as shown below. Ignore the weight of the rope.

Total for Question 1: 13



(a) State the principle of conservation of energy.

**Solution:** The total energy of a closed system remains constant: energy may not be created nor destroyed, but may be transferred from one form to another.

(b) By considering the work done taking a body from rest to speed v with constant acceleration, show that kinetic energy  $= \frac{1}{2}mv^2$ .

**Solution:**  $v^2 = u^2 + 2as$ . For  $u = 0, as = \frac{v^2}{2}$ . Work done =  $Fd \cos \theta$ . Here  $\theta = 0 \rightarrow \cos \theta = 1$ .  $F = ma \rightarrow$  work done =  $mas = \frac{1}{2}mv^2$ 

(c) A force of 500 N must be applied to the handle's end to lift the bucket of water, which has a combined mass of 50 kg. Given that the diameter of the winding axle is 5 cm, calculate the efficiency of the machine.

Solution: 5 %

(d) Bethan lifts the bucket using the handle. She then lets go. Sketch on two graphs, one for time and one for distance, the form of the variation of: (a) kinetic energy with time (b) potential energy with time (c) kinetic energy with distance (d) potential energy with distance. Assume that there are no frictional forces and that air resistance is negligible.

**Solution:** (a) Quadratic dependence on  $t^2$ ; concave-up

(b) Quadratic dependence on  $t^2$ ; concave-down

- (c) linear dependence on distance; positive gradient
- (d) linear dependence on distance; negative gradient

[4]

2. This question explores the different ways in which energy is dissipated by a car. In our simplified model a car of mass  $m_c$  is accelerated rapidly up to its constant cruising speed v, which it travels at until it slams on the brakes after a distance d. The car's passage produces a tube of swirling air with a cross-sectional area  $A_t$  and density  $\rho$ .

Total for Question 2: 17

(a) List one way in which energy is dissipated by a car, other than by creating a swirling wake and by [1] braking.

Solution: Any of the following: rolling resistance, heat, speeding up (or other valid examples)

(b) The kinetic energy acquired by the car as it is accelerated from rest to  $v \text{ ms}^{-1}$  is lost when it brakes. [2] Express the average power dissipation through the brakes between periods of rest in terms of  $m_c$ , v and d.

**Solution:**  $P = \frac{kinetic\ energy}{time\ between\ braking\ events} = \frac{m_c v^3}{2d}$ 

(c) Energy is dissipated into the wake by causing still air to move with swirl with velocity v. By considering the mass of the air tube, express in similar terms the rate of kinetic energy production in the wake.

Solution:  $P = \frac{kinetic \ energy}{time \ between \ braking \ events} = \frac{1}{2}\rho A_t v^3$ 

(d) Andrena and Dave both drive the same route from London to Glasgow. However, Andrena is in a rush and so drives at an average speed of 60 mph. Dave manages to average only 15 mph. Who uses more energy? By what factor are their usages different?

Solution: Andrena uses more by a factor of 16.

(e) In this simplified scenario, show that the dissipation is equally distributed between these two forms only when  $\frac{m_c}{d} = \rho A$ .

**Solution:** Total dissipation  $= \frac{m_c v^3}{2d} + \frac{1}{2}\rho A_t v^3$ Terms are equal only when  $\frac{m_c}{d} = \rho A_t \rightarrow m_c = \rho A d$  $\rho A_t d$  is the mass of the wake.

(f) Since  $m_c, \rho, v$  and  $A_t$  are all constant, there must exist a critical value of d,  $d^*$ , above which dissipation is either drag- or braking-dominated. If  $d < d^*$ , is more power dissipated into the brakes or into the swirling wake?

Solution: Brakes.

[1]

(g) The cross-sectional area of the tube is governed by the frontal area of the car and by its aerodynamic properties. This is quantified using the drag coefficient:  $A_t = c_d A_{car}$ . For a 1.5 ton car with a frontal area of 2 m<sup>3</sup> and a drag coefficient of  $\frac{1}{3}$ , calculate  $d^*$ . Assume  $\rho = 1.2 \ kgm^{-3}$ .

**Solution:** 1875 m

- (h) Which **one** of these statements is true?
  - i. If the distance between stopping events is 10 km, the number of people in the car will not dramatically affect power dissipation.
  - ii. If the distance between stopping events is 10 m, the number of people in the car will not dramatically affect power dissipation.
  - iii. If the distance between stopping events is 10 km, the most significant way in which power consumption can be reduced is by reducing the mass of the car.

**Solution:** (i)  $d \gg d^*$  - the system is drag dominated and so mass is relatively unimportant.

[1]