## GCSE MARKING SCHEME

SUMMER 2023

GCSE<br>MATHEMATICS<br>UNIT 2 - HIGHER TIER 3300U60-1

## INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

## WJEC GCSE MATHEMATICS

## SUMMER 2023 MARK SCHEME

\begin{tabular}{|c|c|c|}
\hline Unit 2: Higher Tier \& Mark \& Comments \\
\hline \begin{tabular}{l}
1.(a) \(7+5 x-10=3 x+8\) or equivalent.
\[
2 x=11 \quad \text { OR }-11=-2 x
\] \\
\(x=\frac{11}{2}\) or \(5 \cdot 5\) or equivalent.
\end{tabular} \& B1
B1

B1 \& | FT until $2^{\text {nd }}$ error. |
| :--- |
| Bracket must be expanded or correct division by 5 e.g $x-2=\frac{3 x}{5}+\frac{1}{5}$ (but not $x-2=\frac{3 x+1}{5}$ ) |
| Or equivalent |
| Correctly simplifying the equation to a single $x$ term and number term (e.g. $2 x-11=0$ ). |
| Mark final answer. |
| Correct answer implies B1B1B1. |
| Do not allow $-x=\frac{-11}{2}$ or $x=\frac{-11}{-2}$ |
| A final answer of ' $11 \div 2$ ' is B1B1B0. |
| If FT leads to a whole number answer, it must be shown as a whole number. Otherwise, accept a fraction. |
| Allow any decimal answer to be rounded or truncated to 1 or more decimal place. |
| Allow B1B1B1 for a correct embedded answer BUT only B1B1B0 if contradicted by $x \neq \frac{11}{2}$ or equivalent. |
| Note: $\begin{aligned} 12 x-24 & =3 x+8 & & \mathrm{B0} \\ 9 x & =32 & & \mathrm{~B} 1(\mathrm{FT}) \\ x & =\frac{32}{9} \text { or } 3 \cdot 5(55 \ldots) \text { or } 3 \cdot 6 . & & \mathrm{B} 1(\mathrm{FT}) \end{aligned}$ |
| If no marks awarded, award SC1 for sight of one of the following: |
| - $5 x-10$ |
| - $12 x-24$. | <br>

\hline | 1.(b) $\begin{gathered} 2 f=13-h \text { or } h-13=-2 f \\ f=\frac{13-h}{2} \text { or } \frac{h-13}{-2}=f \end{gathered}$ |
| :--- |
| or equivalent | \& \[

$$
\begin{aligned}
& \text { B1 } \\
& \text { B1 }
\end{aligned}
$$

\] \& | Or equivalent. |
| :--- |
| Or equivalent. |
| Must not come from incorrect working. |
| Mark final answer. |
| FT only from $\pm 2 f= \pm 13 \pm h$. |
| Unsupported $f=\frac{ \pm 13 \pm h}{ \pm 2}$ implies B0B1 unless B2. |
| Award B 1 B 0 for $-f=\frac{h-13}{2}$ or equivalent. |
| If no marks, award SC1 for a final answer of either: |
| - $f=(13-h) \div 2$ with or without brackets |
| - $f=(h-13) \div-2$ with or without brackets |
| - $\frac{13-h}{2}$ (' $f=$ ' missing). |
| - $\frac{h-13}{-2}$ (' $f=$ ' missing). | <br>

\hline 1.(c) $5(3 x-7 y)$ \& B1 \& Mark final answer. Allow $-5(-3 x+7 y)$ or $5(3 x+-7 y)$. <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline 2.(a) \(\mathrm{P}(\) Bronze \()=0.2\) AND \(\mathrm{P}(\) No Prize \()=0.6\) or equivalent \& B2 \& \begin{tabular}{l}
The values in the table takes precedence. \\
Award B 1 for one of the following: \\
- \(\mathrm{P}(\) Bronze \()=0.2\) (must be clearly identified) \\
- \(P(\) No Prize \()=0.6\) \\
- \(\quad P(\) Bronze \()+P(\) No Prize \()=0.8\) \\
- \(P(\) Bronze \()=1 / 3 P(\) No Prize \()\) provided both \(<1\).
\end{tabular} \\
\hline 2.(b) \(15 \div 0.02 \times 0.18\) or \(15 \times 9\) or equivalent
\[
=135
\] \& M1

A1 \& | Must be for a complete method e.g. |
| :--- |
| - $15 \div 2=7.57 .5 \times 18=135$ |
| - $750-(450+150+15)$ |
| - 0.02:0.18 |
| 15: $135(\mathrm{e} . \mathrm{g} 0 \cdot 18 \times 750$, or $15 \times 9)$ |
| Award M1 A1 for a final answer of $15: 135$. |
| Sight of 135 as a numerator in a fraction $<1$ implies M1A0. | <br>

\hline | 3. |
| :--- |
| One correct evaluation $2 \leq x \leq 3$ |
| 2 correct evaluations $2 \cdot 55 \leq x \leq 2 \cdot 75$, (one evaluation <0, one evaluation $>0$ ) |
| 2 correct evaluations $2 \cdot 55 \leq x \leq 2 \cdot 65$, (one evaluation $<0$, one evaluation $>0$ ) $x=2.6$ | \& B1

B1

M1

A1 \& | Correct evaluation regarded as enough to identify if negative or positive. |
| :--- |
| If evaluations not seen accept 'too high' or 'too low'. Look out for equating $x^{3}-8 x=-3$ |
| Unsupported $x=2 \cdot 6$ is awarded BOBOMOAO. |
| An answer of $x=2.6$ can only be awarded M1A1, following sight of 2 correct evaluations $2.55 \leq x \leq 2.65$ |
| (one evaluation $<0$, one evaluation $>0$ ). | <br>

\hline 4.(a) 1.2 \& B2 \& | Mark final answer. |
| :--- |
| Award B1 for one of the following: |
| - sight of $1 \cdot 1(5519 \ldots .$.$) .$ |
| - an answer of 1-20. |
| Do not award B2 or B1 for answers obtained from incorrect work (e.g. rounding and/or estimating). | <br>


\hline 4.(b) 0.043 \& B2 \& | Mark final answer. |
| :--- |
| Award B1 for sight of one of the following: |
| - $\frac{1}{23}$ |
| - $1 \div 23$ |
| - $0.0434(\ldots)$ |
| - 0.0435 |
| - 0.04 . | <br>

\hline 4.(c)(i) 12 \& B1 \& <br>
\hline 4.(c)(ii) 5 \& B1 \& <br>
\hline
\end{tabular}

| 5. Method to eliminate one variable e.g. equal coefficients AND appropriate intention to add or subtract or use a method of substitution. | M1 | Allow one error in one term (not the term with equal coefficients). |  |  |
| :---: | :---: | :---: | :---: | :---: |
| First variable found $x=4.3$ or $y=2.6$ or equivalent | A1 | CAO <br> Award A0 for expressing the final answers in a form such as $y=\frac{33 \cdot 8}{13}$. |  |  |
| Substitute to find the $2^{\text {nd }}$ variable. | m1 | FT substitution of their ' 1 st variable' if M1 gained. |  |  |
| Second variable found | A1 | No marks for 'trial and improvement'. No marks for an unsupported answer. |  |  |
| $\begin{aligned} & \text { 6.(a) } \\ & (x=) \sin ^{-1} \frac{7.7}{11 \cdot 3} \text { or } \\ & \\ & \\ & \sin ^{-1} \underline{7.7 \times \sin 90} \text { or equivalent } \end{aligned}$ | M2 | Check diagram for answers Award M1 for one of the following: <br> - $\sin x=\frac{7 \cdot 7}{11 \cdot 3}(=0 \cdot 68(1 .)$. <br> - $\frac{\sin x}{7.7}=\frac{\sin 90}{11 \cdot 3}$ or equivalent |  |  |
| Allow an answer between 42.8 and $43\left({ }^{\circ}\right)$ ISW | A1 | Allow correct angles given in radians or gradians: |  |  |
|  |  | Method | Radians | Gradians |
|  |  | $\sin ^{-1} \frac{7.7}{11.3}$ | 0.7496... | 47-727.... |
|  |  | $\sin ^{-1} \frac{7.7 \times \sin 90}{11.3}$ | 0.655... | 47.001 |
| 6. (a) $\frac{\text { Alternative method }}{\text { Correct use of a 'wo-step' method }}$ |  | A partial trigonometric method is MO. |  |  |
| Correct use of a 'two-step' method. | M2 |  |  |  |
| Allow an answer between 42.8 and $43\left({ }^{\circ}\right.$ ) ISW | A1 | Allow 42.8(... ${ }^{\circ}$ ) |  |  |
| ISW |  | Allow correct angles given in radians or gradians. |  |  |


| $6 .(\mathrm{b})$ |
| :---: |
|  |
|  |
|  |

Valid method to find the length $D E$
$D E=13.1 \times \tan 47$

$$
\begin{aligned}
& D E=\frac{\frac{13.1}{\tan 43}}{} \\
& \qquad D E=\frac{13 \cdot 1 \times \sin 47}{\sin 43}
\end{aligned}
$$

$D E$ in the range 14.04 to 14.1 (cm) ISW

Check diagram for answers.

B1

If B 1 already awarded for 'their angle $D B E$ but then 'their angle $B E D$ ' is incorrect and 'their $B E D$ ' is then used (or vice versa) for either M2 or M1, then award B0 previously.

Or award M2 for correct use of a 'two-step' method
(e.g. 'Pythagoras and similar triangles' or 'Pythagoras and correct trigonometric relationship').

FT 'their angle DBE or 'their angle BED' provided not $0^{\circ}, 45^{\circ}, 90^{\circ}$ or $180^{\circ}$. Award M1 for one of the following:

- $\tan 47=\frac{D E}{13 \cdot 1}$
- $\quad \tan 43=\frac{13 \cdot 1}{D E}$
- $\frac{D E}{\sin 47}=\frac{13 \cdot 1}{\sin 43}$ or equivalent

For all M2 or M1 scenarios, FT their clearly stated or shown angle $B E D$ or $D B E$ where appropriate.

For $13.1 \times \sin 47$ FT their clearly stated or shown $\sin 43$
angles $B E D$ and $D B E$ only if $B E D+D B E=90^{\circ}$.
Strict FT for DBE =90-'their $x$ ' or BED = 'their $x$ ', provided 'their $x$ ' $=45^{\circ}$.
Note: DBE must be acute for B1.
May be implied in further work.

Allow 14 from correct workings.

FT from M2 only and provided that angle is acute and leads to a positive answer.

Award B1M2AO for any of the following unsupported answers:

| Method | Radians | Gradians |
| :---: | :---: | :---: |
| $13.1 \times \tan 47$ | -1.63 to $1 . .$. | 11.92 to 12 |
| $\frac{13.1}{\tan 43}$ | -8.743 <br> -5.36 | 16.35 to 16.5 |
| $\frac{13.1 \times \sin 47}{\sin 43}$ | -1.95 to 1.08 | 14.1 to 14.21 |


| 7.(a) | ) $\times 0.95^{4}$ | B1 |  |
| :---: | :---: | :---: | :---: |
| 7.(b) | Sight of 0.83 OR 83\% $\frac{3569}{0.83}$ or $\frac{3569}{83} \times 100$ or equivalent $=4300$ | B1 <br> M1 <br> A1 | Allow (100-17 =) 83 <br> FT 'their $1-0 \cdot 17$ ' provided $<1$ or 'their 100\% - 17\%' provided $<100 \%$. <br> Award B1M1A1 for an embedded answer (e.g. $0.83 \times 4300=3569$ or $\frac{3569}{4300} \times 100=83$ ), <br> BUT only B1M1A0 if contradicted by stating original amount $\neq 4300$. <br> Unsupported 4300 is awarded B1M1A1. |
| $8 .$ | $\begin{aligned} & \frac{\pi \times r^{2}}{2}=77 \text { or equivalent } \\ & \qquad \begin{array}{r} r^{2}=49(\cdot 0 \ldots) \text { or } r^{2}=\frac{154}{\pi} \\ r=7(\cdot 0 \ldots) \\ \text { (Area of trapezium }=) \frac{2 \times 7(\cdot 0 \ldots)+22}{2} \times 7(\cdot 0 \ldots) \\ \text { or equivalent } \\ \\ =126 \cdot 0(\ldots)\left(\mathrm{cm}^{2}\right) \end{array} \end{aligned}$ | M1 <br> m1 <br> A1 <br> M1 <br> A1 | Check diagram for answers. <br> Sight of $49(\cdot 0 \ldots)$ implies M1m1. <br> FT 'their $r^{2}$ ' provided M1 awarded. 7 must not be from incorrect working. <br> FT 'their derived or stated $r$ '. <br> Accept 126.1 or $126\left(\mathrm{~cm}^{2}\right)$ <br> Mark final answer. |
| $9 .$ |  | B2 | B1 for 2 correct vertices within a triangle, e.g. A and 1 other vertex <br> OR for a triangle of correct shape, size and orientation in incorrect position <br> OR consistent correct use of an incorrect negative scale factor OR for 3 correct vertices (A implied) in the correct location not joined to form the triangle. |

\begin{tabular}{|c|c|c|}
\hline \[
\begin{aligned}
\& \text { 10.(a) (Reflex angle D }=\text { ) } \\
\& 180+360 / 5 \text { OR } 360-3 \times 180 / 5 \\
\& =252\left(^{\circ}\right) \\
\& \text { (Arc length CE }=) 252 / 360 \times \pi \times 2 \times 11 \\
\& 48.3(\ldots) \text { OR } 48 \cdot 4 \text { OR } 77 \pi / 5 \text { OR } 15-\pi \text { - OR } 15 \cdot 4 \pi(\mathrm{~cm}) \\
\& \text { o.e. }
\end{aligned}
\] \& \begin{tabular}{l}
M1 \\
A1 \\
M1 \\
A1
\end{tabular} \& \begin{tabular}{l}
Or any other complete method. May be seen in stages. \\
Award for sight of \(252\left({ }^{\circ}\right)\) if not contradicted by further incorrect work. \\
FT for M1 for 'their derived angle'. \\
ISW \\
Accept 48(cm) from correct working. \\
12089/250 from using \(3 \cdot 14\) \\
Allow a FT for this mark provided the angle used is \(>180^{\circ}\). \\
If no marks award SC1 for sight of \(108\left({ }^{\circ}\right)\) OR \(72\left({ }^{\circ}\right)\)
\end{tabular} \\
\hline \begin{tabular}{l}
Organisation and Communication \\
Accuracy of writing
\end{tabular} \& OC1

W1 \& | For OC1, candidates will be expected to: |
| :--- |
| - present their response in a structured way |
| - explain to the reader what they are doing at each step of their response |
| - lay out their explanation and working in a way that is clear and logical |
| - write a conclusion that draws together their results and explains what their answer means |
| For W1, candidates will be expected to: |
| - show all their working |
| - make few, if any, errors in spelling, punctuation and grammar |
| - use correct mathematical form in their working |
| - use appropriate terminology, units, etc. | <br>

\hline $$
\text { 10.(b) } \begin{aligned}
\left(\frac{671}{11}\right)^{2}(=450241 / 121) & \text { OR } 61^{2} \\
& =3721
\end{aligned}
$$ \&  \& Answer in the sentence takes precedence. If no marks, allow SC1 for $3721 / 25$ OR $148 \frac{21}{25}$ OR 148.84 [from (671/55)²]. <br>

\hline 11.

$$
\begin{gathered}
x^{2}(a+1)=b \text { OR }-x^{2}(a+1)=-b \text { OR } \\
x^{2}(-a-1)=-b \\
x^{2}=\frac{b}{a+1} \text { OR }-x^{2}=\frac{-b}{-a-1} \\
x= \pm \sqrt{\frac{b}{a+1}}
\end{gathered}
$$ \& B1

B1

B1 \& | FT until $2^{\text {nd }}$ error. $x^{2}$ or $-x^{2}$ factorised. |
| :--- |
| Isolating the $x^{2}$ or $-x^{2}$. |
| Allow a FT from $2 x^{2}=\frac{b}{a}$. |
| B0 for $\sqrt{ }[b \div(a+1)]$ (use of the division sign). |
| Allow omission of $\pm$. |
| Mark final answer. |
| Note: $\begin{array}{rlrl} 2 x^{2} & =\frac{b}{a} & & \mathrm{~B} 0 \\ x^{2} & =\frac{b}{2 a} & \mathrm{~B} 1(\mathrm{FT}) \\ x & =( \pm) \sqrt{\frac{b}{2 a}} & \mathrm{~B} 1(\mathrm{FT}) \end{array}$ | <br>

\hline
\end{tabular}

| 12.(a) $2(2 x+3)(2 x-3)$ | B3 | Award B3-1 for a correct answer followed by further incorrect work. <br> Award B2 for the sight of any one of the following: <br> - $(4 x+6)(2 x-3)$ <br> - $(4 x-6)(2 x+3)$ <br> - $8(x+3 / 2)(x-3 / 2)$ <br> - $(2 x+3)(2 x-3)$ <br> - $2(2 x+3)(2 x+3)$ <br> - $2(2 x-3)(2 x-3)$ <br> Award B 1 for the sight of any one of the following: <br> - $2\left(4 x^{2}-9\right)$ <br> - $8\left(x^{2}-9 / 4\right)$ <br> - $(4 x+6)(2 x+3)$ <br> - $(4 x-6)(2 x-3)$ <br> - $(x+3 / 2)(x-3 / 2)$ <br> If no marks: <br> Allow SC2 for $(2 \sqrt{2} x+3 \sqrt{2})(2 \sqrt{2} x-3 \sqrt{2})$ o.e. OR other valid, equivalent 'factorisation', e.g. $(8 x-12)(x+1.5) \text { o.e. }$ <br> Allow SC1 for $(\sqrt{8} x+\sqrt{18})(\sqrt{8} x-\sqrt{18})$ o.e. |
| :---: | :---: | :---: |
| 12.(b) 3/2 AND -3/2 | B1 | Or equivalent for either roots. <br> FT if possible, provided exactly 2 possible distinct solutions. |
| 12.(c) A positive quadratic curve passing through $(0,-18)$ as a minimum with -18 indicated on the $y$-axis AND passing through $(-3 / 2,0)$ and $(3 / 2,0)$ which are indicated on the $x$-axis. | B2 | FT for $x$-axis intersections, provided exactly 2 possible distinct solutions. <br> Award B1 for any one of the following: <br> A positive quadratic curve passing through $(0,-18)$ as a minimum with -18 indicated on the $y$-axis OR <br> A quadratic curve (either positive or negative) passing through $(-3 / 2,0)$ and $(3 / 2,0)$ which are indicated on the $x$-axis. <br> If the conditions for B 2 are met, then only allow B1 for concave and/or convex curvature above the $x$-axis. |

\begin{tabular}{|c|c|c|}
\hline 13.(a)
\[
\begin{aligned}
\& \frac{2}{4} \times \frac{3}{5} \times \frac{3}{6} \\
\& =\frac{18}{120} \text { ISW }\left(=\frac{3}{20} \text { or } 0.15\right) \text { oe }
\end{aligned}
\] \& M1 \& \begin{tabular}{l}
If no marks, award SC1 for: \\
\(\frac{a}{4} \times \frac{b}{5} \times \frac{c}{6}\) correctly evaluated with at least two of \(\mathrm{a}, \mathrm{b}\) and c correct \\
OR \\
an answer of \(\frac{48}{120}\) ( \(=\frac{2}{5}\) or 0.4 or equivalent) from assuming that 1 is prime, \(\frac{3}{4} \times \frac{4}{5} \times \frac{4}{6}\) OR an answer of \(\frac{4}{120}\) ( \(=\frac{1}{30}\) or \(0.03(3 \ldots)\) or equivalent) from excluding 2 as prime, \(\frac{1}{4} \times \frac{2}{5} \times \frac{2}{6}\)
\end{tabular} \\
\hline 13.(b)
\[
\begin{aligned}
\& {[1-P(1,1,1)-3 \times P(1,1,2)=]} \\
\& 1-4 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6} \text { OR } \frac{120-4}{120}
\end{aligned}
\]
\[
=\frac{116}{120} \quad \text { ISW } \quad\left(=\frac{29}{30}\right) \text { oe }
\] \& M2 \& \begin{tabular}{l}
Award M1 for sight of: \\
- \(4 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}\) or equivalent (from \(\mathrm{P}(1,1,1)+3 \times \mathrm{P}(1,1,2)\) ) OR \\
- \(1-2 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}\) or equivalent (from \(1-\mathrm{P}(1,1,1)-\mathrm{P}(1,1,2)\) ) \(O R\) \\
- \(1-3 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}\) or equivalent (from \(1-3 \times \mathrm{P}(1,1,2)\) ) OR \\
- \(1-\frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}\) or equivalent (from \(1-\mathrm{P}(1,1,1)\) ), note: this may include further incorrect work, e.g. \(\frac{117}{120}\left(=\frac{39}{40}\right)\) from \(1-\frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}-\frac{1}{4} \times \frac{1}{5} \times \frac{2}{6}\) or equivalent OR \\
- 120-4 \\
(120 permutations - the 4 permutations not allowed) \\
CAO \\
Allow 0.96́ OR 0.966(6...) OR 0.967 oe. \\
Allow 0.97 from correct working. \\
If no marks award SC1 for an answer of \(\frac{114}{120}\left(=\frac{19}{20}\right)\) from \(1-3 \times P(1,1,1)-3 \times P(1,1,2)\) \\
*Note: award M2 for a fully correct method of listing all the ways leading to totals greater than 4, but M0 for a partial attempt.
\end{tabular} \\
\hline \begin{tabular}{l}
14. \(\quad\) Sight of \(22(\mathrm{~cm})\) OR \(\sqrt[3]{10648}(\mathrm{~cm})\)
\[
\sqrt{ }\left(22^{2}+22^{2}+22^{2}\right) \text { OR } \sqrt{ }(1452)
\]
\[
=38 \cdot 1(\ldots \mathrm{~cm}) \text { OR } 22 \sqrt{3}(\mathrm{~cm}) \text { OR }
\] \\
a value that rounds to \(38 \cdot 1(\mathrm{~cm})\), e.g. \(38 \cdot 09(\mathrm{~cm})\)
\end{tabular} \& B1
M2

A1 \& | FT 'their $22^{\prime}$ for M2. May be seen in stages. (If the answer is completed in stages, then any arithmetic errors in intermediate answers can be ignored, allowing for a FT for the M marks). M1 for $22^{2}+22^{2}+22^{2}(=1452)$ |
| :--- |
| CAO. |
| Allow $38(\mathrm{~cm})$ from correct working provided their answer would round to 38.1 . | <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline \begin{tabular}{l}
15.
\[
3 x-7+x-2=(x-2)(3 x-7) \text { oe }
\] \\
Sight of \(3 x^{2}-7 x-6 x+14\)
\[
\begin{aligned}
\& 3 x^{2}-17 x+23=0 \text { OR }-3 x^{2}+17 x-23=0 \\
\& x=\frac{-(-17) \pm \sqrt{(-17)^{2}-4 \times 3 \times 23}}{2 \times 3}
\end{aligned}
\]
\[
\begin{aligned}
x= \& \frac{17 \pm \sqrt{13}}{6} \\
\& x=3.43 \text { with } x=2.23
\end{aligned}
\]
\end{tabular} \& B1
B1
B1

M1

A1

A1 \& | A correct common denominator may be shown throughout for first 3 marks. |
| :--- |
| Also, look out for alternative, correct methods leading to the same quadratic equated to zero. |
| For multiplying throughout the numerator of all terms by the common denominator. |
| Or equivalent. May be seen in the denominator. '= 0 ' required, but may be implied by an attempt to use the quadratic formula or if $a=3, b=-17$, $c=23$ used in the quadratic formula. |
| FT from B1B0 from one error only. |
| This substitution into the formula must be seen for M1, otherwise award MOAOA0. |
| FT 'their derived quadratic equation equated to 0 ', (but not $3 x^{2}-13 x+14=0$ ), provided of equivalent difficulty ( $a, b$ and $c$ must be non-zero). |
| Allow one slip in substitution for M1 only, but must be correct formula. |
| Can be implied from at least one correct value of $x$ evaluated. |
| CAO for their quadratic equation. |
| Answers must be given to 2 decimal places. |
| On FT solutions must require rounding. | <br>

\hline | 16. Initially calculating angle BDC $\begin{gathered} \cos B D C=\frac{5^{2}+7^{2}-11^{2}}{2 \times 5 \times 7} \quad(=-47 / 70) \\ (B D C=) 132\left(\cdot 17 \ldots .^{\circ}\right) \end{gathered}$ $\sin B A D=\frac{7 \times \sin \left[180-132\left(\cdot 17 \ldots{ }^{\circ}\right)\right]}{13}$ |
| :--- |
| $(\mathrm{BAD}=)$ Answers in the range $23 \cdot 4\left({ }^{\circ}\right)$ to $23 \cdot 6\left({ }^{\circ}\right)$ | \& M2

A1

M2

A1 \& | M1 for $11^{2}=5^{2}+7^{2}-2 \times 5 \times 7 \times \cos B D C$ |
| :--- |
| The correct angles given in: |
| Radians (2•3038... to $2 \cdot 3073 \ldots$ ) or |
| Gradians (146•6666... to $146 \cdot 8888 \ldots$...). |
| Allow a trial and improvement method for M2A1 but length of side BC must be in the range 10.95(cm) to 11.05(cm). |
| Sight of 28(•13... $)$ OR 132(•17... $)$ OR 19( $\left.68 \ldots{ }^{\circ}\right)$, calculated from one use of the rearranged cosine rule in triangle BCD and attributed to the wrong angle, stated or seen on the diagram, is M0 SC1. |
| FT 'their stated or derived BDC'. |
| M 1 for $\frac{\sin \operatorname{BAD}}{7}=\frac{\sin \left[180-132\left(\cdot 17 \ldots{ }^{\circ}\right)\right]}{13}$ o.e. |
| Mark final answer for angle BAD. |
| Allow a correctly rounded answer to a whole number provided an answer in the range $23.4\left({ }^{\circ}\right)$ to $23 \cdot 6\left({ }^{\circ}\right)$ is seen. | <br>

\hline
\end{tabular}

Alternative method 1
16. Initially calculating angle BCD $\cos B C D=\frac{5^{2}+11^{2}-7^{2}}{2 \times 5 \times 11} \quad(=97 / 110)$ $(B C D=) 28\left(\cdot 13 \ldots{ }^{\circ}\right)$
$\sin B A D=\frac{\left.11 \times \sin 28(\cdot 13 \ldots)^{\circ}\right)}{13}$
$(B A D=)$ Answers in the range $23.4\left(^{\circ}\right)$ to $23.6\left({ }^{\circ}\right)$

Alternative method 2
16. Multistep method to calculate angle BDC (First BCD, then CBD or vice versa)
$\cos B C D=\frac{5^{2}+11^{2}-7^{2}}{2 \times 5 \times 11} \quad(=97 / 110) O R$ $\cos C B D=\frac{7^{2}+11^{2}-5^{2}}{2 \times 7 \times 11}(=145 / 154)$
$(B C D=) 28\left(\cdot 13 \ldots{ }^{\circ}\right) \quad O R \quad(C B D=) 19\left(\cdot 68 \ldots{ }^{\circ}\right)$
$\sin C B D=\frac{5 x \sin [28(\cdot 13 \ldots)]}{7} O R$

$$
\sin B C D=\frac{7 \times \sin 19(\cdot 68 \ldots)}{5}
$$

$(C B D=) 19\left(\cdot 68 \ldots{ }^{\circ}\right) \quad O R \quad(B C D=) 28\left(\cdot 13 \ldots{ }^{\circ}\right)$
$(B D C=) 180-\left[28\left(\cdot 13 \ldots{ }^{\circ}\right)+19\left(\cdot 68 \ldots{ }^{\circ}\right)\right]$

$$
=132\left(\cdot 17 \ldots{ }^{\circ}\right)
$$

$\sin B A D=\frac{7 \times \sin \left[180-132\left(\cdot 17 \ldots{ }^{\circ}\right)\right]}{13}$
$(B A D=)$ Answers in the range $23.4\left({ }^{\circ}\right)$ to $23 \cdot 6\left({ }^{\circ}\right)$

The correct angles given in:
Radians ( $0.4886 \ldots$ to $0.4911 \ldots$ ) or
Gradians (31-1111... to 31-2666...).
Allow a trial and improvement method for M2A1 but length of side BD must be in the range 6.95(cm) to 7.05(cm).

Sight of 28(•13... ${ }^{\circ}$ ) OR 132(•17... ${ }^{\circ}$ ) OR 19(.68... $)$, calculated from one use of the rearranged cosine rule in triangle BCD and attributed to the wrong angle, stated or seen on the diagram, is M0 SC1.

M2 FT 'their stated or derived BCD'.
M1 for $\frac{\sin B A D}{11}=\frac{\sin 28\left(\cdot 13 \ldots{ }^{\circ}\right)}{13} \quad$ o.e.

A1 Mark final answer for angle BAD.
Allow a correctly rounded answer to a whole number provided an answer in the range $23.4\left(^{\circ}\right)$ to $23.6\left(^{\circ}\right.$ ) is seen.

A complete, correct method must be seen for the first M2.
M1 for one of the following:
$7^{2}=5^{2}+11^{2}-2 \times 5 \times 11 \times \cos B C D \quad O R$

$$
5^{2}=7^{2}+11^{2}-2 \times 7 \times 11 \times \cos C B D
$$

The correct angles given in:
Radians (BCD $\approx 0.49$ or $C B D \approx 0.34$ ) or
Gradians $(B C D \approx 31.2$ or $C B D \approx 21.9)$.
M1 for $7^{2}=5^{2}+11^{2}-2 \times 5 \times 11 \times \cos B C D$
.

This M2 (or M1 below) is for a complete multistep method, i.e. it includes all the previous steps.
FT 'their stated or derived BDC'.
$M 1$ for $\frac{\sin B A D}{7}=\frac{\left.\sin \left[180-132(\cdot 17 \ldots)^{\circ}\right)\right]}{13}$ o.e.
1 Mark final answer for angle BAD.

Allow a correctly rounded answer to a whole number provided an answer in the range $23.4\left({ }^{\circ}\right)$ to $23.6\left({ }^{\circ}\right)$ is seen.

Alternative method 3
16. Multistep method to calculate angle BDC
(First $B C D$, then $B D C$ OR first $C B D$, then $B D C$ )

$$
\begin{aligned}
& \cos B C D=\frac{5^{2}+11^{2}-7^{2}}{2 \times 5 \times 11} \quad(=97 / 110) O R \\
& \cos C B D=\frac{7^{2}+11^{2}-5^{2}}{2 \times 7 \times 11} \quad(=145 / 154)
\end{aligned}
$$

$$
(B C D=) 28\left(\cdot 13 \ldots{ }^{\circ}\right) O R(C B D=) 19\left(\cdot 68 \ldots{ }^{\circ}\right)
$$

$\sin B D C=\frac{11 \times \sin [28(\cdot 13 \ldots)]}{7}\left(=47.82 \ldots{ }^{\circ}\right) O R$

$$
\begin{aligned}
& \sin B D C=\frac{\left.11 \times \sin 19(\cdot 68 \ldots)^{\circ}\right)}{5}\left(=47 \cdot 82 \ldots{ }^{\circ}\right) \\
& \left(B D C=180-47\left(\cdot 82 \ldots{ }^{\circ}\right)\right)=132\left(\cdot 17 \ldots{ }^{\circ}\right) \\
& \sin B A D=\frac{7 \times \sin \left[180-132\left(\cdot 17 \ldots .^{\circ}\right)\right]}{13}
\end{aligned}
$$

$(B A D=)$ Answers in the range $23 \cdot 4\left(^{\circ}\right)$ to $23 \cdot 6\left({ }^{\circ}\right)$

Ambiguous case

M2 A complete, correct method must be seen for the first M2.
M1 for one of the following.

$$
\begin{aligned}
& 7^{2}=5^{2}+11^{2}-2 \times 5 \times 11 \times \cos B C D O R \\
& 5^{2}=7^{2}+11^{2}-2 \times 7 \times 11 \times \cos C B D
\end{aligned}
$$

A1 The correct angles given in:
Radians ( $B C D \approx 0.49$ or $C B D \approx 0.34$ )
Gradians ( $B C D \approx 31 \cdot 2$ or $C B D \approx 21.9$ ).

This M2 (or M1 below) is for a complete multistep method, i.e. it includes all the previous steps.
FT 'their stated or derived BDC'.
$M 1$ for $\underline{\sin B A D}=\underline{\left.\sin \left[180-132(\cdot 17 \ldots)^{\circ}\right)\right]}$ o.e.

A1 Mark final answer for angle BAD.
Allow a correctly rounded answer to a whole number provided an answer in the range $23.4\left(^{\circ}\right)$ to $23.6\left({ }^{\circ}\right)$ is seen.

