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OCR

A Level

A Level Physics

Gravitational Fields 2 (Answers)

Name:

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Total Marks: /30

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1. Kepler's laws were developed long before Newton's era and are based purely on empirical observation. From them it is possible to determine key orbit characteristics, especially if theoretical models are developed in accordance with observations.

Total for Question 1: 10

(a) State the following:

i. Kepler's first law. [1]

Solution: Orbit of a planet is an ellipse with the sun at one of the two foci.

ii. Kepler's second law. [1]

Solution: A line segment joining a planet and the sun sweeps out equal areas in equal time intervals.

iii. Kepler's third law. [1]

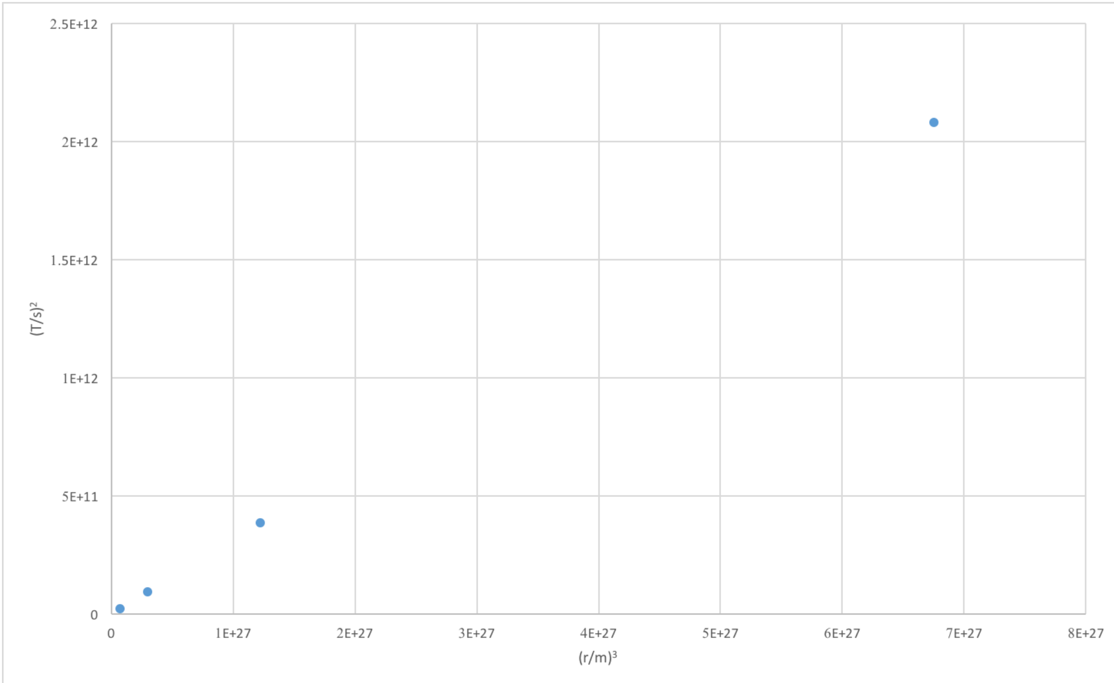
Solution: Square of the orbital period is proportional to the cube of the average distance from the sun.

(b) What provides a planet's centripetal force? [1]

Solution: Gravitational force of attraction.

(c) Derive an expression that gives a theoretical justification for Kepler's third law. [3]

Solution: $\frac{mv^2}{r} = \frac{GmM}{r^2} \rightarrow v^2 = \frac{GM}{r}$
Since the planet is assumed to be moving in a circle, $v = \frac{2\pi r}{T}$
Combining gives: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$



(d) The graph above shows the orbital properties of Jupiter's moons. Use it to calculate Jupiter's mass. [3]

Solution: 1.9×10^{27} kg

2. Satellites can also be analysed using the various laws of gravitation and circular motion. For this question, assume that Earth's mass is 6.0×10^{24} kg and that it has a radius of 6400 km.

Total for Question 2: 10

(a) By equating gravitational and centripetal forces, show that the mass of a satellite in orbit does not affect its speed. [2]

Solution: $\frac{mv^2}{r} = -GmM/r^2 \rightarrow v = \sqrt{\frac{GM}{r}}$

(b) Calculate the speed at which a satellite must be released into orbit if it is to maintain a height of 100 km above Earth's surface. [2]

Solution: 7.8 kms^{-1}

(c) Define a geostationary orbit. [3]

Solution: 1/ directly above the equator
2/ rotates in same direction as earth
3/ period of orbit is 24hrs.

(d) Calculate the altitude of a geostationary orbit. [3]

Solution: 3600 km

3. All vector fields have an associated scalar potential. For this question, assume Earth has a radius of 6400 km and a mass of 6.0×10^{24} kg.

Total for Question 3: 10

(a) Define, in words, the gravitational potential. [1]

Solution: At a point, the energy required, per unit mass, to bring a body from infinity to that point.

(b) Given that the gravitational potential, V_g, is 63 MJkg^{-1} at Earth's surface, calculate the following:

i. V_g at infinity. [1]

Solution: Zero

ii. V_g at an altitude equal to Earth's radius. [1]

Solution: 31.5 MJkg^{-1}

(c) Calculate the gravitational potential energy of a 10 kg ball at an altitude equal to three times Earth's radius. [2]

Solution: 210 MJ

(d) Sketch a graph to show how the magnitude of the gravitational force varies with the distance from the centre of the spherical object creating the field. What is represented by the area underneath the graph? [1]

> **Solution:** Should take form of $y \propto 1/x$
> Work done.

(e) The average kinetic energy of an H_2 molecule is given by the equation $\frac{1}{2}m\bar{c^2} = \frac{3}{2}kT$, where m is the mass of the molecule, c is the r.m.s. speed, k is the Boltzmann constant and T is temperature. By calculating the r.m.s. speed and the escape velocity, determine whether or not a helium molecule at 300 K can escape Earth's atmosphere. The mass of one atom of helium is 6.6×10^{-27} kg. [4]

> **Solution:** In theory it cannot: r.m.s. speed is 970 ms^{-1} and escape velocity is 11.2 kms^{-1}.