

## Tuesday 20 June 2023 – Afternoon

### A Level Mathematics A

#### H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator



#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document has **12** pages.

#### ADVICE

- Read each question carefully before you start your answer.

**Formulae**  
**A Level Mathematics A (H240)**

**Arithmetic series**

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small angle approximations**

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

**Section A**  
**Pure Mathematics**

- 1 Using logarithms, solve the equation

$$4^{2x+1} = 5^x,$$

giving your answer correct to **3** significant figures. [3]

- 2 (a) Express  $3 \sin x - 4 \cos x$  in the form  $R \sin(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the value of  $\alpha$  correct to **4** significant figures. [3]
- (b) Hence solve the equation  $3 \sin x - 4 \cos x = 2$  for  $0^\circ < x < 90^\circ$ , giving your answer correct to **3** significant figures. [2]

- 3 The cubic polynomial  $f(x)$  is defined by  $f(x) = x^3 + px + q$ , where  $p$  and  $q$  are constants.

- (a) (i) Given that  $f'(2) = 13$ , find the value of  $p$ . [2]
- (ii) Given also that  $(x - 2)$  is a factor of  $f(x)$ , find the value of  $q$ . [2]

The curve  $y = f(x)$  is translated by the vector  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ .

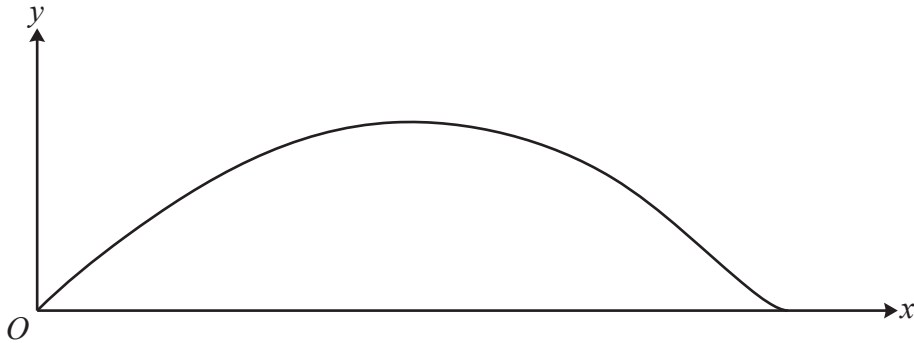
- (b) Using the values from part (a), determine the equation of the curve after it has been translated. Give your answer in the form  $y = x^3 + ax^2 + bx + c$ , where  $a$ ,  $b$  and  $c$  are integers to be found. [4]

- 4 A circle  $C$  has equation  $x^2 + y^2 - 6x + 10y + k = 0$ .

- (a) Find the set of possible values of  $k$ . [2]
- (b) It is given that  $k = -46$ .

Determine the coordinates of the **two** points on  $C$  at which the gradient of the tangent is  $\frac{1}{2}$ . [5]

- 5 A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the  $x$ -axis and the curve with parametric equations

$$x = 1 + 0.2t - \cos t, \quad y = k \sin^2 t,$$

where  $k$  is a positive constant and  $0 \leq t \leq \pi$ .

Lengths are in metres and the area of the emblem must be  $1 \text{ m}^2$ .

(a) Show that  $k \int_0^\pi (0.2 + \sin t - 0.2 \cos^2 t - \sin t \cos^2 t) dt = 1$ . [3]

(b) Determine the exact value of  $k$ . [6]

- 6 The first, third and fourth terms of an arithmetic progression are  $u_1$ ,  $u_3$  and  $u_4$  respectively, where

$$u_1 = 2 \sin \theta, \quad u_3 = -\sqrt{3} \cos \theta, \quad u_4 = \frac{7}{2} \sin \theta,$$

and  $\frac{1}{2}\pi < \theta < \pi$ .

(a) Determine the exact value of  $\theta$ . [3]

(b) Hence determine the value of  $\sum_{r=1}^{100} u_r$ . [3]

- 7 A car  $C$  is moving horizontally in a straight line with velocity  $v \text{ ms}^{-1}$  at time  $t$  seconds, where  $v > 0$  and  $t \geq 0$ . The acceleration,  $a \text{ ms}^{-2}$ , of  $C$  is modelled by the equation

$$a = v \left( \frac{8t}{7+4t^2} - \frac{1}{2} \right).$$

- (a) In this question you must show detailed reasoning.

Find the times when the acceleration of  $C$  is zero. [3]

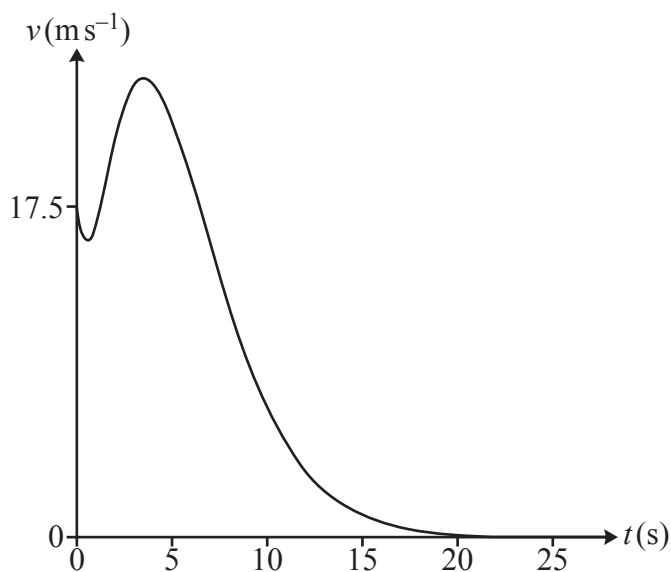
At  $t = 0$  the velocity of  $C$  is  $17.5 \text{ ms}^{-1}$  and at  $t = T$  the velocity of  $C$  is  $5 \text{ ms}^{-1}$ .

- (b) By setting up and solving a differential equation, show that  $T$  satisfies the equation

$$T = 2 \ln \left( \frac{7+4T^2}{2} \right). \quad [6]$$

- (c) Use an iterative formula, based on the equation in part (b), to find the value of  $T$ , giving your answer correct to 4 significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process. [2]

- (d) The diagram below shows the velocity-time graph for the motion of  $C$ .



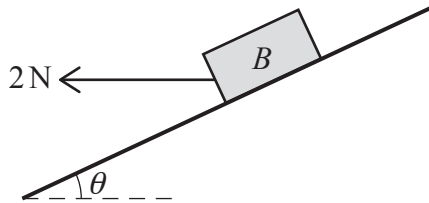
Find the time taken for  $C$  to decelerate from travelling at its maximum speed until it is travelling at  $5 \text{ ms}^{-1}$ . [1]

**Section B**  
**Mechanics**

- 8 A particle  $P$  moves with constant acceleration  $(3\mathbf{i} - 2\mathbf{j})\text{ms}^{-2}$ . At time  $t = 4$  seconds,  $P$  has velocity  $6\mathbf{i}\text{ms}^{-1}$ .

Determine the speed of  $P$  at time  $t = 0$  seconds. [4]

9



A block  $B$  of weight  $10\text{N}$  lies at rest in equilibrium on a rough plane inclined at  $\theta$  to the horizontal. A horizontal force of magnitude  $2\text{N}$ , acting above a line of greatest slope, is applied to  $B$  (see diagram).

- (a) Complete the diagram in the Printed Answer Booklet to show all the forces acting on  $B$ . [1]

It is given that  $B$  remains at rest and the coefficient of friction between  $B$  and the plane is  $0.8$ .

- (b) Determine the greatest possible value of  $\tan \theta$ . [5]

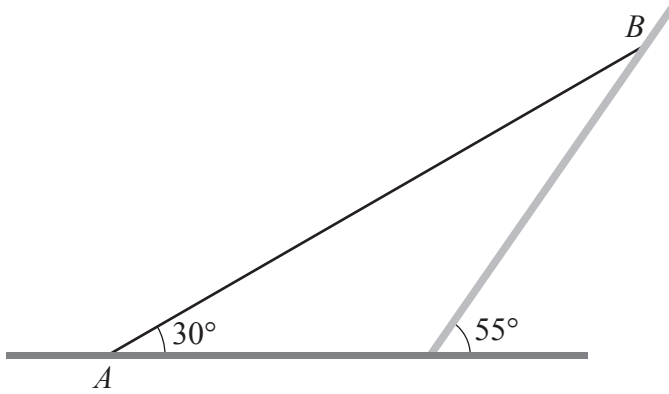
- 10 A particle  $P$  of mass  $m\text{kg}$  is moving on a smooth horizontal surface under the action of two constant horizontal forces  $(-4\mathbf{i} + 2\mathbf{j})\text{N}$  and  $(a\mathbf{i} + b\mathbf{j})\text{N}$ . The resultant of these two forces is  $\mathbf{R}\text{N}$ . It is given that  $\mathbf{R}$  acts in a direction which is parallel to the vector  $-\mathbf{i} + 3\mathbf{j}$ .

- (a) Show that  $3a + b = 10$ . [3]

It is given that  $a = 6$  and that  $P$  moves with an acceleration of magnitude  $5\sqrt{10}\text{ms}^{-2}$ .

- (b) Determine the value of  $m$ . [4]

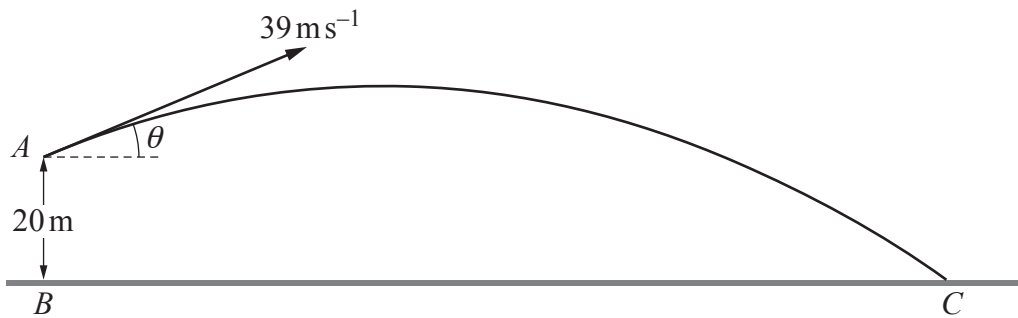
11



A uniform rod  $AB$ , of weight  $20\text{ N}$  and length  $2.8\text{ m}$ , rests in equilibrium with the end  $A$  in contact with rough horizontal ground and the end  $B$  resting against a smooth wall inclined at  $55^\circ$  to the horizontal. The rod, which rests in a vertical plane that is perpendicular to the wall, is inclined at  $30^\circ$  to the horizontal (see diagram).

- (a) Show that the magnitude of the force acting on the rod at  $B$  is  $9.56\text{ N}$ , correct to **3** significant figures. [3]
- (b) Determine the magnitude of the contact force between the rod and the ground. Give your answer correct to **3** significant figures. [5]

12 In this question you should take the acceleration due to gravity to be  $10 \text{ m s}^{-2}$ .



A small ball  $P$  is projected from a point  $A$  with speed  $39 \text{ m s}^{-1}$  at an angle of elevation  $\theta$ , where  $\sin \theta = \frac{5}{13}$  and  $\cos \theta = \frac{12}{13}$ . Point  $A$  is 20 m vertically above a point  $B$  on horizontal ground. The ball first lands at a point  $C$  on the horizontal ground (see diagram).

The ball  $P$  is modelled as a particle moving freely under gravity.

(a) Find the maximum height of  $P$  above the ground during its motion. [3]

The time taken for  $P$  to travel from  $A$  to  $C$  is  $T$  seconds.

(b) Determine the value of  $T$ . [3]

(c) State **one** limitation of the model, other than air resistance or the wind, that could affect the answer to part (b). [1]

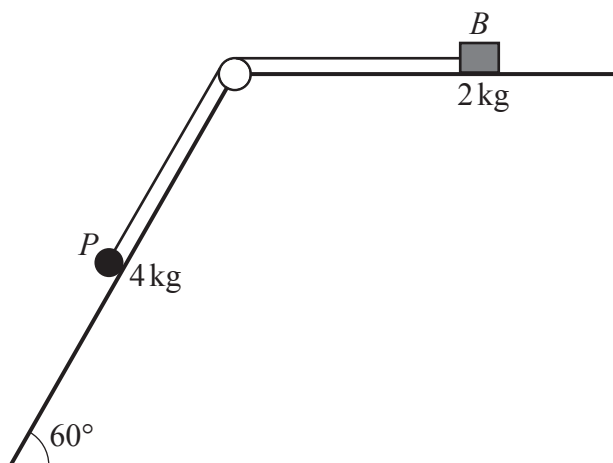
At the instant that  $P$  is projected, a second small ball  $Q$  is released from rest at  $B$  and moves towards  $C$  along the horizontal ground.

At time  $t$  seconds, where  $t \geq 0$ , the velocity  $v \text{ m s}^{-1}$  of  $Q$  is given by

$$v = kt^3 + 6t^2 + \frac{3}{2}t,$$

where  $k$  is a positive constant.

(d) Given that  $P$  and  $Q$  collide at  $C$ , determine the acceleration of  $Q$  immediately before this collision. [6]



The diagram shows a small block  $B$ , of mass  $2\text{ kg}$ , and a particle  $P$ , of mass  $4\text{ kg}$ , which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane. The particle can move on the inclined plane, which is rough, and which makes an angle of  $60^\circ$  with the horizontal. The block can move on the horizontal surface, which is also rough.

The system is released from rest, and in the subsequent motion  $P$  moves down the plane and  $B$  does not reach the pulley.

It is given that the coefficient of friction between  $P$  and the inclined plane is twice the coefficient of friction between  $B$  and the horizontal surface.

(a) Determine, in terms of  $g$ , the tension in the string. [7]

When  $P$  is moving at  $2\text{ ms}^{-1}$  the string breaks. In the  $0.5$  seconds after the string breaks  $P$  moves  $1.9\text{ m}$  down the plane.

(b) Determine the deceleration of  $B$  after the string breaks. Give your answer correct to 3 significant figures. [5]

**END OF QUESTION PAPER**



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