A-level

## MATHEMATICS

7357/3
Paper 3
Mark scheme
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

## Key to mark types

| $M$ | mark is for method |
| :--- | :--- |
| $R$ | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

## Key to mark scheme abbreviations

| CAO | correct answer only |
| :--- | :--- |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | Indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |
| ISW | Ignore Subsequent Working |

## AS/A-level Maths/Further Maths assessment objectives

| AO |  |  |
| :--- | :--- | :--- |
| AO1 | AO1.1a | Select routine procedures |
|  | AO1.1b | Correctly carry out routine procedures |
|  | AO1.2 | Accurately recall facts, terminology and definitions |
|  | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
|  | AO2.2a | Make deductions |
|  | AO2.2b | Make inferences |
|  | AO2.4 | Explain their reasoning |
|  | AO2.5 | Use mathematical language and notation correctly |
|  | Translate problems in mathematical contexts into mathematical processes |  |
|  | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
|  | AO3.2a | Interpret solutions to problems in their original context |
|  | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
|  | AO3.3 | Translate situations in context into mathematical models |
|  | AO3.4 | Use mathematical models |
|  | AO3.5a | Evaluate the outcomes of modelling in context |
|  | AO3.5b | Recognise the limitations of models |
|  | AO3.5c | Where appropriate, explain how to refine models |

Examiners should consistently apply the following general marking principles

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

## Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

## Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

## Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1}$ | Ticks correct box | 2.2 a | R 1 | $y=\|x-2\|-3$ |
|  |  | Question 1 Total |  | $\mathbf{1}$ |


| $\mathbf{Q}$ | Marking instructions | $\mathbf{A O}$ | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{2}$ | Ticks correct box |  |  |  |
|  |  | 2.2 a | R 1 | $y$ |
|  |  |  |  |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 3 | Circles correct answer | 2.2 a | R 1 | $x=3$ |
|  | Question 3 Total |  | $\mathbf{1}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{4}$ | Obtains $5 x^{-2}$ or $x^{\frac{1}{3}-2}$ <br> PI by $p=-2$ or $q=-\frac{5}{3}$ | 1.1 a | M1 | $5 x^{-2}-x^{-\frac{5}{3}}$ |
|  | Obtains $5 x^{-2}-x^{-\frac{5}{3}}$ <br> PI by $p=-2$ and $q=-\frac{5}{3}$ OE <br> Allow -1.67 or better for $-\frac{5}{3}$ | 1.1 b | A1 |  |
|  | Do not ISW incorrect algebra |  |  |  |
|  | Question 4 Total |  | $\mathbf{2}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 5 | Obtains $2 \times 3 e^{2 x}$ or $6 e^{2 x}$ or $2 y$ PI by correct answer | 1.1b | B1 | $\frac{\mathrm{d} y}{\mathrm{~d} x}=2 \times 3 e^{2 x}$$y=10 \Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=2 \times 10=20$ |
|  | Substitutes $y=10$ or $3 e^{2 x}=10$ in their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ or substitutes $x=[0.6,0.602]$ or $x=\frac{1}{2} \ln \left(\frac{10}{3}\right)$ OE in their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ | 1.1a | M1 |  |
|  | Obtains 20 <br> CAO <br> 20 cannot come from a rounded value for 20 seen | 1.1b | A1 |  |
|  | Question 5 Total |  | 3 |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| $\mathbf{6 ( a )}$ | Draws cubic curve in the correct <br> orientation | 1.1 a | M 1 |  |
|  | Deduces minimum or maximum <br> at (0,0) on their curve | 2.2 a | M 1 |  |
|  | Draws a fully correct cubic curve <br> with $x$-intercept at $-\frac{a}{2}$ shown on <br> the curve | 2.2 a | A 1 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 6(b)(i) | Substitutes $x=-3$ into $\mathrm{p}(x)$ <br> Condone missing bracket for $(-3)^{2}$ <br> Must see an expression in terms of $a$ | 1.1a | M1 | $\begin{aligned} (-3)^{2}(2 \times-3+a)+36 & =0 \\ -54+9 a+36 & =0 \\ 9 a-18 & =0 \\ a & =2 \end{aligned}$ |
|  | Completes reasoned argument with at least one correct intermediate step and no error seen to show $a=2$ AG <br> Must set an expression for $p(-3)=0$ <br> Condone recovery of missing bracket for $(-3)^{2}$ to get 9 <br> Do not condone any other missing bracket | 2.1 | R1 |  |
|  | Subtotal |  | 2 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{6 ( b ) ( i i ) )}$ | States 'translation' or 'translate' <br> or 'translated' <br>  <br>  <br>  <br>  <br>  <br>  <br> Must not have other <br> transformation other than <br> translation | 1.1 b | B1 |  |
|  | States the vector $\binom{0}{36}$ or 36j | 1.1 b | B1 |  |
|  | Subtotal |  | $\mathbf{2}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{6 ( b ) ( \text { (iii) }}$ | Explains that the translated <br> graph only has one real solution <br> or only has a root at -3 <br> Condone missing 'real' | 2.4 | E1 | The translated graph will only have <br> one real solution. <br> $b^{2}-4 a c<0$ |
|  | Deduces that the discriminant of <br> $2 x^{2}+b x+c$ must be negative <br> and shows the required result <br> Do not allow the use of $a=2$ <br> with reference to part (b)(i) <br> Allow $b^{2}-8 c<0$ following <br> from $b^{2}-4 a c$ seen | 2.2 E | Hence $b^{2}-4 \times 2 \times c<0$ |  |


|  | Question 6 Total |  | 9 |  |
| :--- | :--- | :--- | :--- | :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 7(a) | Forms an expression for the area of one sector or both sectors <br> e.g $\frac{1}{2} r^{2}\left(\frac{\pi-\theta}{2}\right)$ or $r^{2}\left(\frac{\pi-\theta}{2}\right)$ <br> or $\frac{1}{2} r^{2}(\pi-\theta)$ <br> OE <br> Allow substitution of $r=2.5$ <br> Condone $r=5$ <br> Condone missing brackets | 3.1b | M1 | Area of sectors = $2 \times \frac{1}{2}(2.5)^{2}\left(\frac{\pi-\theta}{2}\right)$ <br> Area of rhombus = $\begin{aligned} & 2 \times \frac{1}{2}(2.5)^{2} \sin \theta \\ & \mathrm{~A}=(2.5)^{2}\left(\frac{\pi-\theta}{2}\right)+(2.5)^{2} \sin \theta \end{aligned}$ $A=\frac{25}{8}(\pi-\theta)+\frac{25}{4} \sin \theta$ |
|  | Forms an expression for the area of half rhombus or full rhombus e.g $\frac{1}{2} r^{2} \sin \theta$ or $r^{2} \sin \theta$ <br> Allow substitution of $r=2.5$ Condone $r=5$ | 3.1b | M1 | $\mathrm{A}=\frac{25}{8}(\pi-\theta+2 \sin \theta)$ |
|  | Substitutes $r=2.5$ to get a correct expression for area of both sectors or full rhombus <br> Condone missing brackets | 1.1b | A1 |  |
|  | Completes reasoned argument by calculating correct total area with at least one correct intermediate step and no error seen to show the given result. AG <br> Allow recovery of missing brackets | 2.1 | R1 |  |
|  | Subtotal |  | 4 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 7(b)(i) | Differentiates wrt $\theta$ Condone sign errors and omission of $\frac{25}{8}$ | 3.1a | M1 | $\frac{\mathrm{dA}}{\mathrm{~d} \theta}=-\frac{25}{8}+\frac{25}{4} \cos \theta$ |
|  | Obtains $\frac{25}{8}(-1+2 \cos \theta)$ OE | 1.1b | A1 | Max area occurs when $\frac{\mathrm{dA}}{\mathrm{d} \theta}=0$ |
|  | Explains maximum or stationary or turning occurs when $\frac{\mathrm{dA}}{\mathrm{d} \theta}=0$ Label $\frac{\mathrm{dA}}{\mathrm{d} \theta}$ must be seen | 2.4 | E1 | $\begin{array}{r} \frac{25}{4} \cos \theta=\frac{25}{8} \\ \cos \theta=\frac{1}{2} \quad \therefore \theta=\frac{\pi}{3} \end{array}$ |
|  | Equates their $\frac{25}{8}(-1+2 \cos \theta)$ to 0 and rearranges to obtain a value for $\cos \theta$ or $\theta$ when $\cos$ is not seen <br> Condone omission of $\frac{25}{8}$ | 1.1a | M1 | When $\theta=\frac{\pi}{3}$ $\frac{\mathrm{d}^{2} \mathrm{~A}}{\mathrm{~d} \theta^{2}}=-5.41<0$ so maximum. |
|  | Obtains $\cos \theta=\frac{1}{2}$ or $\cos ^{-1}\left(\frac{1}{2}\right)$ OE and shows that $\theta=\frac{\pi}{3}$ AG | 2.2a | A1 |  |
|  | Uses second derivative to obtain $-\frac{25 \sqrt{3}}{8}$ or AWRT -5 and completes argument to show maximum occurs when $\theta=\frac{\pi}{3}$ <br> Allow gradient test <br> To be awarded R1, marks M1A1M1A1 must be scored as the minimum | 2.4 | R1 |  |
|  | Subtotal |  | 6 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 7(b)(ii) | Substitutes $\theta=\frac{\pi}{3}$ into <br> $\mathrm{A}=\frac{25}{8}(\pi-\theta+2 \sin \theta)$ fully <br> or AWFW [11.9, 12] | 3.4 | M 1 |  |
|  | Obtains the correct exact area <br> ACF with $\sin \frac{\pi}{3}$ evaluated <br> ISW | 1.1 b | A 1 | $=\frac{25}{8}\left(\frac{2 \pi}{3}+\sqrt{3}\right)$ |
|  | Subtotal |  | $\mathbf{2}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 7(c) | States the angle would be the <br> same or the angle will still be $\frac{\pi}{3}$ <br> or (b)(i) stays the same <br> Condone the answer will be the <br> same | 3.5 c | E1 | The angle would be the same. |
|  | The area would be quadrupled. <br> States the area would be <br> quadrupled or area is <br> $\frac{25}{2}\left(\frac{2 \pi}{3}+\sqrt{3}\right)$ or their answer <br> in (b)(ii) multiplied by 4 <br> OE <br> Allow (b)(ii) increased by scale <br> factor of 4 | E1.5c |  |  |


|  | Question 7 Total |  | 14 |  |
| :--- | :--- | :--- | :--- | :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 8 | Obtains $5 x^{4}$ <br> PI by $\frac{1}{5}(u-2)^{-\frac{4}{5}}$ | 1.1b | B1 | $\begin{aligned} & u=x^{5}+2 \\ & \frac{d u}{d x}=5 x^{4} \end{aligned}$ |
|  | Substitutes for denominator and dx operator <br> PI by fully correct substitution <br> Condone any limits or missing integral sign or $d u$ Condone $d x$ in place $d u$ | 1.1a | M1 | $\begin{aligned} & \int \frac{x^{9}}{u^{3}} \frac{1}{5 x^{4}} d u \\ & \frac{1}{5} \int_{2}^{3} \frac{u-2}{u^{3}} d u \\ & =\frac{1}{5} \int_{2}^{3} u^{-2}-2 u^{-3} d u \end{aligned}$ |
|  | Substitutes $x^{5}=u-2$ or $x=(u-2)^{\frac{1}{5}}$ in at least one place | 1.1a | M1 | $\frac{1}{5}\left[-u^{-1}+u^{-2}\right]_{2}^{3}$ |
|  | Obtains $\frac{1}{5} \int \frac{u-2}{u^{3}} d u$ <br> Condone missing or incorrect $\frac{1}{5}$ or any limits <br> Must have $d u$ | 1.1b | A1 | $\begin{aligned} & =\frac{1}{5}\left(\left(\frac{1}{9}-\frac{1}{3}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)\right) \\ & =\frac{1}{180} \end{aligned}$ |
|  | Integrates $u^{-2}$ or $u^{-3}$ correctly | 1.1a | M1 |  |
|  | Obtains $\frac{1}{5}\left[-u^{-1}+u^{-2}\right]$ Condone any limits | 1.1b | A1 |  |
|  | Completes reasoned argument by substituting correct limits consistent with their variable to show the given result AG <br> R 1 could be scored if $d u$ is missing throughout | 2.1 | R1 |  |
|  | Question 8 Total |  | 7 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 9(a) | Obtains $y=10.2$ or $y=\frac{11}{3}$ OE <br> AWFW [3.6, 3.7] for $\frac{11}{3}$ | 3.4 | B1 | $\begin{aligned} & \text { When } t=0.2, \quad y=10.2 \\ & t=3, \quad y=\frac{11}{3} \\ & 10.2-\frac{11}{3}=6.53<7 \end{aligned}$ <br> The slide is safe. |
|  | Finds the difference between their two values of $y$ | 1.1b | M1 |  |
|  | Makes a comparison between 6.53 and 7 and states that the safety requirement is met. <br> For 6.53, accept AWFW [6.5, 6.53ं] OE | 3.2a | R1 |  |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 9(b)(i) | Obtains $1+t^{-2}$ or $1-2 t^{-2}$ OE Ignore labels | 1.1b | B1 | $\frac{\mathrm{d} x}{\mathrm{~d} t}=1+t^{-2}$$\frac{\mathrm{d} y}{\mathrm{~d} t}=1-2 t^{-2}$$\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} t} \times \frac{\mathrm{d} t}{\mathrm{~d} x}$$\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1-2 t^{-2}}{1+t^{-2}}$ |
|  | Uses chain rule to obtain $\frac{\mathrm{d} y}{\mathrm{~d} x}$ using their $\frac{\mathrm{d} x}{\mathrm{~d} t}$ and $\frac{\mathrm{d} y}{\mathrm{~d} t}$ <br> Condone missing brackets | 1.1a | M1 |  |
|  | Obtains a correct expression ISW | 1.1b | A1 |  |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 9(b)(ii) | Forms equation for appropriate derivative equal to zero. <br> Their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ or their $\frac{\mathrm{d} y}{\mathrm{~d} t}=0$ | 3.1a | M1 | $\begin{aligned} 1-2 t^{-2} & =0 \\ t^{2} & =2 \\ t & =\sqrt{2} \\ y & =\sqrt{2}+\frac{2}{\sqrt{2}} \\ & =2 \sqrt{2} \end{aligned}$ |
|  | Obtains $t=\sqrt{2}$ <br> Allow 1.4 or better for $\sqrt{2}$ <br> $t=\sqrt{2}$ must come from correct $\frac{\mathrm{d} y}{\mathrm{~d} x} \text { or } \frac{\mathrm{d} y}{\mathrm{~d} t}$ <br> PI by substituting $\sqrt{2}$ into $y$ | 1.1b | A1 |  |
|  | Substitutes their value for $t$ into $y$ and obtains a value for $y$ provided $0.2<t<3$ | 3.4 | M1 | Length of $R S=2.83$ metres |
|  | Obtains correct length with unit e.g $2 \sqrt{2}$ metres or 2.8 metres or or AWFW [2.82, 2.83] metres <br> Allow equivalent correct length in different units <br> Do not ignore subsequent incorrect rounding | 3.2a | A1 |  |
|  | Subtotal |  | 4 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 9(b)(iii) | States $\tan \theta=$ value of their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ <br> at $t=3$ <br> OE <br> PI by correct answer or 0.61 or <br> better or $55^{\circ}$ | 3.1 a | M 1 | When $t=3, \frac{\mathrm{~d} y}{\mathrm{~d} x}=0.7$ |
|  | Obtains $35^{\circ}$ <br> CAO | 3.2 a | A 1 | $\tan \theta=0.7$ <br> $\theta=35^{\circ}$ |
|  | Subtotal |  | $\mathbf{2}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 0}$ | Circles correct answer | 1.1 b | B 1 | $-\frac{6}{5}$ |
|  |  |  |  | $\mathbf{1}$ |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 11 | Ticks correct box | 2.2 a | B 1 | $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=0$ |
|  | Question 11 Total |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 12(a) | States one of the following <br> assumptions in context <br> probability of passing the <br> test is constant or 0.4 or <br> fixed or the same or does <br> not change <br> passing the test occurs <br> independently <br> only two outcomes of <br> passing or failing test | 3.5b | E1 | The probability of passing the <br> driving test is constant |
| Do not ignore incorrect <br> statements about any of the <br> above <br> Must use 'test' <br> Condone 'exam' for 'test' <br> Allow equivalent statements for <br> failing for the reference to <br> probability or independence <br> Do not allow probability being <br> independent <br> Do not allow fixed number of <br> drivers or tests | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 12(b) | Obtains correct probability <br> AWFW [0.0156, 0.016] | 1.1 b | B1 | 0.0157 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 12(c) | Obtains correct probability <br> AWFW [0.908, 0.91] | 1.1 b | B1 | 0.908 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| 12(d) | States $P(X \geq 13)$ or <br> $P(13 \leq X \leq 32)$ or $1-P(X \leq 12)$ <br> or $1-[0.46,0.462]$ <br> PI by correct answer | 1.1 a | M 1 | $\mathrm{P}(X>12)=1-\mathrm{P}(X \leq 12)$ <br> $=1-4618$ <br> $=0.538$ |
|  | Obtains correct probability <br> AWFW [0.538, 0.54] | 1.1 b | A1 |  |
|  | Subtotal |  | $\mathbf{2}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 2 ( e ) ~}$ | Obtains 12.8 <br> Do not ISW | 1.1b | B1 | 12.8 |
|  | Subtotal |  |  | $\mathbf{1}$ |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 12(f) | Uses the correct formula for <br> variance or standard deviation <br> with 32, 0.4 and 0.6 substituted <br> OE <br> PI by 7.68 or AWFW [2.77, 2.8] <br> or $\frac{8 \sqrt{3}}{5}$ <br> Ignore incorrect labels <br> Condone missing brackets | 1.1 a | M1 | Variance $=32 \times 0.4 \times 0.6=7.68$ |
|  | Obtains the correct standard <br> deviation <br> AWFW [2.77, 2.8] or $\frac{8 \sqrt{3}}{5}$ <br> Do not ignore incorrect labels <br> Do not ISW <br> Do not allow $\sqrt{7.68}$ | 1.1 b | A1 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 13(a) | Finds P(both bronze) or <br> P(both silver) <br> or calculates $1-2 \times 0.2 \times 0.8$ <br> PI by correct answer | 3.1 b | M1 | P (both bronze) $=0.2 \times 0.2=0.04$ <br>  <br> Obtains the correct probability <br> Ignore incorrect rounding after <br> correct probability seen |
| Subtotal | 1.1 b | A1 | P(both same type) $=0.68$ |  |
|  |  | $\mathbf{2}$ |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 13(b) | Finds P(at least one of the coins <br> is bronze) | 3.1 b | M1 | P (at least one bronze) <br> $=1-0.8 \times 0.8$ <br> $=0.36$ |
|  | Obtains the correct probability <br> Allow 0.11 or better for $\frac{1}{9}$ <br> Ignore incorrect rounding after <br> correct probability seen | 2.2 a | A1 | P (both bronze I at least one <br> bronze) <br> $=\frac{0.2 \times 0.2}{0.36}=\frac{1}{9}$ |
|  | Subtotal |  | $\mathbf{2}$ |  |


|  | Question 13 Total |  | 4 |  |
| :--- | :--- | :--- | :--- | :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 14(a) | Obtains 1 or 100\% | 1.2 | B1 | 1 |
|  |  | Subtotal |  | $\mathbf{1}$ |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 14(b) | States both hypotheses correctly for two-tailed test | 2.5 | B1 | $\begin{aligned} & \mathrm{H}_{0}: \mu=24500 \\ & \mathrm{H}_{1}: \mu \neq 24500 \end{aligned}$ |
|  | Obtains 26730 or 26700 | 1.1a | B1 | $\bar{X}=26730$ |
|  | States or uses correct model PI by normal with mean 24500 and variance $\frac{5200^{2}}{24}$ or 1126666.6 or standard deviation $\frac{5200}{\sqrt{24}}$ or 1061 or better OE <br> or by correct probability AWFW [0.017, 0.02] <br> or test statistic $( \pm) \frac{\text { their } 26730-24500}{5200 \div \sqrt{24}} \text { or }$ <br> or test statistic value AWRT ( $\pm$ )2.1 <br> or AWRT 22400 or AWRT 26600 | 1.1a | M1 | $\begin{aligned} & \mathrm{P}(\bar{X}>26730)=0.018 \\ & 0.018<0.025 \end{aligned}$ <br> Reject $\mathrm{H}_{0}$ <br> There is sufficient evidence to suggest that the mean daily mass of aluminium cans recycled has changed. |
|  | Obtains AWFW [0.017, 0.02] <br> or the correct value of the test statistic AWRT 2.1 <br> or acceptance region AWRT22 $400 \leq \bar{X} \leq$ AWRT26 600 <br> or critical region $\geq$ AWRT 26600 ignore reference to the lower region | 1.1b | A1 |  |



| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 4 ( c )}$ | Explains that a different sample <br> is likely to produce a different <br> sample mean <br> OE <br> e.g sample mean could be the <br> same, sample mean could be <br> different, sample mean will be <br> different | 3.5b | E1 | Sample mean could be different. |
| Must refer to the sample mean <br> or mean of the new 24 days | The result could be different so the <br> claim could be wrong. |  |  |  |
| Explains that the result in part <br> (b) could be different so the <br> claim is incorrect <br> OE <br> e.g the result might be different <br> so the claim is invalid <br> the result could be the same so <br> the claim is invalid <br> Statement on the result of the | 2.2 b | E1 |  |  |
| hypothesis test must not be <br> definite | 2 Subtotal |  |  |  |


|  | Question 14 Total |  | 10 |  |
| :--- | :--- | :--- | :--- | :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 15(a)(i) | Finds IQR <br> PI by correct expression or value for the lower or upper limit | 1.1b | B1 | $\begin{aligned} & I Q R=1570-1167=403 \\ & \\ & 1393-1.5 \times 403=788.5 \\ & 1393+1.5 \times 403=1997.5 \end{aligned}$ <br> Hence 2040 should be removed |
|  | Substitutes their IQR and obtains a value for the lower or upper limit PI by correct value for the lower or upper limit | 1.1a | M1 |  |
|  | Obtains correct lower and upper limits and selects mass 2040 | 3.2a | A1 |  |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 15(a)(ii) | States 'outlier' <br> ISW | 1.2 | B1 | Outlier |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 15(b) | Forms the equation for total probability <br> PI by $\mathrm{k}=0.08 \mathbf{0 E}$ | 3.1b | M1 | $\begin{aligned} & 0.14+0.37+0.9 \mathrm{k}+0.25+ \\ & 0.4 \mathrm{k}+1.7 \mathrm{k}=1 \end{aligned}$ |
|  | Obtains the correct value of $k$ OE | 1.1b | A1 | $\mathrm{P}(1 \leq N<5)=0.37+0.9 \times 0.08+$ |
|  | Forms a correct expression for $\mathrm{P}(1 \leq N<5)$ with or without $k$ substituted $\text { e.g } 0.37+0.9 k+0.25+0.4 k$ <br> or $0.62+1.3 \mathrm{k}$ or $1-0.14-1.7 \mathrm{k}$ <br> OE | 1.1a | M1 |  |
|  | Obtains correct probability | 1.1b | A1 |  |
|  | Subtotal |  | 4 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 5 ( c ) ( i ) ~}$ | Identifies the LDS contains cars <br> from 2 years or chooses 100 <br> cars from each year <br> or identifies the LDS contains 5 <br> makes of car or chooses 40 <br> from each make | 2.4 | M1 | Select 20 of each of the five makes |
| of car in each of the two years. |  |  |  |  |
| Condone statement 20 of each <br> car or 10 groups |  | R1 |  |  |
|  | Concludes that 20 cars selected <br> from each of the 5 makes of car <br> for both years | 2.4 | Subtotal |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 15(c)(ii) | States that the disadvantage of <br> quota sampling in LDS is that it <br> is biased or not random or not <br> proportionate. | 3.5 b | E1 | Could produce a biased sample |
|  | Subtotal |  | 1 |  |
|  | Question 15 Total |  | 11 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 16(a)(i) | Obtains correct probability <br> AWFW [0.037, 0.038] <br> lgnore incorrect rounding after <br> correct probability seen | 1.1 b | B1 | 0.0375 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 16(a)(ii) | Obtains correct probability <br> AWFW [0.246, 0.25] <br> lgnore incorrect rounding after <br> correct probability seen | 1.1b | B1 | 0.2467 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 16(a)(iii) | Obtains correct probability <br> AWFW [0.96, 0.9602] <br> lgnore incorrect rounding after <br> correct probability seen | 3.3 | B1 | 0.9601 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 16(b) | Obtains either $z$-value from inverse normal distribution AWFW [0.25, 0.26] or AWFW [0.84, 0.85] <br> Ignore signs | 3.1b | B1 | $\begin{aligned} & \mathrm{P}\left(z<\frac{5.9-\mu}{\sigma}\right)=0.6 \\ & \mathrm{P}\left(z>\frac{6.1-\mu}{\sigma}\right)=0.2 \\ & z=0.2533 \text { and } z=0.8416 \\ & \frac{5.9-\mu}{\sigma}=0.2533 \\ & \frac{6.1-\mu}{\sigma}=0.8416 \\ & \mu=5.81 \text { and } \sigma=0.34 \end{aligned}$ |
|  | Forms an equation with unknown $\mu$ and $\sigma$ using standardised result and $z$-value for 0.6 <br> Accept $z=$ AWFW $[-4,4]$ but do not allow $0, \pm 0.2, \pm 0.4, \pm 0.6$ or $\pm 0.8$ <br> Condone $\mu$ - 5.9 <br> Must use 5.9 | 3.3 | M1 |  |
|  | Forms an equation with unknown $\mu$ and $\sigma$ using standardised result and $z$-value for 0.2 <br> Accept $z=$ AWFW $[-4,4]$ but do not allow $0, \pm 0.2, \pm 0.4, \pm 0.6$ or $\pm 0.8$ <br> Condone $\mu-6.1$ <br> Must use 6.1 | 3.3 | M1 |  |
|  | Obtains both equations correctly | 1.1 b | A1 |  |
|  | Obtains correct value of $\mu$ AWFW [5.8, 5.82] ISW | 1.1b | A1 |  |
|  | Obtains correct value of $\sigma$ AWFW [0.33, 0.35] ISW | 1.1 b | A1 |  |
|  | Subtotal |  | 6 |  |
|  | Question 16 Total |  | 9 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 17 | States both hypotheses correctly for one-tailed test 0.7 OE | 2.5 | B1 | $\begin{aligned} & \mathrm{H}_{0}: p=0.7 \\ & \mathrm{H}_{1}: p>0.7 \end{aligned}$ |
|  | States or uses correct model PI by calculation of one of the probabilities below $\begin{aligned} & \mathrm{P}(X \leq 19)=[0.806,0.807] \\ & \mathrm{P}(X \leq 20)=[0.909,0.91] \\ & \mathrm{P}(X \leq 21)=[0.966,0.967] \\ & \mathrm{P}(X \geq 20)=[0.193,0.1935] \\ & \mathrm{P}(X \geq 21)=[0.09,0.091] \\ & \mathrm{P}(X \geq 22)=[0.033,0.0333] \\ & \mathrm{P}(X \geq 23)=[0.0089,0.00896] \end{aligned}$ <br> or critical value of 23 or critical region $\geq 23$ <br> condone missing or incorrect labels | 3.3 | M1 | Under null hypothesis $X \sim \mathrm{~B}(25,0.7)$ $\begin{aligned} \mathrm{P}(X \geq 21) & =1-\mathrm{P}(X \leq 20) \\ & =1-0.9095 \\ & =0.0905 \end{aligned}$ $0.0905>0.025$ <br> Do not reject $\mathrm{H}_{0}$ <br> There is insufficient evidence of an increase in the proportion of local businesses that made a profit in their first year. |
|  | Obtains [0.09, 0.091] or [0.909, 0.91] or obtains critical value 23 or critical region $\geq 23$ | 1.1b | A1 |  |
|  | Evaluates binomial model by correctly comparing their $\mathrm{P}(X \geq 21)$ or [0.09, 0.091] with 0.025 <br> or <br> evaluates binomial model by correctly comparing their $\mathrm{P}(X<21)$ with 0.975 <br> or evaluates binomial model by correctly determining if 21 is in their critical region | 3.5a | M1 |  |
|  | Infers $\mathrm{H}_{0}$ or null hypothesis not rejected <br> Condone $\mathrm{H}_{0}$ accepted <br> All figures must be correct Ignore reference to $\mathrm{H}_{1}$ | 2.2b | A1 |  |


| Concludes correctly in context <br> that there is insufficient <br> evidence of an increase in the <br> proportion of local businesses <br> that made a profit in their first <br> year. <br> To be awarded R1, marks <br> M1A1M1A1 must be scored as <br> the minimum <br> Labels of probability calculations <br> must be correct <br> Conclusion must not be definite | R.2a | R 1 |
| :--- | :--- | :--- |
| Question 17 Total |  | $\mathbf{6}$ |


|  | Question Paper Total | 100 |  |
| :--- | :--- | :--- | :--- |

