OCR
Oxford Cambridge and RSA

Tuesday 4 June 2024 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

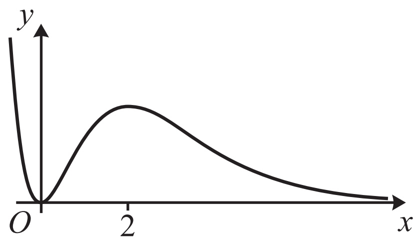
$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

1



The diagram shows part of the curve $y = x^2e^{-x}$.

(a) Use the trapezium rule with 4 intervals of equal width to find an estimate for $\int_0^2 x^2e^{-x}dx$. Give your answer correct to **3** significant figures. **[4]**

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for $\int_0^2 x^2e^{-x}dx$. **[1]**

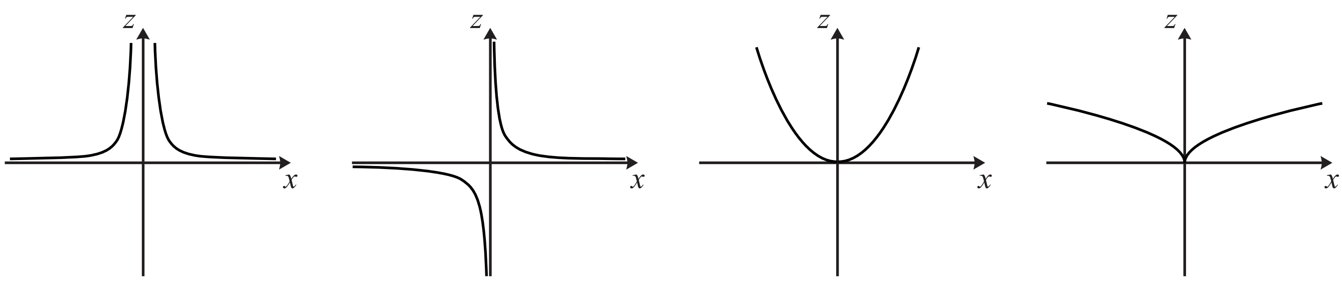
(c) Explain why it is not clear from the diagram whether the value from part **(a)** is an under-estimate or an over-estimate for $\int_0^2 x^2e^{-x}dx$. **[2]**

2 You are given that y is inversely proportional to x^6 and z is directly proportional to the cube root of y.

(a) **(i)** Find an equation for z in terms of x and k, where k is a constant of proportionality. **[2]**

(ii) State which of the diagrams below could represent the graph of z against x. **[1]**

Fig. 1.1 **Fig. 1.2** **Fig. 1.3** **Fig. 1.4**



(b) Given that $z = 3$ when $x = 4$, determine the values of x when $z = 12$. **[3]**

3 **(a)** Find a counterexample to disprove the statement that the product of two prime numbers is always odd. **[1]**

(b) In each of the following cases write one of the symbols $\Rightarrow$, $\Leftrightarrow$, $\Leftarrow$ in the box in the Printed Answer Booklet to make each statement correct.

(i)	$x^2 = 3x$	☐	$x = 3$	**[1]**
(ii)	$x > 4$	☐	$x^3 > 64$	**[1]**
(iii)	$x° = 45°$	☐	$\tan x° = 1$	**[1]**

(c) Prove that the sum of the squares of **any** two odd numbers is always a multiple of 2 but never a multiple of 4. **[4]**

4 A sequence has terms $u_1, u_2, u_3, \ldots$ defined by $u_1 = 2$ and $u_{n+1} = 1 - \frac{1}{u_n}$ for $n \geqslant 1$.

(a) Find the values of u_2, u_3 and u_4. **[2]**

(b) Describe the behaviour of the sequence. **[1]**

(c) Given that $\sum_{n=1}^{k} u_n = 73$, determine the value of k. **[3]**

5 The line $x + 13y = 108$ is the normal to the curve $y = ax^2 + b\sqrt{x}$ at the point (4, 8).

Determine the values of the constants a and b. **[8]**

6 **In this question you must show detailed reasoning.**

The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 4x^3 - 25x^2 - 58x + 16$.

(a) Show that $x = \frac{1}{4}$ is a root of the equation $\mathrm{f}(x) = 0$. **[1]**

(b) Hence express $\mathrm{f}(x)$ as the product of a linear factor and a quadratic factor, with all terms in the factors having integer coefficients. **[3]**

(c) Solve the equation $4\mathrm{e}^{3y} - 25\mathrm{e}^{2y} - 58\mathrm{e}^{y} + 16 = 0$, giving each root in the form $y = k\ln 2$ where k is a constant. **[4]**

7 The point A has coordinates (1, 7), and the point B has coordinates (h, 10).

(a) You are given that the gradient of the line AB is 2.

Find the value of h. **[2]**

(b) You are given that B is the midpoint of AC.

Find the coordinates of the point C. **[2]**

(c) You are given that the straight line through the points A, B and C has two distinct points of intersection with the curve $y = x^2 - 4x + k$.

Determine the set of possible values of k. **[6]**

8 **(a)** State the set of values for which $|x| > x$. **[1]**

(b) You are given that n is an integer such that $|n| \leqslant 9$.

(i) Find the maximum value of $|2n-1|$. **[1]**

(ii) Find the minimum value of $|2n-1|$. **[1]**

(c) **(i)** Solve the equation $\left|\frac{1}{2}x-1\right| = |2x-3|$. **[3]**

(ii) Explain why the equation $\left|\frac{1}{2}x-1\right| = 2x-3$ has only one solution, and state the value of this solution. **[1]**

9 The depth of the water, d metres, in a tidal river during a given day is modelled by the equation

$$d = 1.9 + 1.1\cos(30t - 60)°$$

where t is the number of hours after midnight.

(A tidal river is one whose level is influenced by tides.)

(a) **(i)** Find the minimum depth of water given by this model. **[1]**

(ii) Find the value of t when the minimum depth first occurs. **[2]**

(b) A boat can only enter the river when the depth of water is at least 1 metre.

Determine the two periods of time during the day between which this boat will **not** be able to enter the river. Give your answers correct to the **nearest minute**. **[5]**

In reality the depth of the river decreases as this boat travels along the river. An improved model uses the equation

$$d = e^{-cp}(1.9 + 1.1\cos(30t - 60)°)$$

where c is a positive constant and p is the distance, in kilometres, travelled along the river after entering it.

(c) Explain how this new equation could give an improved model. **[1]**

10 **In this question you must show detailed reasoning.**

The first three terms of a convergent geometric progression are $2x + 3$, $x + 9$ and $2x - 6$ respectively.

Determine the sum to infinity of this geometric progression. **[8]**

Turn over for questions 11 and 12

11 A curve has equation $y = 5\ln(1-\cos 2x)$, where x is in radians.

(a) State the values of x for which $5\ln(1-\cos 2x)$ is not defined. **[2]**

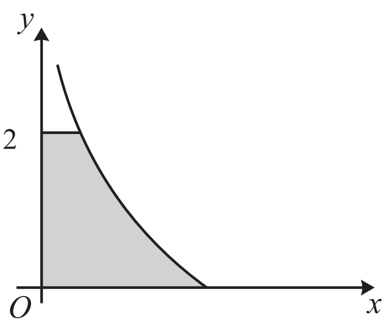
(b) P is the stationary point on the curve that has the smallest positive x-coordinate.

Determine the exact coordinates of P. **[4]**

(c) **(i)** Show that $\frac{d^2y}{dx^2} + 20e^{-\frac{1}{5}y} = 0$. **[5]**

(ii) State what can be deduced about all of the stationary points on this curve, giving a reason for your answer. **[1]**

12 **In this question you must show detailed reasoning.**



The diagram shows the curve with parametric equations $x = \frac{2}{(2t+1)^4}$, $y = 2t^2 + 3t$ for $t \geqslant 0$.

The shaded region is enclosed by the curve, the x-axis, the y-axis and the line $y = 2$.

(a) Show that the area of the shaded region is given by $\int_a^b \frac{8t+6}{(2t+1)^4}\,dt$, where a and b are constants to be determined. **[5]**

(b) Determine the exact area of the shaded region. **[6]**

END OF QUESTION PAPER