AS

MATHEMATICS

7356/2

Paper 2

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	$3a$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$-\frac{3}{5}$
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Integrates with at least one term in x correct	1.1a	M1	$\frac{1}{2}x^4 - \frac{8}{x} + c$
	Obtains correct integral Condone omission of $+c$ Condone inclusion of integral sign ACF	1.1b	A1	
	Includes $+ c$ FT their integral Must be some evidence of integration e.g., a power increased by 1	1.1b	B1F	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Substitutes $x = 2$ and $y = 0$ into their integral from part **(a)** to find a value of c PI by their correct value of c	1.1a	M1	$0 = \frac{1}{2}2^4 - \frac{8}{2} + c$ $c = -4$
	Finds correct value of c for their equation and states equation FT their integral from (a) but must have + c	1.1b	A1F	$y = \frac{1}{2}x^4 - \frac{8}{x} - 4$
	Subtotal		**2**	

	Question 3 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
4	Obtains ln$(x + 1)(x - 1)$ Or ln$(x^2 - 1)$ PI by correct equation in x^2 Condone missing brackets	1.1b	B1	ln$(x + 1)(x - 1)$ $= \ln 15 - \ln 49$ $= \ln \frac{15}{49}$ $x^2 - 1 = \frac{15}{49}$ $x^2 = \frac{64}{49}$ $x = \frac{8}{7}$ x cannot be $-\frac{8}{7}$ because the ln functions would not exist with this value
	Obtains ln 49 or ln7^2 for 2 ln 7 PI by correct equation in x^2	1.1b	B1	
	Applies subtraction rule for ln to right-hand side PI by correct equation in x^2	1.1a	M1	
	Obtains correct exact value for x^2 PI	1.1b	A1	
	Explains why $x = -\frac{8}{7}$ is not a valid solution. Must refer to ln(-ve)	2.4	E1	
	Question 4 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5	Recalls tan 15° as $\frac{\sin 15°}{\cos 15°}$ OE PI by division of given surd expressions ACF	1.2	B1	tan 15° $= \frac{\sin 15°}{\cos 15°}$ $= \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ $\frac{6-2\sqrt{12}+2}{6-2} = \frac{8-4\sqrt{3}}{4}$ $= 2 - \sqrt{3}$
	Multiplies top and bottom by conjugate of their denominator	1.1a	M1	
	Expands either the denominator or the numerator correctly ACF	1.1b	A1	
	Completes derivation of required expression from correct numerator and denominator AG	2.1	R1	
	Question 5 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6	Equates the equation of the curve to the equation of the line	1.1a	M1	$2x^2 + px + 1 = 5x - 2$ $2x^2 + (p - 5)x + 3 = 0$ Discriminant is $(p - 5)^2 - 24$ $= p^2 - 10p + 1$ $p^2 - 10p + 1 > 0$ $p > 5 + 2\sqrt{6}$ or $p < 5 - 2\sqrt{6}$
	Obtains the correct quadratic in form f(x) = 0 ACF	1.1b	A1	
	Obtains $(p - 5)^2 - 24$ ACF	1.1b	A1	
	Sets their discriminant to be > 0 Condone non-strict inequality here, but discriminant cannot contain terms in x Or Solves their discriminant = 0 to obtain exact values of p	1.1a	M1	
	Obtains correct inequalities ACF but must be exact	1.1b	A1	
	Question 6 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States correct coordinate Condone no brackets	1.1b	B1	$(a - 2, 2b)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical Solution
7(b)	States correct coordinates Condone no brackets	1.1b	B1	$(a, 8b)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical Solution
7(c)	States correct scale factor	1.1b	B1	$\frac{1}{3}$
	Subtotal		**1**	

	Question 7 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8	Expresses two consecutive odd numbers as $(2k \pm 1)$ or $(2k + 1)$ & $(2k + 3)$ or $(k \pm 1)$ where k is even or $(k + 1)$ & $(k + 3)$ where k is even OE	2.1	M1	Let the consecutive odd numbers be $(2k + 1)$ and $(2k - 1)$ where k is an integer $(2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1$ $(2k - 1)^3 = 8k^3 - 12k^2 + 6k - 1$ Sum $= 16k^3 + 12k$ $= 4k(4k^2 + 3)$ Factor of 4 shows that this is a multiple of 4
	Expands at least one **odd-numbered** cubic expression – allow one slip	1.1a	M1	
	Expands both of their two different odd-numbered cubic expressions correctly	1.1b	A1	
	Simplifies the sum of their cubic expansions correctly	1.1a	M1	
	Identifies common factor of 4 and completes proof. If $(k \pm 1)$ or $(k + 1)$ & $(k + 3)$ have been used, reference must be made to k being even, to clearly identify the factor of 4. CSO N.B. $(2k + 3)^3 = 8k^3 + 36k^2 + 54k + 27$ $(k - 1)^3 = k^3 - 3k^2 + 3k - 1$ $(k + 1)^3 = k^3 + 3k^2 + 3k + 1$ $(k - 3)^3 = k^3 - 9k^2 + 27k - 27$ $(2k + 1)^3 + (2k + 3)^3 =$ $16k^3 + 48k^2 + 60k + 28$	2.1	R1	
	Question 8 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Forms inverse proportion equation for P and substitutes given values to obtain given result AG	2.1	B1	$P = \dfrac{k}{n}$ $24 = \dfrac{k}{10}$ $k = 240$ $P = \dfrac{240}{n}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Forms inverse proportion equation for C and substitutes given value PI by sight of 25 with or without inequalities	1.1a	M1	$C = \dfrac{l}{n^2}$ $60 = \dfrac{l}{100}$ $l = 6000$ $\dfrac{240}{n} > \dfrac{6000}{n^2}$ $240n > 6000$ $n > 25$
	Obtains correct value of constant of proportionality PI by sight of 25 with or without inequalities	1.1b	A1	
	Forms inequality linking P and their C. Condone equality at this stage. Or Shows an attempt at trial and error to solve the inequality Condone equality at this stage. Or States n > 25 n = 25 or n ≥ 25 only	1.1a	M1	
	Obtains n > 25 ignore any extra inequality containing 0 CAO	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Identifies correctly the number of items that need to be **sold** to make a **profit** corresponding to **their** range of $\boldsymbol{n}$ from part **(b)** Providing n > 0 OE	3.5a	E1F	The artist makes a profit if they sell more than 25 items
	Subtotal		**1**	

	Question 9 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Explains the result using $\frac{1}{2}ab\sin C$ or other trigonometry AG	2.4	E1	Area of a triangle is $\frac{1}{2}ab\sin C$ Here a and b are both x and $C =$ 60º $A = \frac{1}{2}x^2\frac{\sqrt{3}}{2} = \frac{1}{4}\sqrt{3}x^2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Obtains $3x + 2y = 66$ OE	3.1a	B1	$3x + 2y = 66$ $y = 33 - \frac{3}{2}x$
	Uses their expression for y to give an expression for xy in terms of x ACF	1.1a	M1	$xy = (33 - \frac{3}{2}x)x$ $A = (33 - \frac{3}{2}x)x + \frac{1}{4}\sqrt{3}x^2$
	Completes correct derivation of given formula AG	2.1	R1	$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Differentiates, at least one term correct	1.1a	M1	$\frac{\mathrm{d}A}{\mathrm{d}x} = 33 - \frac{1}{2}(6 - \sqrt{3})x$
	Obtains correct derivative	1.1b	A1	At a stationary point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$
	States that for a stationary (OE) point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ Must state $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ not $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ but allow use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ elsewhere	2.4	E1	$33 - \frac{1}{2}(6 - \sqrt{3})x = 0$ $(6 - \sqrt{3})x = 66$ $x = \frac{66}{6 - \sqrt{3}}$ $x = 12 + 2\sqrt{3}$
	Solves their linear equation = 0 to obtain x PI ACF	1.1a	M1	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = -\frac{1}{2}(6 - \sqrt{3})$ Negative so maximum
	Obtains correct exact value of x ACF	1.1b	A1	
	Differentiates their $\frac{\mathrm{d}A}{\mathrm{d}x}$ to obtain an expression independent of x Or Tests x values either side of $12 + 2\sqrt{3}$ in $\frac{\mathrm{d}A}{\mathrm{d}x}$	1.1a	M1	
	Completes reasoned argument to deduce the maximum. Must state that $x = 12 + 2\sqrt{3}$ CSO Can be obtained independently of E1	2.2a	R1	
	Subtotal		**7**	

	Question 10 Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Obtains correct gradient of L_1 PI by fully rearranged equation of L_1 Condone slip in $\frac{41}{7}$ term	1.1b	B1	$y = -\frac{1}{7}x + \frac{41}{7}$
	Uses perpendicular gradients rule on their gradient of L_1	1.1a	M1	Gradient of L_1 is $\frac{-1}{7}$ Gradient of radius is 7 Equation of radius is $(y - 5) = 7(x - 6)$
	Completes reasoned argument to obtain given equation AG	2.1	R1	Equation of radius is $y = 7x - 37$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Obtains correct equation for radius through (0, 3)	1.1b	B1	Gradient of L_2 is 1 Gradient of radius is –1 Equation of radius is $y = -x + 3$
	Solves for $y = 7x - 37$ and their **linear** radius equation for L_2 to find the coordinates of their intersection of the radii	3.1a	M1	Radii intersect when $7x - 37 = -x + 3$ $x = 5, y = -2$
	Calculates distance from their centre to either contact point PI by $\sqrt{50}$ or $5\sqrt{2}$ or 50 seen in equation of C	1.1a	M1	Distance from (5, –2) to (0, 3) is $\sqrt{5^2 + 5^2} = \sqrt{50}$
	Obtains correct equation for C	2.1	R1	Equation of C is $(x - 5)^2 + (y + 2)^2 = 50$
	Subtotal		**4**	

	Question 11 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
12	Ticks the correct box	1.2	B1	The mass of an individual bag of nuts
	Question 12 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13	Circles correct answer	1.1b	B1	2
	Question 13 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Sets up the numbering of the population (allow 00 to 92) Allow 01 to 93	2.4	E1	Number the employees from 1 to 93
	Explains how the calculator (random number generator) will be used to generate random numbers (two-digit PI by between 1 and 93 or in this range or ignore numbers out of range or using these numbers) If using random number tables must state 2-digit numbers OE	2.4	E1	Choose random numbers from the calculator in this range
	Explains how to deal with repeats (PI) and identify 20 different employees/numbers	2.4	E1	Ignore repeats and continue until 20 different employees have been selected
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	States either Stratified or Quota	1.2	B1	Stratified sampling
	Subtotal		**1**	

	Question 14 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Obtains the correct probability ACF CAO	3.1b	B1	$\text{P(Even)} = \frac{3}{9} = \frac{1}{3}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Multiplies 3 fractions with at least 2 correct Condone the correct 3 fractions in a single calculation added	3.1b	M1	$\text{P}(703) = \frac{1}{4} \times \frac{2}{8} \times \frac{3}{9}$
	Obtains correct answer ACF Accept AWRT 0.021	1.1b	A1	$= \frac{1}{48}$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Multiplies three correct probabilities together for either P(222) or P(333) PI by $\frac{1}{288}$ or $\frac{1}{96}$ ACF	3.1b	M1	P(divisible by 111) = P(222) or P(333) $= \frac{1}{4} \times \frac{1}{8} \times \frac{1}{9} + \frac{1}{4} \times \frac{1}{8} \times \frac{3}{9}$
	Obtains correct answer ACF Accept AWRT 0.014	1.1b	A1	$= \frac{1}{72}$
	Subtotal		**2**	

	Question 15 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
16	Obtains at least one of **P($X \geq 3$)** = 0.6 Or **P($Y \leq 4$)** = 0.5 + p PI by 0.57	1.1b	B1	$0.3 + p + 0.2 + 0.1 + p + 3p + 0.05$ $= 1$ $0.65 + 5p = 1$ $5p = 0.35$ $p = 0.07$ P($X \geq 3$) = 0.6 P($Y \leq 4$) = 0.57 As 0.6 > 0.57 the claim is correct.
	Sums the y probabilities and compares to 1 Allow one slip Or Sets up a correct inequality for p P($X \geq 3$) > P($Y \leq 4$) PI by $p < 0.1$ ACF	3.1a	M1	
	Obtains the correct value for p Or Obtains $p < 0.1$ ACF	1.1b	A1	
	Obtains 0.6 for P($X \geq 3$) and 0.57 for P($Y \leq 4$), and shows that the claim is correct. Or States that if the claim is correct then $p < 0.1$, and uses a value of $p = 0.1$ to show that the sum of the probabilities for Y is greater than 1 and concludes that the claim is correct. Condone P($X \geq 3$) is greater than P($Y \leq 4$) for 'claim'	2.1	R1	
	Question 16 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)	Obtains correct mean CAO	3.4	B1	Mean $= 30 \times 0.79$ $= 23.7$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)	States correct answer AWRT 0.12	3.4	B1	0.124
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(c)	States correct answer AWRT 0.014	3.4	B1	$P(X \le 18) = 0.01399$ $= 0.014$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
17(d)	States or finds 1 – $P(X \le 25$ or $26)$ PI by correct answer or AWRT 0.21(5)	3.4	M1	$P(X > 26) = P(X \ge 27)$ $= 1 - P(X \le 26)$ $= 1 - 0.9015596$ $= 0.0984404$ $= 0.0984$
	Obtains correct answer AWRT 0.098	1.1b	A1	
	Subtotal		**2**	

	Question 17 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	States both hypotheses correctly for a one-tailed test. Accept equivalent in words. Accept population proportion for p. Accept 25%, but not $x =$ or $\bar{x} =$ or $\mu =$	2.5	B1	H_0: p = 0.25 H_1: p > 0.25
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Infers that that the Null Hypothesis is rejected PI by wording of the conclusion	2.2b	B1	There is sufficient evidence to suggest that the proportion of customers buying a loaf of bread has increased
	Concludes correctly in context. Must include '**proportion**' OE, not 'probability' or 'number of' '**Sufficient evidence'** OE required. Condone idea of causality if suggested	3.2a	E1	
	Subtotal		**2**	

	Question 18 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
19(a)	Estimates median correctly AWFW 1650 to 1675 Condone missing units	1.1b	B1	Median = 1662.5kg
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(i)	Correctly identifies zero mass implied from the Box Plot Accept masses should be at least 75kg (due to inclusion of mass of driver of 75kg in data)	2.2a	B1	The Box Plot implies that there is at least one car of zero mass which is not possible
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(ii)	Argues that the claim is incorrect with the correct reason	2.4	E1	There are car masses of zero in the Large Data Set So, the claim is incorrect
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
19(c)	Explains that the 2002 data is the only one of the two data sets to contain zero masses Or Gives a valid reason for Box Plot B being from the 2002 data or for Box Plot A being from 2016. The reason must be justified from the relevant Box Plot	2.4	E1	The 2002 data is the only one of the two data sets to contain zeros in the masses of cars field
	Subtotal		**1**	

	Question 19 Total		**4**	

	Question Paper Total		**80**	