Oxford Cambridge and RSA

## Monday 23 May 2022 - Afternoon

## AS Level Further Mathematics A

## Y532/01 Statistics

Time allowed: 1 hour 15 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. When a numerical value is needed use $g=9.8$ unless a different value is specified in the question.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.


## INFORMATION

- The total mark for this paper is $\mathbf{6 0}$.
- The marks for each question are shown in brackets [ ].
- This document has 8 pages.


## ADVICE

- Read each question carefully before you start your answer.

Answer all the questions.
1 A geography student chose a certain point in a stream and took measurements of the speed of flow, $v \mathrm{~ms}^{-1}$, of water at various depths, $d \mathrm{~m}$, below the surface at that point. The results are shown in the table.

| $d$ | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v$ | 0.8 | 0.5 | 0.7 | 1.2 | 1.1 | 1.3 | 1.6 | 1.4 | 0.4 |

$n=9 \quad \sum d=2.7 \quad \sum v=9.0 \quad \sum d^{2}=0.96 \quad \sum v^{2}=10.4 \quad \sum d v=2.85$
(a) (i) Explain why $d$ is an example of an independent, controlled variable.
(ii) Use two relevant terms to describe the variable $v$ in a similar way.

A statistician believes that the point $(0.5,0.4)$ may be an anomaly.
(b) Calculate the equation of the least squares regression line of $v$ on $d$ for all the points in the table apart from $(0.5,0.4)$.
(c) Use the equation of the line found in part (b) to estimate the value of $v$ when $d=0.5$.
(d) Use your answer to part (c) to comment on the statistician's belief.
(e) Use the diagram in the Printed Answer Booklet (which does not illustrate the data in this question) to explain what is meant by "least squares regression line".

2 Eight runners took part in two races. The positions in which the runners finished in the two races are shown in the table.

| Runner | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| First race | 3 | 1 | 5 | 6 | 2 | 8 | 7 | 4 |
| Second race | 4 | 3 | 8 | 7 | 2 | 5 | 6 | 1 |

Test at the $5 \%$ significance level whether those runners who do better in one race tend to do better in the other.

3 A discrete random variable $X$ has the following probability distribution.

| $x$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $p$ | 0.31 | 0.3 | $p^{2}$ |

(a) Determine the value of $p$.
(b) It is given that $\mathrm{E}(a X+b)=\operatorname{Var}(a X+b)=23.19$, where $a$ and $b$ are positive constants. Determine the value of $a$ and the value of $b$.

4 A school pupil keeps a note of whether her journeys to school and from school are delayed. The results for a random sample of journeys are shown in the table.

|  | Direction of journey |  |
| :---: | :---: | :---: |
|  | To school | From school |
| Delayed | 64 | 56 |
| Not delayed | 74 | 106 |

Test at the $10 \%$ significance level whether there is association between delays and the direction of the journey.

5 The manager of an emergency response hotline believes that calls are made to the hotline independently and at constant average rate throughout the day. From a small random sample of the population, the manager finds that the mean number of calls made in a 1 -hour period is 14.4.

Let $R$ denote the number of calls made in a randomly chosen 1-hour period.
(a) Using evidence from the small sample, state a suitable distribution with which to model $R$. You should give the value(s) of any parameter(s).
(b) In this part of the question, use the distribution and value(s) of the parameter(s) from your answer to part (a).
(i) Find $\mathrm{P}(R>20)$.
(ii) Given that $\mathrm{P}(R=r)>\mathrm{P}(R=r+1)$, show algebraically that $r>13.4$.
(iii) Hence write down the mode of the distribution.

The manager also finds, from records over many years, that the modal value of $R$ is 10 .
(c) Use this result to comment on the validity of the distribution used in part (b).
(d) Assume now that the type of distribution used in part (b) is valid. Find the range(s) of values of the parameter(s) of this distribution that would correspond to the modal value of $R$ being 10 .

6 A teacher has 10 different mathematics books. Of these books, 5 are on Algebra, 3 are on Calculus and 2 are on Trigonometry.

The teacher chooses 5 of the books at random.
(a) Find the probability that 3 of the books are on Algebra.

The teacher now arranges all 10 books in random order on a shelf.
(b) Find the probability that the Calculus books are next to each other and the Trigonometry books are next to each other.

## In this question you must show detailed reasoning.

(c) Find the probability that 2 of the Calculus books are next to each other but the third Calculus book is separated from the other 2 by at least 1 other book.

7 Each of three students, X, Y and Z, was given an identical pack of 48 cards, of which 12 cards were red and 36 were blue. They were each told to carry out a different experiment, as follows:

Student X: Choose a card from the pack, at random, 20 times altogether, with replacement. Record how many times you obtain a red card.

Student Y: Choose a card from the pack, at random, 20 times altogether, without replacement. Record how many times you obtain a red card.

Student Z: Choose single cards from the pack at random, with replacement, until you obtain the first red card. Record how many cards you have chosen, including the first red card.
(a) Find the probability that student Z has to choose more than 8 cards in order to obtain the first red card.

Each student carries out their experiment 30 times.
The frequencies of the results recorded by each student are shown in the following table, but not necessarily with the rows in the order $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ :

|  | Number recorded | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $\geqslant 9$ | Observed Mean | Observed <br> Variance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Observed <br> Frequencies | Student 1 | 0 | 0 | 1 | 3 | 7 | 8 | 6 | 4 | 1 | 0 | 5.03 | 1.97 |
|  | Student 2 | 0 | 8 | 5 | 4 | 2 | 3 | 3 | 2 | 1 | 2 | 4.03 | 11.57 |
|  | Student 3 | 0 | 1 | 2 | 5 | 4 | 6 | 5 | 3 | 4 | 0 | 4.97 | 3.70 |

(b) In this question you must show detailed reasoning.

Two other students make the following statements about the results. For each of the statements, explain whether you agree with the statement. Do not carry out any hypothesis tests, but in each case you should give two justifications for your answer.
(i) "The second row is a good match with the expected results for student Z."
(ii) "The third row is definitely student X 's results."

## END OF QUESTION PAPER

## BLANK PAGE

BLANK PAGE

Oxford Cambridge and RS

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.
If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.
OCR is part of Cambridge University Press \& Assessment, which is itself a department of the University of Cambridge

