| Surname |
| :--- |
| Other Names |


| Centre <br> Number | Candidate <br> Number |
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| 0 |  |

## GCSE



MATHEMATICS - Component 1
Non-Calculator Mathematics HIGHER TIER

TUESDAY, 6 NOVEMBER 2018

- MORNING

2 hours 15 minutes

## ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination. A ruler, protractor and a pair of compasses may be required.

## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer all the questions in the spaces provided.
If you run out of space, use the continuation page at the back of the booklet, taking care to number the question(s) correctly.
Take $\pi$ as $3 \cdot 14$.

## INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the need for good English and orderly, clear presentation in your answers.

| For Examiner's use only |  |  |
| :---: | :---: | :---: |
| Question | Maximum Mark | Mark Awarded |
| 1. | 3 |  |
| 2. | 5 |  |
| 3. | 2 |  |
| 4. | 2 |  |
| 5. | 4 |  |
| 6. | 5 |  |
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| 9. | 6 |  |
| 10. | 3 |  |
| 11. | 4 |  |
| 12. | 5 |  |
| 13. | 6 |  |
| 14. | 4 |  |
| 15. | 11 |  |
| 16. | 7 |  |
| 17. | 4 |  |
| 18. | 5 |  |
| 19. | 6 |  |
| 20. | 5 |  |
| 21. | 2 |  |
| 22. | 7 |  |
| 23. | 3 |  |
| 24. | 4 |  |
| 25. | 4 |  |
| 26. | 3 |  |
| Total | 120 |  |

## Formula list

## Area and volume formulae

Where $r$ is the radius of the sphere or cone, $l$ is the slant height of a cone and $h$ is the perpendicular height of a cone:

$$
\begin{gathered}
\text { Curved surface area of a cone }=\pi r l \\
\text { Surface area of a sphere }=4 \pi r^{2} \\
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
\end{gathered}
$$

## Kinematics formulae

Where $a$ is constant acceleration, $u$ is initial velocity, $v$ is final velocity, $s$ is displacement from the position when $t=0$ and $t$ is time taken:

$$
\begin{gathered}
v=u+a t \\
s=u t+\frac{1}{2} a t^{2} \\
v^{2}=u^{2}+2 a s
\end{gathered}
$$

1. The scatter diagram shows the height and mass of 10 dogs. These dogs are all West Highland Terriers.

(a) Draw a line of best fit on the scatter diagram.
(b) Estimate the mass of a West Highland Terrier that has a height of $26 \cdot 7 \mathrm{~cm}$.
Estimated mass is .....................................
(c) Another dog has a mass of $8 \cdot 2 \mathrm{~kg}$ and a height of 35 cm .

Is this dog likely to be a West Highland Terrier?


Explain your answer.
$\qquad$
2. The graph shows the cost of delivery for goods bought at a furniture shop.

(a) Ceri buys a sofa from this shop and has it delivered.

Ceri lives 4 miles from the shop.
How much does Ceri pay for the delivery?
$\qquad$
$\qquad$
(b) (i) Tick the correct statement about the distance and cost.

They are not in proportion.


They are in inverse proportion.
They are in direct proportion.


They are in indirect proportion.

(ii) Find the gradient of the line and state what it represents in this context.
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$\qquad$
$\qquad$
3. Light travels at $3 \times 10^{5}$ kilometres per second.

The circumference of the Earth at the equator is 40000 km .
Show that, in theory, a beam of light could circle the Earth at the equator more than 7 times in 1 second.
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$\qquad$
4. A sphere has a radius of $\frac{3}{4}$ metre.

Work out the surface area of the sphere.
Give your answer in its simplest form as a multiple of $\pi$.
便
5.

$$
\mathbf{p}=\binom{-1}{5}, \quad \mathbf{q}=\binom{7}{-2} \quad \text { and } \quad \mathbf{r}=\binom{19}{4}
$$

(a) Work out the column vector $4(\mathbf{p}+\mathbf{q})$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

$$
4(\mathbf{p}+\mathbf{q})=(\quad)
$$

(b) When $2 \mathbf{p}+x \mathbf{q}=\mathbf{r}$, find the value of $x$.
6. It takes

- 2 park keepers 1 hour to weed 2 flowerbeds
- 3 park keepers 2 hours to prune 6 trees.

At 9 a.m. one morning, 5 park keepers start work as follows.

| Number of park keepers | Activity |  |
| :---: | :--- | :--- |
| 2 | Weed: | 2 flowerbeds |
| 3 | Prune: | 13 trees |

When one activity has been completed all the park keepers work on the other activity. You may assume that all the park keepers work at the same rate and are equally skilled.

How long does it take for the park keepers to complete the pruning and weeding? You must show all your working.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$ hours
7. (a)


Use the diagram to solve the simultaneous equations

$$
\begin{gather*}
y-x=3 \\
x+2 y=1 \tag{2}
\end{gather*}
$$

Give your answers correct to 1 decimal place.
(b) A theme park sells entrance tickets.

2 adult tickets and 3 child tickets would cost a total of $£ 72$.
3 adult tickets and 1 child ticket would cost a total of $£ 66$.
A family ticket costs $£ 45$ and allows entry for 2 adults and 2 children.
How much cheaper is it to buy a family ticket than it would be to buy 2 adult and 2 child tickets?
$\qquad$
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$\qquad$
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$\qquad$

It is $£$ cheaper to buy a family ticket.
Examiner
8. (a) Factorise $x^{2}+2 x-15$.
(b) Write down the solutions of the equation $x^{2}+2 x-15=0$.
9. Thirty shoppers were asked which of apples, oranges and bananas, they regularly liked to eat. Of these 30 shoppers

- 2 liked all three fruits,
- 5 liked oranges but did not like apples or bananas,
- 6 liked both oranges and bananas,
- 17 liked apples,
- 10 liked bananas.
(a) Extra information from the 30 shoppers has been shown in the Venn diagram below. Complete the Venn diagram using the information given above.

(b) One shopper is chosen at random.

Find the probability that this shopper does not like any of the three fruits.
$\qquad$
$\qquad$
(c) One shopper is chosen at random from those who like apples. Find the probability that this shopper does not like oranges.
$\qquad$
$\qquad$
10. Mike wants to find out how many fish there are in his lake.

On Monday evening,
Mike captured a random sample of 100 fish and tagged them. He then released them back into the water.

On Tuesday evening,
Mike captured a second random sample of 50 fish and counted the number that had been tagged.

He found that 10 of the fish in the second sample had been tagged.
Mike will allow fishing at his lake when there are more than 800 fish.
You may assume that the number of fish in the lake stays the same between the two samples being taken.

Should Mike allow fishing at his lake?
Show calculations to justify your decision.
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$\qquad$
$\square$ Do not allow fishing $\square$
11. Alec bought a house.

By the end of the first year, the value of Alec's house had increased by $1 \%$.
By the end of the second year, its value had decreased by $10 \%$ of its value at the end of the first year.

Use multipliers to work out the overall percentage decrease in the value of Alec's house for this two-year period.
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$\qquad$
12. (a) Write 80 as a product of prime factors using index notation.
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$\qquad$
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$\qquad$
$\qquad$
$\qquad$
(b) $8112=2^{4} \times 3 \times 13^{2}$

Find the value of the smallest square number that is a multiple of 8112.
13. (a) Jo estimates the value of $2 \cdot 75^{4}$ to be less than 81 .

Show that Jo is correct.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Find the value of each of the following.
(i) $\sqrt{32} \times \sqrt{2}$
$\qquad$
$\qquad$
$\qquad$
(ii) $5^{11} \div 5^{8}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(iii) $16^{\frac{3}{4}}$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
14. On the graph paper below, draw the region which satisfies all of these inequalities.

$$
y \geqslant-2 \quad y \geqslant 2 x-4 \quad y<4-x \quad x \geqslant-1
$$

Indicate clearly the region that is your answer.
$\qquad$


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15. Sita is investigating the popularity of two social media websites, $A$ and $B$.

She asked 60 technology students how much time they each spent using social media website A yesterday.
The table summarises the data she collected.

| Time spent, <br> $t$ (minutes) | $t \leqslant 20$ | $t \leqslant 30$ | $t \leqslant 40$ | $t \leqslant 50$ | $t \leqslant 60$ | $t \leqslant 70$ | $t \leqslant 80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> frequency | 0 | 5 | 25 | 45 | 52 | 58 | 60 |

(a) How many students spent more than 60 minutes using social media website $A$ ?
(b) Draw a cumulative frequency diagram to represent Sita's information.

(c) Use your diagram to find an estimate of the median time and the interquartile range of the times the students spent using social media website A.
$\qquad$

Median $\qquad$ minutes Interquartile range $\qquad$ minutes
(d) Sita asked the same 60 technology students how much time they each spent using social media website B yesterday. She used her data to draw the histogram shown below.

Frequency density

(i) Calculate an estimate for the difference between the number of students who use social media website B for more than 60 minutes and those who use social media website $A$ for more than 60 minutes.
$\qquad$
$\qquad$
$\qquad$
(ii) Sita uses her results to state which social media website is more popular. Is her conclusion likely to be reliable?
Explain how you decide.
$\qquad$
$\qquad$
16. (a)


Diagram not drawn to scale
$A, B, C$ and $D$ are points on a circle.
The line $P Q$ is a tangent to the circle at $B$.
$P \widehat{B} A=51^{\circ}, B \widehat{C D}=118^{\circ}$ and $A \widehat{B D}=w^{\circ}$.
Find the value of $w$.
You must show all your working.
$\qquad$
(b)


## Diagram not drawn to scale

$E, F$ and $G$ are points on a circle with centre $H$.
$E \widehat{F H}=x$ and $G \hat{F} H=y$.
Complete the proof of the following statement:
The angle at the centre is twice the angle at the circumference.

Proof:

$$
\begin{aligned}
& E \widehat{F G}=x+y \\
& F \widehat{E} H=x \text { (triangle } F E H \text { is isosceles) } \\
& F \widehat{F H E}=
\end{aligned}
$$

17. 



At a factory, clay travels through a pipe with rectangular uniform cross-section measuring 25 cm by 12 cm .

The pipe is always full of clay which travels along the pipe at a constant speed of 5 cm per second.

How long does it take a 3-litre block of clay to travel completely through the 40 cm section of pipe from $A$ to exit the pipe at $B$ ?

18.


The intensity of a light source, $I$ candela, is inversely proportional to the square of the distance, $d$ metres, from the light source.

From a distance of 2 metres, a torch has a light intensity of 7 candela.
Find the light intensity of the torch from a distance of 4 metres.
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19. (a) Simplify $\frac{3}{\sqrt{3}}$.
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(b) Write $\sqrt{63}+\sqrt{175}$ in the form $k \sqrt{7}$, where $k$ is an integer.
$\qquad$
$\qquad$
$\qquad$
(c) Expand and simplify $(4-2 \sqrt{5})(2+\sqrt{5})$.
$\qquad$
$\qquad$
$\qquad$
(d) Complete the following identity.

$$
x^{2}-3 \equiv(x-\ldots \ldots . .)(x+\ldots \ldots . .)
$$

20. (a) Write $\frac{5}{12}$ as a recurring decimal.
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$\qquad$
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$\qquad$
(b) (i) Write $1 . \ddot{2} \dot{7}$ as an improper fraction.
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$\qquad$
(ii) Hence calculate $(1 \cdot \dot{2} \dot{7})^{2}$, giving your answer as a mixed number in its simplest form.
21. (a) Janesh wants to draw the graph of $y=f(x)$ and the graph of $y=-f(x)$.

Janesh says,
'To draw $y=-f(x)$ I must reflect the graph of $y=f(x)$ in the $y$-axis because the $y$-coordinates all change sign.'

Is Janesh correct?


Explain how you decide.
$\qquad$
$\qquad$
(b) Yasmin wants to draw the graph of $y=f(x)$ and the graph of $y=-f(x)$
when $f(x)=\frac{1}{x}$.
Yasmin draws both of her graphs on this diagram.


Make one criticism of Yasmin's diagram.
$\qquad$
$\qquad$
22. The functions $g(x)$ and $h(x)$ are defined for all real $x$ by

$$
\begin{aligned}
& g(x)=4-3 x, \\
& h(x)=x^{2}+5 .
\end{aligned}
$$

(a) Solve $g^{-1}(x)=x+2$.
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(b) Find and simplify an expression for $g g(x)$.
$\qquad$
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$\qquad$
(c) Explain why $g h(-1)=g h(1)$.
$\qquad$
$\qquad$
23. (a) On the axes below, sketch the graph of $y=\sin x$ for $0^{\circ} \leqslant x \leqslant 540^{\circ}$.

(b) Write down the number of solutions of the equation $\sin x=0.25$ for $0^{\circ} \leqslant x \leqslant 540^{\circ}$.
24. Anna has 7 photographs.
(a) Find the number of different ways that Anna can arrange her 7 photographs in a line on a table.
(b) Find the number of different ways that Anna can randomly select 3 of her 7 photographs and arrange them in a line on a shelf.
25. A solution of the equation $x^{2}-0 \cdot 5=0$ lies between 0 and 1 . Jane uses the method of decimal search to find the solution correct to one decimal place.
(a) Complete Jane's table.
[1]

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x^{2}-0.5$ | -0.5 |  |  | -0.41 | -0.34 | -0.25 | -0.14 | -0.01 | 0.14 | 0.31 | 0.5 |

(b) Explain why Jane's solution lies between 0.7 and 0.8 .
$\qquad$
$\qquad$
$\qquad$
(c) You are given that $74^{2}=5476$.

Show that Jane's solution is 0.7 correct to one decimal place.
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$\qquad$
$\qquad$
$\qquad$
26. In this question all dimensions are in centimetres.

The diagram shows a sketch of two circles, both with centre 0 .


Diagram not drawn to scale

The equation of the inner circle is $x^{2}+y^{2}=4$.
The equation of the outer circle is $x^{2}+y^{2}=144$.
The points $A$ and $B$ lie on the inner circle and outer circle respectively.
$O A B$ is a straight line.
(a) Find the length of the line $A B$.

You must show all your working.
(b) Write down the equation of the tangent to the outer circle at the point $B$.
$\qquad$
$\qquad$

For

