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AQA

A Level

A Level Mathematics

Appropriate Probability Distributions (Answers)

Name:

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Total Marks:

N3- Appropriate Probability Distributions- Answers

AQA

1) **For the following state whether it could be modelled with a binomial or normal distribution.**

[1 mark for each correct answer- 5 max]

i) The number of heads from 10 flips a fair two-sided coin

Binomial. Sample size fixed. Either a head or not a head.

ii) The number of sisters you have

Binomial. Sample size fixed. Either a sister or not a sister.

iii) The time a processor lasts

Normal. Continuous random variable.

iv) The concentration of chemical x in blood (mg/ml)

Normal. Continuous random variable.

v) The probability of getting a six on a die from 100 rolls.

Binomial. Sample size fixed. Either a six or not a six.

2) **A company makes biscuits and has a set of roller cutters. For efficiency reasons, once one roller stops produces biscuits or the correct dimensions the whole set are replaced. The variable B represents the scaled number of biscuits each cutter makes before it is replaced. The distribution of B has a mean of 25 and standard deviation of 2.**

i) **Find the probability that a randomly selected cutter, B, is less than 23.**

[1 mark for normal distribution]

This is a normal distribution and can be modelled as

$$B \sim N(25, 2^2)$$

[1 mark for correct methodology]

The standard normal variable for B is

$$z = \frac{B - 25}{2}$$

For $P(less\ than\ 23\ bisuits) = P(B < 23)$

[1 mark for correct B probability]

When $\mathrm{B} = 23$

$$z = \frac{23 - 25}{2} = -1$$

So when $P(B < 23) = P(z < -1) = 0.1587$

ii) **95% of cutters last for more than n bisuits. Find n.**

[1 mark for inverse value]

The normal standard inverse of 95% is 1.644853627.

[1 mark for correct methodology]

$$\frac{n - 25}{2} = 1.6448$$

[1 mark for correct answer]

$$n = 21.71$$

iii) **An inspector takes five samples. Determine the probability that fewer than two of them will have a life span less than 23.**

[1 mark for spotting it is binomial]

This is a binomial problem with $p = 0.1587, q = \ 0.8413\ and\ n = 5.$

[1 mark for each correct line – 3 max]

So, $P(0) = q^5 = 0.8413^5 = 0.4215$

And $P(1) = 5q^4p = 5\ \times\ 0.501\ \times\ 0.1587 =\ 0.3975$

$\therefore P(< 2) = 0.4215 + 0.3975 = 0.819$

3) **A stratified sample of information regarding 100 wild hares has been given to you.**

Experts agree that this sample is representative of the whole population.

Colour	Grey	Black	Brown	White
Frequency	8	21	62	9

i) **State the probability that the next hare observed will have a grey coat.**

[1 mark]

This is the probability on an individual trial.

$$P(grey) = 0.08$$

ii) **Find the probability of 9 rabbits in the next 100 being white.**

[1 mark for correct formula]

Using binominal probability formula we get

[1 mark for correct answer]

$$P(X = 9) = 0.13810605686364$$

The same set of rabbits also had their weight w recorded. These records are shown in the table below.

Weight (kg)	$w < 1.5$	$1.5 \leq w < 2$	$2 \leq w < 2.5$	$2.5 \leq w < 3$
Frequency	**15**	**32**	**29**	**24**

iii) State, or other, the probability of picking fewer than 320 rabbits from the next 1,000 with a weight greater than 1.5 kg but less than 2 kg.

[1 mark for observing can be estimated using normal distribution]

[1 mark for each of correct statements – 3 max]

$$p(1.5 \leq w < 2) = 0.32$$

$$np = 0.32 \times 1000 = 320$$

$$nq = 0.68 \times 1000 = 680$$

Both np and $nq > 10$, therefore, this can be approximated using the normal distribution.

The mean of the distribution is

$$\mu = np = 320$$

and the variance is

[1 mark for correct mean and variance- 2 max]

[1 mark for correct standard normal variable]

[1mark for correct answer]

$$\sigma^2 = np(1-p) = 320(1-0.32) = 217.6$$

$$\therefore \sigma = \sqrt{217.6}$$

$$X \sim N(320, 217.6)$$

$$z = \frac{320 - 320}{\sqrt{217.6}} = 0$$

$$p(z < 0) = 0.5$$