## GCE

# Further Mathematics A 

Y532/01: Statistics

AS Level

Mark Scheme for June 2022

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

## Text Instructions

## 1. Annotations and abbreviations

| Annotation in RM assessor | Meaning |
| :--- | :--- |
| $\checkmark$ and $\mathbf{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0,1 |
| SC | Special case |
| $\wedge$ | Omission sign |
| MR | Misread |
| BP | Blank Page |
| Seen |  |
| Highlighting |  |
|  | Meaning |
| Other abbreviations <br> mark scheme |  |
| dep* | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This question included the instruction: In this question you must show detailed reasoning. |

## 2. Subject-specific Marking Instructions for A Level Mathematics A

Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ${ }^{\wedge}$ ) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.
Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')

OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question
Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).
If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.
c The following types of marks are available.
M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.
Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the $M$ marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only - differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
$\mathrm{f} \quad$ We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to $\mathbf{3}$ s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range
NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads " 2 s.f".
Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80 , 9.81 or 10 for $g$ should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

| Question |  |  | Solution | Marks | AO | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | (a) | (i) | The values of $d$ do not depend on any other variable in the experiment and they may be (or are) selected by the experimenter. | $\begin{aligned} & \text { B1 } \\ & {[1]} \end{aligned}$ | 1.2 | Either of these points. | Not just "equal increments". Need to use the nature of $d$, not its values |
|  | (a) | (ii) | $v$ is a dependent, response variable | $\begin{aligned} & \hline \mathbf{B 1} \\ & {[1]} \\ & \hline \end{aligned}$ | 2.5 | Both needed. Not "uncontrolled" |  |
|  | (b) |  | $v=0.329+2.71 d$ | B2 <br> [2] | $\begin{aligned} & \hline 1.1 \\ & 1.1 \end{aligned}$ | One error, e.g. wrong letters: B1 In [0.328, 0.329] and [2.71, 2.72] | SR all 9 points: $v=d+0.7$ B1. SR: if B0, give M1 for correct subst into $b$ seen |
|  | (c) |  | 1.69 (1.6857) | B1 <br> [1] | 1.1 | In range [1.68, 1.69] | No FT |
|  | (d) |  | Quite a big difference (between 0.4 and their 1.69) so statistician is likely to be right/it may well be an anomaly | $\begin{array}{r} \hline \text { B1ft } \\ {[1]} \end{array}$ | 2.4 | Comment on statistician's statement, not over-certain, ft on their 1.69. | Ignore any comment about extrapolation (irrelevant here) or outliers. B0 if clearly wrong comparison used |
|  | (e) |  | Draw regression line and 2 or more verticals State that equation minimises sum of squares of vertical distances ("residuals") | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \end{aligned}$ [2] | $\begin{aligned} & \hline 1.1 \\ & 2.4 \end{aligned}$ | Needs "minimises", "sum", and "squares" or "residuals". Allow "minimise $\Sigma d^{2}$ " if clear from diagram | Allow "average" or "total" instead of "sum". Distances shown not vertical: B0 Not just "minimise sum of squares" |
| 2 |  |  | $\mathrm{H}_{0}: \rho_{s}=0, \mathrm{H}_{1}: \rho_{s}>0$, where $\rho_{s}$ is the population value of Spearman's rank correlation coefficient $\begin{aligned} & \Sigma d^{2}=1+4+9+1+0+9+1+9=34 \\ & r_{s}=1-6 \Sigma d^{2} /(8 \times 63) \text { or } 3.125 / \sqrt{ } 5.25^{2} \\ & \quad=0.595(238) \text { or } \frac{25}{42} \\ & <0.6429 \end{aligned}$ <br> Do not reject $\mathrm{H}_{0}$. Insufficient evidence that those who do well in one race tend to do well in the other. | A1 <br> A1 <br> M1ft <br> A1ft <br> [7] | $\begin{gathered} \hline 1.1 \\ 2.5 \\ \\ \\ 1.1 \\ 1.1 \\ 1.1 \\ 1.1 \\ 2.2 b \end{gathered}$ | One error, e.g. $\rho_{s}$ not defined in terms of population: B1. <br> Allow $\rho$ or $r_{s}$ rather than $\rho_{s}$ <br> Use $\Sigma d^{2}$ and correct formula <br> Exact or awrt 0.595 <br> Not < 0.6215 (pmcc) <br> Consistent, ft on their TS <br> Contextualised, not over-assertive. <br> Not "sig evidence that $\mathrm{H}_{0}$ true" | SR: $\mathrm{H}_{0}$ : No association between orders in the two races, $\mathrm{H}_{1}$ : positive association: <br> B1 max <br> Allow use of pmcc formula here <br> Needs recognisable attempt at $r_{s}$ and -1 $\leq r_{s} \leq 1$. Allow from $r_{s}<0.7381$ <br> (2-tailed test) or from < 0.6215 (pmcc). |
| 3 | (a) |  | $\begin{aligned} & p+p^{2}=0.39 \\ & p \neq-1.3 \text { as } 0<p<1 \\ & p=0.3 \end{aligned}$ | M1 <br> B1 <br> A1 <br> [3] | $\begin{aligned} & 1.1 \\ & 2.3 \\ & \\ & 1.1 \end{aligned}$ | Use $\Sigma p=1$ <br> Explicitly reject -1.3 , with reason, allow $0 \leq p \leq 1$ etc Final answer 0.3 oe and nothing else | E.g. "must be positive". Not just "reject" or "invalid" Can get this even if B1 not gained |


|  | (b) |  | $\begin{aligned} & \mathrm{E}(X)=\Sigma x P(X=x)=2.18 \\ & \operatorname{Var}(X)=5.68-2.18^{2}(=0.9276) \\ & 0.9276 a^{2}=23.19 \\ & \Rightarrow a=5 \\ & 2.18 a+b=23.19 \\ & \Rightarrow b=12.29 \end{aligned}$ | M1 M1 M1 A1 M1 A1 $[6]$ | $\begin{gathered} \hline 1.1 \\ 1.1 \\ 1.1 \\ 2.2 \mathrm{a} \\ 2.1 \\ 1.1 \end{gathered}$ | $\begin{aligned} & \text { Use } \Sigma x P(X=x) \text { to get } \mathrm{E}(X) \\ & \text { Use } \Sigma x^{2} \mathrm{P}(X=x)-[\mathrm{E}(X)]^{2} \\ & a^{2} \times \text { their } \operatorname{Var}(X)=23.19\left(\operatorname{not} a^{2} \mathrm{E}(X)\right) \\ & a=5 \text { or awrt } 5.00[\text { no ft }] \\ & \text { Use their } a \times \text { their } 2.18+b=23.19 \\ & 12.3 \text { or awrt } 12.29 \end{aligned}$ | $\operatorname{Var}(X)=3.5$ from $5.68-2.18:$ M0 <br> $2.18 a^{2}=23.19$ : usually M1, 000, M1A0 <br> Ignore negative $a$ and any value of $b$ obtained from it, e.g. $(-5,34.09)$ <br> CWO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Question |  |  | Solution | Marks | AO | Guidance |  |
| 4 |  |  | $\mathrm{H}_{0}$ : no association between delay and direction of journey. $\mathrm{H}_{1}$ : there is association $\begin{aligned} & E_{f}: \quad 55.2,64.8,82.8,97.2 \\ & X^{2}=1.248+1.063+0.832+0.709 \\ & \quad=3.85 \\ & >2.706 \end{aligned}$ <br> Reject $\mathrm{H}_{0}$. Significant evidence of association between delays and direction of journey. | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \\ & \text { M1 } \\ & \\ & \text { A1 } \\ & \text { A1 } \\ & \text { M1ft } \\ & \text { A1ft } \\ & {[7]} \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 1.1 \\ \\ 1.1 \\ 1.1 \\ \\ 1.1 \\ 1.1 \\ 1.1 \\ 2.2 \mathrm{~b} \end{gathered}$ | Or: delay and direction of journey are independent, etc. Context needed At least 2 correct to 2 SF M0 if formula stated incorrectly Allow from no Yates (4.33). No Yates: loses this mark only Explicit comparison with 2.706 Consistent, ft on their TS Context needed, not over-assertive | Ignore any statements involving parameters (e.g. $\rho$ ) <br> Can be implied by correct $X^{2}$ <br> E.g. 5.045 from no modulus: M0 $1.403+1.195+0.935+0.797=4.33:$ <br> M1A0 <br> M1 and A1 need comparison with 2.7(06) or 3.84(1) |
| 5 | (a) |  | Po(14.4) | $\begin{aligned} & \hline \text { B1 } \\ & {[1]} \\ & \hline \end{aligned}$ | 3.3 | Stated or implied | Allow just "Poisson" if 14.4 used in (b) |
|  | (b) | (i) | $\begin{aligned} & 1-\mathrm{P}(\leq 20) \\ & =0.0604(0.060351 \ldots) \end{aligned}$ | $\begin{gathered} \hline \text { M1 } \\ \text { A1 } \\ {[2]} \\ \hline \end{gathered}$ | $\begin{aligned} & 1.1 \\ & 1.1 \end{aligned}$ | $\begin{aligned} & \hline 0.094(0) \text { or } 0.0372: \text { M1A0 } \\ & \text { In range [0.0603, } 0.0604] \end{aligned}$ | 0.06 or 0.060: M1A0. <br> NB: be aware that 0.94 is M0. |
|  | (b) | (ii) | $\begin{align*} & \mathrm{e}^{-14.4} \frac{14.4^{r}}{r!}>\mathrm{e}^{-14.4} \frac{14.4^{r+1}}{(r+1)!} \\ & \Rightarrow r+1>14.4 \text { so } r>13.4 \tag{AG} \end{align*}$ | M1 <br> A1 <br> [2] | $\begin{aligned} & \hline 1.2 \\ & 2.1 \end{aligned}$ | Correct inequality <br> Correctly derive AG, allow = throughout until conclusion | One further intermediate line needed, condone omissions of brackets |
|  | (b) | (iii) | 14 | B1 <br> [1] | 3.4 | 14 only |  |
|  | (c) |  | 14 (or 14.4) is some way away from 10 (and 10 is based on a large sample) so a Poisson distribution (or $\lambda=14.4$ ) is unlikely. | $\begin{array}{r} \text { B1ft } \\ {[1]} \end{array}$ | 3.5a | OE. FT on their mode. Not too definite - not just "invalid". $\mathrm{P}(R=10)=0.0589$ irrelevant | Allow "the population parameter may not be close to 14 ", but not "the mode may not be $10^{\prime \prime}$. Not $10 \neq 14$. B0 if other wrong or irrelevant statements seen |
|  | (d) |  | $\begin{aligned} & \mathrm{P}(10)>\mathrm{P}(11) \text { (and }>\mathrm{P}(9)) \Rightarrow \lambda<11 \\ & 10<\lambda<11 \end{aligned}$ | $\begin{gathered} \hline \text { M1 } \\ \text { A1 } \\ {[2]} \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 3.4 \\ & 1.1 \end{aligned}$ | Either $\lambda<11$ or $\lambda>10($ or $\lambda \geq 10)$ This or exact equivalent only, but allow $10 \leq \lambda<11$, allow $\mu$ | Allow M1A1 if both $\lambda<11$ and $\lambda>10$ seen but wrongly combined at end |


| Question |  | Solution | Marks | AO | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | (a) | $\begin{aligned} & { }^{5} C_{3} \times{ }^{5} C_{2} \div{ }^{10} C_{5} \\ & =\frac{25}{63} \text { or } 0.397(0.396825 \ldots) \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ {[3]} \end{gathered}$ | $\begin{aligned} & 1.1 \\ & 1.1 \\ & 1.1 \end{aligned}$ | Two correct terms: M1A0 Completely correct expression Answer, exact or awrt 0.397 | $\begin{aligned} & \text { Numerator (= 100) can be } \\ & { }^{5} C_{3} \times{ }^{3} C_{2}+{ }^{5} C_{3} \times{ }^{3} C_{2} \times{ }^{2} C_{1}+{ }^{5} C_{3} \times{ }^{2} C_{2} \end{aligned}$ |
|  | OR | $\begin{aligned} & \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} \quad\left[=\frac{5}{126}\right] \\ & \times{ }^{5} C_{3} \quad[=10] \\ & =\frac{25}{63} \text { or } 0.397(0.396825 \ldots) \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  | Answer, exact or awrt 0.397 |  |
|  | (b) | $\begin{aligned} & 7!\times 3!\times 2!\div 10! \\ & =\frac{1}{60} \text { or } 0.0167(0.01666 \ldots) \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ \quad[3] \end{gathered}$ | $\begin{aligned} & 1.1 \\ & 1.1 \\ & 1.1 \end{aligned}$ | Two correct terms: M1A0 <br> Correct expression <br> Answer, exact or awrt 0.0167 | E.g. $(6 \times 3!) \times(6 \times 2!) \div 10!$ M1A0 |
|  | OR | $\begin{aligned} & \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} \times \frac{2}{7} \times \frac{1}{6} \quad\left[=\frac{1}{2520}\right] \\ & \times{ }^{7} P_{2} \quad[=42] \\ & =\frac{1}{60} \text { or } 0.0167(0.01666 \ldots) \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  | Five equivalent probs multiplied <br> Needs at least 3 probs <br> Answer, exact or awrt 0.0167 |  |
|  | (c) | $\begin{array}{\|l} \hline \text { DR } \\ \|x\| x\|x\| x\|x\| x\|x\| \quad \text { (x = non-Calculus) } \end{array}$ <br> Pair can go in 8 places, single in 7 , and $3 \times 2$ ways of choosing the pair in order <br> Total $7!\times 8 \times 7 \times(3 \times 2) \div 10$ ! $=\frac{7}{15}$ or $0.466 \ldots$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ {[4]} \end{gathered}$ | $\begin{gathered} 3.3 \\ 3.1 \mathrm{~b} \\ 2.1 \\ 2.2 \mathrm{a} \end{gathered}$ | (Decide first whether candidate is using direct calc or the complement) Use of barriers clearly indicated Correct application of strategy $\times 7!\div 10!$ <br> Final answer, exact or awrt 0.467 <br> SR: If M0, 7 ! or $8!\times$ integers $\div 10$ !: B1 | Equivalent: <br> (same barrier idea M1) <br> Pair can go in 8 places, single in 7 <br> AAAAATT arranged in ${ }^{7} \mathrm{C}_{2}$ ways M1 <br> This gives $21 \times 56=1176$ ways A1 <br> Total arrangements $10!/(5!3!2!)=2520$ <br> Therefore answer is $\frac{1176}{2520}=\frac{7}{15} \quad$ A1 |
|  | OR | $P(\text { all } 3 \text { Calculus together })=\frac{8!3!}{10!}\left[=\frac{1}{15}\right]$ <br> P (no Calculus book is next to another) $\begin{align*} & =\frac{\left(8^{\prime} 7^{\prime} 6\right)^{\prime} 7!}{10!} \quad\left[=\frac{7}{15}\right] \\ & 1-\mathrm{P}(\text { all } 3 \text { together })-\mathrm{P}(\text { all separate }) \\ & =1-\frac{1}{15}-\frac{7}{15}=\frac{7}{15} \text { or } 0.466 \ldots \tag{A1} \end{align*}$ | B1 <br> B1 <br> M1 <br> A1 | OR | Two calculus books at end: $\left[\frac{3}{10} \times \frac{2}{9} \times \frac{7}{8}\right] \times 2$ <br> M1 <br> Two calcs books together, not at end: $\begin{array}{rlrl} {\left[\frac{7}{10}\right.} & \left.\times \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7}\right] \times 7 & & \text { M1 } \\ \text { Total } & =\frac{7}{60}+\frac{7}{20} & & \text { A1 } \\ & =\frac{7}{15} \text { or } 0.466 \ldots & & \text { A1 } \\ \hline \end{array}$ | Or other valid strategy. <br> (A1 for both expressions correct) |


| Question |  |  | Solution | Marks | AO | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | (a) |  | Geo(0.25) $\begin{aligned} \mathrm{P}(>8)= & 0.75^{8} \\ & =0.1(00113) \text { or } \frac{6561}{65536} \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { M1 } \\ \text { A1 } \\ {[3]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline 3.1 \mathrm{~b} \\ 1.1 \\ 1.1 \end{gathered}$ | Stated or implied <br> Or $1-0.25\left(1+0.75+\ldots+0.75^{7}\right)$ <br> Exact or awrt 0.100 , allow " 0.1 " | NB: $12 / 36=1 / 3$ may be MR. See below $0.75^{9}=0.075$ or equivalent addition: M1M1A0 |
|  | (b) | (i) | DR <br> Either: Geometric stated or implied <br> $\mathrm{E}(Z)=4$ which is close to 4.03 <br> $\operatorname{Var}(Z)=12$ which is close to 11.57 <br> So second row is a good match for expected results for student $Z$ | $\begin{gathered} \text { M1 } \\ \\ \mathbf{A 1} \\ \mathbf{A 1} \\ \mathbf{A 1} \\ {[4]} \end{gathered}$ | $\begin{gathered} 3.1 \mathrm{~b} \\ \\ 1.1 \\ 2.4 \\ 3.2 \mathrm{a} \end{gathered}$ | Identify geometric for Z <br> One correct calculation interpreted Another correct calc interpreted Needs two pieces of correct evidence, and a conclusion | SR if M0: <br> B1 for one clear correct comparison, B1 for a second one; all correct B1; conclusion "agree/good match" B1 <br> Accept "The statement is true" |
|  |  |  | Or: Geometric stated or implied Frequencies are (generally) decreasing $4.03 \approx 4$ <br> so second row is a good match for expected results for student Z | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  | Needs adequate correct evidence conclusion needed! | Or equivalent with variance |
|  | (b) | (ii) | DR $X \sim \mathrm{~B}(20,0.25)$ <br> $\mathrm{E}(X)=5$ which is close to 4.97 <br> $\operatorname{Var}(X)=3.75$ which is close to 3.7 <br> Therefore quite strong indication that the third row is X , but not definite | M1 <br> A1 <br> A1 <br> [3] | 3.1b <br> 1.1 <br> 3.2b | Stated or implied <br> (B(30, 0.25): M1A0A0) <br> Two correct comparisons Consistent conclusion, indicate uncertainty, must deny "definitely". Needs both previous marks | SR if M0: <br> B1 for one clear correct comparison, B1 a second one; all correct and conclusion "very likely but not definite" B1 [e.g. sampling without replacement reduces probabilities of higher results] Needn't actually say "very likely" as denying "definitely" is enough! |

NB: MR $12 / 36=1 / 3($ instead of $12 / 48=1 / 4)$;
(a) $\quad 256 / 6561=0.0390$ M1M1A0(MR)
(b)(i) 3, 6 and not a very good match: full marks
(b)(ii) $\mathrm{B}(20,1 / 3), \mathrm{E}(X)=6^{2} / 3, \operatorname{Var}(X)=40 / 9$, not very close etc: full marks

## APPENDIX

Exemplar responses for various questions

## Q1(a)

|  | Response | Mark |
| :---: | :--- | :---: |
| $\alpha$ | The depth of the river cannot be changed [or similar wrong statement] | B 0 |
| $\beta$ | Fixed intervals between values [no mark for just this] | B 0 |
| $\gamma$ | It increases at a constant average rate (0.05 each time) | B 0 |
| $\delta$ | It doesn't change if $v$ changes | $\mathrm{B} 0 ?$ |
| $\varepsilon$ | Because specific values for the depth were chosen [anything along these lines] | B 1 |
| $\zeta$ | It is the thing that is being changed (in increments of 0.05) | B 1 |
| $\eta$ | Depth of water does not rely on any external factors | B 1 |
| $\theta$ | It is the variable they are altering | B 1 |
| $\mathbf{l}$ | The depth doesn't depend on $v \quad$ [focus on independent rather than controlled] | B 1 |
| $\kappa$ | Depth doesn't depend on $v ;$ depth is controlled as values differ by 0.05 [give credit for first point] | B 1 |

## Q1(d)

| $\alpha$ | The statistician's belief is correct [not enough, and too definite] | B0 |
| :---: | :--- | :---: |
| $\beta$ | It seems that the measurement is anomalous [not enough] | B0 |
| $\gamma$ | The statistician is probably right as $1.69 \neq 0.4$ [no reason for them to be exactly equal] | B0 |
| $\delta$ | I agree as 1.69 is very different from $0.4 \quad$ [enough] | B1 |
| $\varepsilon$ | 1.69 is much greater than 0.4 so it is probably an anomaly [good] | B1 |

Q2 Hypotheses (same principles apply also in Q4, although there is only 1 mark there and the hypotheses are easier to state)

| $\alpha$ | $\mathrm{H}_{0}:$ no association between variables, $\mathrm{H}_{1}:$ positive association between variables [no context] | B 0 |
| :--- | :--- | :--- | :--- |
| $\beta$ | $\mathrm{H}_{0}:$ no association between runners' positions, $\mathrm{H}_{1}:$ association between runners' positions [not one-tailed: needs "positive"] | B 0 |
| $\gamma$ | $\mathrm{H}_{0}:$ no evidence of association between runners' positions, $\mathrm{H}_{1}:$ evidence of positive association between runners' positions <br> ["evidence" does not belong in hypotheses] | B 0 |
| $\delta$ | $\mathrm{H}_{0}: \rho_{s}=0, \mathrm{H}_{1}: \rho_{s} \geq 1 \quad$ [two errors: $H_{l}$ wrong, no interpretation of $\rho$ ] | B 0 |
| $\varepsilon$ | $\mathrm{H}_{0}: \rho_{s}=0, \mathrm{H}_{1}: \rho_{s} \neq 0 \quad$ [two errors: $H_{l}$ wrong, no interpretation of $\rho$ ] | B 0 |
| $\zeta$ | $\mathrm{H}_{0}: \rho=0, \mathrm{H}_{1}: \rho>0 \quad$ [one error: no interpretation of $\rho$, but allow use of $\rho$ instead of $\rho_{s,}$. Ditto for $r_{s}$, or $r$ ] | B 1 |
| $\eta$ | $\mathrm{H}_{0}: \rho=0$, there is no association between runners' places; $\mathrm{H}_{1}: \rho>0$, there is positive association [two different answers, either of |  |
| which would score 1 but neither gains 2 and we don't give $1+1$ here] | B 1 |  |

Conclusions (In general, allow "Accept $H_{0}$ " as a synonym for "Do not reject $H_{0}$ ", etc.)

| a | Wrong but validly obtained TS leading consistently to "Reject $\mathrm{H}_{0}$. There is significant evidence that runners who do better in one race tend to do better in the other" [standard FT] | M1A1 |
| :---: | :---: | :---: |
| $\beta$ | Do not reject $\mathrm{H}_{0}$. Runners who do better in the first race do not tend to do better in the second [too definite] | M1A0 |
| $\gamma$ | Do not reject $\mathrm{H}_{0}$. There is insufficient evidence of association [no context] | M1A0 |
| $\delta$ | Insufficient evidence to reject $\mathrm{H}_{0}$. Runners who do better in the first race do not tend to do better in the second ["Evidence" used, albeit in wrong sentence, but BOD] | M1A1 |
| $\varepsilon$ | Do not reject $\mathrm{H}_{0}$. There is evidence that runners who do well in the first race do not do any better than others in the second race. [Nonrejection doesn't give positive evidence that $H_{0}$ is correct. When $H_{0}$ is not rejected, candidates should aim for a "double negative"] | M1A0 |
| $\zeta$ | (from correct TS and CV) Reject $\mathrm{H}_{0}$. (+ anything) [inconsistent] | M0A0 |
| $\eta$ | If $H_{0}, H_{l}$ the wrong way round, but calculations right: $\quad$ Do not reject $\mathrm{H}_{0}$. (+ anything) | M1A0 |

Q4 Conclusions (Similar general principles apply here as in Q2)

| $\alpha$ | (from miscalculated comparison) Reject $\mathrm{H}_{0}$. There is evidence of an association between direction of journey and timing |  | M1A0 |
| :---: | :---: | :---: | :---: |
| $\beta$ | Significant evidence to reject $\mathrm{H}_{0}$. There is association between direction of journey and delays | [as in Q2 ex $\delta$ ] | M1A1 |
| $\gamma$ | Reject $\mathrm{H}_{0}$. There is association between direction of journey and delays [too definite] | [as in Q2 ex $\beta$ ] | M1A0 |
| $\delta$ | Reject $\mathrm{H}_{0}$. There is association between them [no context] |  | M1A0 |
| $\varepsilon$ | (from correct comparison) Do not reject $\mathrm{H}_{0}$. (+ anything) |  | M0A0 |
| $\zeta$ | If $H_{0}, H_{l}$ the wrong way round but calculations right: $\mathrm{Reject} \mathrm{H}_{0}$. (+ anything) |  | M1A0 |
| $\eta$ | Reject $\mathrm{H}_{0}$. There is significant evidence of association between direction of journey and delays | [best answer] | M1A1 |

## Q5(c)

| $\alpha$ | Not valid as it is based on a small sample |  | B0 |
| :---: | :---: | :---: | :---: |
| $\beta$ | 14 is not equal to 10 so it is unlikely to be valid | [don't expect them to be exactly equal] | B0 |
| $\gamma$ | 14 is not close to 10 so it may not be valid | [dubious about this as "may not be" is always true, but BOD] | B1 |
| $\delta$ | Not too far from part (iii), so valid enough but not | mpletely reliable [allow this judgement] | B1 |
| $\varepsilon$ | It is not very valid as the values should be closer | [BOD. I don't want to penalise the idea of something being "partly valid"] | B1 |
| $\zeta$ | Probably valid as 14 is not far from 10 |  | B1 |
| $\eta$ | Less valid as 14 is not close to 10 |  | B1 |

## Q7(b)(i)

| $\alpha$ | Second row is a good match. Average number of cards chosen before red is 4, as only $1 / 4$ of the pack is red and the student continuously <br> picks up cards (especially with a 10\% chance of picking up 8 cards before getting a red) it is likely that getting more than 9 reds twice is <br> possible since there is a 25\% chance of all the reds being close together <br> [enough to imply Geo( $1 / 4)$ (even though wrong\} but no more] | M1A0 <br> A1A0 |
| :---: | :--- | :--- | :--- |
| $\beta$ | I agree. The expected number for Geo( $1 / 4)$ is 4 which is very similar to the mean (4.03). The variance is much higher which is accurate <br> for geometric distributions as it is not averaged over a certain number of trials. [I suspect candidate has forgotten variance formula!] | M1A1 <br> A0A0 |
| $\gamma$ | $4 \approx 4.03$ and $12 \approx 11.57$ so the student is correct $\quad$ [i.e., "it is a good match". Allow this] | M1A1 <br> A1A1 |
| $\delta$ | In geometric, prob of getting red is same on each attempt and since replacement is occurring the number of reds is likely to be less than <br> number of reds for student $Y$ since there is no replacement. So student's statement is likely to be true. ["geometric" confused, so use SR] | M0 <br> SR B1 |

Q7(b)(ii)

| $\alpha$ | Some differences, but very similar, and more similar than other rows, so highly likely that row 3 is student X | M0 |
| :---: | :--- | :---: |
| $\beta$ | I agree. E $(X)=5$ which is close to 4.97. Var $(X)=3.75$ which is close to 5 [5 and 3.75 imply M1 but wrong number (5) then used] | M1A0 |
| $\gamma$ | Very likely that student X's results are the third row since the number of reds in the pack remains constant, there is a 12/48 chance that <br> the student would pick a red. Therefore it is likely that out of 20 times cards are picked, 4.97 is close to 5 (which is $1 / 4$ of 20) | M1A0 |
| $\delta$ | I agree. E $(X)=5$ which is close to 4.97; Var $(X)=3.75$ which is close to 3.7 [correct apart from no rejection of "definite"] | M1A1A0 |
| $\varepsilon$ | It cannot be said that row 3 is definitely Student X's results, since the observed values of mean and variance may be unlikely results <br> from Y or Z's distributions. It's however likely that row 3 is student X since the observed mean and variance are very similar to the <br> expected mean and variance (5 and 3.75) of X's distribution | M1A1A1 |
| $\zeta$ | I agree that it could be student X's results as the expected number of reds is 5 which is similar to the observed mean of 4.97. However, I <br> disagree that it is definitely student X's results as student 1 has an observed mean of 5.03 which is just as close. <br> [I think this is a good answer and I am happy to treat the two means as two different facts] | M1A1A1 |
| $\eta$ | There are more 8s recorded for student 3 than for student 1, as expected as the cards are replaced, but it cannot be said for certain that X <br> is student 3. Additionally student 3 has a lower mean than student 1, but this would not be expected for student X. [SC: no dist implied] | B1B0B0 |


| Response | Mark |  |
| :--- | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223553998
Alternatively, you can email us on
support@ocr.org.uk
For more information visit

ocr.org.uk/qualifications/resource-finder
ocr.org.uk
f Twitter/ocrexams
3) locrexams
in /company/ocr

- locrexams


## 바웅 CAMBRIDGE <br> unvurrtit press a smsssment

OCR is part of Cambridge University Press \& Assessment, a department of the University of Cambridge.
For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored. © OCR 2022 Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee. Registered in England. Registered office The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA.

Registered company number 3484466 . OCR is an exempt charity.
OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.

