OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y531/01: Pure Core

AS Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		x-coord (or y-coord): $4 + 3\lambda = 19$ (or $-2 + -2\lambda = -12$) => $\lambda = 5$	M1	1.1	Forming and solving an equation in λ for any coordinate	
			z-coord: $7 + 4\lambda = 17$ => $\lambda = 2.5$ (or if $\lambda = 5$ then $7 + 4\lambda = 27$) (or $7 + 4 \times 5 \neq 17$)	M1	1.1	Either forming and solving an equation (with a different correct solution) in λ or using the previous value to demonstrate an inconsistency	This second M mark is for considering a coordinate with a different λ (So M2 for z and one other)
			Inconsistency so point does **not** lie on the line	A1	2.2a	Full marks can be gained for correctly identifying an inconsistency even if there is an error in solving the third equation	Correct conclusion e.g. "point not on line is enough here" as long as with 2 correct different values of λ – no need to see the word "inconsistency"
				[3]			
	(b)	(i)	**a.b** $= 1\times-3 + -2\times6 + 2\times2$	M1	1.1	Forming the dot product. Can be implied by –11 (but not 11)	
			$\cos\theta = \dfrac{"-11"}{\sqrt{1^2+(-2)^2+2^2}\sqrt{(-3)^2+6^2+2^2}}$	M1	1.1	Their dot product divided by the product of the (correctly formed) moduli. Allow sin/cos confusion here	$-\frac{11}{21}$ but can be awarded for $\frac{11}{21}$
			$\theta = 122°$ (3 sf)	A1	1.1	Do not ISW (so e.g. **M2A0** for working leading to final answer of 58.4°)	121.5881… If more accurate than 3s.f. accept answers in range [121.4 , 121.8]
				[3]			

	(b)	**(ii)**	Required vector is **a×b** $= \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} -16 \\ -8 \\ 0 \end{pmatrix}$	**M1** **A1**	1.1 1.1	This mark can be awarded for any non-zero multiple, even if non-numerical. or any numerical non-zero multiple e.g. $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	Some evidence of method needed for M1. Could be one correct value, or correct calculation for one value. Ignore errors in attempts to simplify. Condone incorrect statements e.g. $\begin{pmatrix} -16 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$
	(b)	**(ii)**	**Alternative method:** Assume vector is of the form $\begin{pmatrix} 1 \\ p \\ q \end{pmatrix}$. Then: $1 - 2p + 2q = 0$ **and** $-3 + 6p + 2q = 0$ $2p - 2q = 1$ and $6p + 2q = 3 \Rightarrow p = ½, q = 0$ so required vector is $\begin{pmatrix} 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}$.	**M1** **A1**	 1.1	Need both 'dot product' equations. Ignore lack of consideration of $\begin{pmatrix} 0 \\ p \\ q \end{pmatrix}$. or any numerical non-zero multiple e.g. $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	Could also be awarded for considering e.g. $\begin{pmatrix} t \\ p \\ q \end{pmatrix}$ and deriving $t - 2p + 2q = 0$ **and** $-3t + 6p + 2q = 0$ Eliminating e.g. q leads to $t = 2p$ and then $q = 0$ and so to $\begin{pmatrix} 2p \\ p \\ 0 \end{pmatrix}$ but a (non-zero) value for p must chosen so that the final answer is a vector and not a family of vectors.
				[2]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$\mathbf{A}+\mathbf{B}=\begin{pmatrix} a-2 & 6 \\ -2 & 3 \end{pmatrix}$	**B1**	1.1	Any double signs must be simplified correctly	
			$\mathbf{AB}=\begin{pmatrix} -2a-1 & 5a \\ -1 & -5 \end{pmatrix}$	**B1**	1.1		
			$\boldsymbol{A}^2=\begin{pmatrix} a^2-1 & a+3 \\ -a-3 & 8 \end{pmatrix}$	**B1** **[3]**	1.1		
	(b)	(i)	(det**A**) = $a\times3-1\times-1$ $3a+1=25 \Rightarrow a=8$	**M1** **A1** **[2]**	1.1 1.1	Correct expansion of determinant of **A**	
		(ii)	(System reduces to $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -2 \\ -6 \end{pmatrix}$ so no unique solution =>) (det**A**) = $3a+1=0$ $\therefore a=-⅓$	**M1** **A1**	3.1a 1.1	Setting their determinant to 0 if it is a linear function of a.	Answer only is ok here
			Alternate solution: Multiplying the first equation by 3 gives: $3ax+3y=-6$ $-x+3y=-6$ These two equations are the same if $3a=-1\rightarrow a=\frac{-1}{3}$	**M1** **A1**		Or subtracting gives $(3a+1)x=0\rightarrow a=\frac{-1}{3}$	
				[2]			

Question			Answer	Marks	AO	Guidance	
3			**DR** $\Sigma\alpha' = \alpha\beta + \beta\gamma + \gamma\alpha = -2/5$	**B1**	1.1	Quantity must be either identified as, or used as, sum of new roots	NB for reference: $\sum\alpha = \frac{3}{5},\ \sum\alpha\beta = -\frac{2}{5},\ \alpha\beta\gamma = -\frac{9}{5}$
			$\alpha'\beta'\gamma' = (\alpha\beta)(\beta\gamma)(\gamma\alpha) = (\alpha\beta\gamma)^2$...	**M1**	1.1	For expressing product of new roots in terms of old roots	
			...$= (-9/5)^2 = 81/25$	**A1**	1.1	For expressing product of new roots in terms of old roots	Condone $(9/5)^2$ if seen. Do not condone $-9/5^2$ or $-(9/5)^2$ unless recovered
			$\Sigma\alpha'\beta' = (\alpha\beta)(\beta\gamma) + (\beta\gamma)(\gamma\alpha) + (\gamma\alpha)(\alpha\beta)$ $= \alpha\beta\gamma(\alpha + \beta + \gamma)$...	**M1**	1.1	Finding sum of products and rewriting into symmetric form	
			$= (-9/5)(3/5) = -27/25$	**A1**	1.1		
			$a = 25 \Rightarrow 25x^3 + 10x^2 - 27x - 81 = 0$	**A1**	1.1	Or any non-zero integer multiple	Needs to be an equation.
			Alternative method $\alpha\beta\gamma = -9/5$	**B1**			
			$u = \alpha\beta\gamma/x = -9/(5x)$	**B1**		SOI	
			When $x=\alpha$, $u = \beta\gamma$, and similar for other roots	**B1**			
			$5\left(\frac{-9}{5u}\right)^3 - 3\left(\frac{-9}{5u}\right)^2 - 2\left(\frac{-9}{5u}\right) + 9 = 0$	**M1**			
			$\frac{-729}{25u^3} - \frac{243}{25u^2} + \frac{18}{5u} + 9 = 0$	**M1**			
			$25x^3 + 10x^2 - 27x - 81 = 0$	**A1**			
				[6]			

Question			Answer	Marks	AO	Guidance	
4			If $n = 4$, LHS = $3^4 = 81$ RHS = $10 \times 4 = 40 < 81$ = LHS So true for $n = 4$	B1	2.5	Basis case. Comparison must be explicit and correct	An assertion without calculation such as e.g. $3^4 > 10 \times 4$ is insufficient for **B1**. BOD statements such as "therefore true when n=1"
			(Assume that) $3^k > 10k$ (for some integer $k \geq 4$).	M1	2.1	Inductive hypothesis set up	
			$3^{k+1} = 3 \times 3^k > 3 \times 10k$...	M1	1.1	Considering for $k + 1$ and using inductive hypothesis correctly	Asserting $3^{k+1} > 10(k+1)$ without justification gets **M0**.
			... $= 30k = 10(k+1) + 10(2k-1) > 10(k+1)$ since $2k - 1 > 0$ since $k \geq 4$ i.e. if $3^k > 10k$ then $3^{k+1} > 10(k+1)$	A1	2.2a	Showing enough working to establish statement for $k + 1$. Must be justified but justification could be e.g. $k > 1$.	Could compare $3k$ and k+1
			So true for $n = k \Rightarrow$ true for $n = k + 1$. But true for $n = 4$. So true for all integers $n \geq 4$	A1	2.4	Clear and complete conclusion, following a **correct and complete** proof with no incorrect statements. **Must** be $n \geq 4$ not eg 0 or 1 for **A1**.	This mark must only be awarded if the language and notation in the whole proof and conclusion is correct.
				[5]			

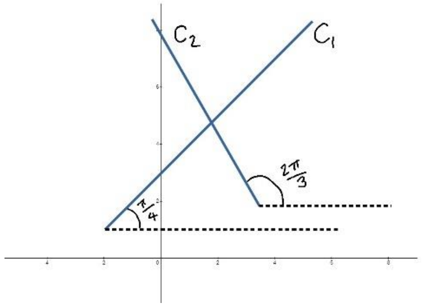
Question			Answer	Marks	AO	Guidance	
5	(a)		**DR** $(a + b\mathrm{i})^2 = a^2 - b^2 + 2ab\mathrm{i}$	B1	1.1	Seen or implied in solution	
			$a^2 - b^2 = -16$ and $2ab = 30$ (where a and b are real)	M1	1.1	Comparing real and imaginary parts (no i unless later recovered) from a 3 (or 4) term expansion. Allow sign slips	
			$b = \frac{15}{a} \Rightarrow a^2 - \left(\frac{15}{a}\right)^2 = -16$ $\Rightarrow a^4 + 16a^2 - 225 = 0$	M1	1.1	Eliminating b or a to obtain 3 term quadratic in a^2 or b^2. Unknowns must not be in denominator. Must be an equation.	($b^4 - 16b^2 - 225 = 0$) Factorised forms: $(a^2 - 9)(a^2 + 25)$ $(b^2 - 25)(b^2 + 9)$
			a (b) real so $a^2 = 9$ (or $b^2 = 25$) only	A1	1.1	Rogue solutions; $a^2 = -25$, $b^2 = -9$	
			$3 + 5\mathrm{i}$ and $-3 - 5\mathrm{i}$	A1	2.2a	Both roots. Can be $\pm(3 + 5\mathrm{i})$ but not $\pm 3 \pm 5\mathrm{i}$ or $\pm 3 + 5\mathrm{i}$.	Note: 4/5 possible following B0
				[5]			

5	**(b)**		**DR** $(3 + 5i)^3 = (-16 + 30i)(3 + 5i) = -198 + 10i$	**M1**	1.1	Can awarded this for cubing an incorrect answer to 5(a) Could be done by expansion either in two steps or binomial: $3^3 + 3\times3^2\times5i + 3\times3\times(5i)^2 + (5i)^3$ Do not need to see intermediate step before correct answer. If incorrect need to see proof of expanding three brackets	Or $(-3 - 5i)^3 = 198 - 10i$... For binomial want to see 4 terms and either 2nd or 3rd term correct (up to sign error)
			$= \frac{-99+5i}{4}\times 2^3$ so $\frac{3}{2}+\frac{5i}{2}$	**A1** **[2]**	3.1a	Correct answer must follow a correct root	$= \frac{-99+5i}{4}\times(-2)^3$

Question			Answer	Marks	AO	Guidance	
6	(a)		T is a reflection (in 2-D) and in any reflection any point on the mirror line remains invariant...	B1	2.4	Any point on the mirror line stays where it is...	
			...and so the mirror line must itself be a line of invariant points.	B1	2.2a	...so the mirror line is a line of invariant points. Accept "so the line of invariant points is the mirror line"	If B0B0 then SC1 for any answer which is, in effect, a statement that the mirror line is an invariant line.
				[2]			
	(b)		For line of invariant points **Ar** = **r**	B1	1.1		
			$\frac{1}{13}\begin{pmatrix}5 & 12\\12 & -5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{13}\begin{pmatrix}5x+12y\\12x-5y\end{pmatrix}$	M1	1.1	Multiplying general point into **A**	Could be awarded for sight of $5x + 12y$ or $12x - 5y$ o.e.
			$=\begin{pmatrix}x\\y\end{pmatrix}\Rightarrow\frac{1}{13}(5x+12y)=x$ or $\frac{1}{13}(12x-5y)=y$	M1	1.1	Equating and deriving an equation relating x and y	
			$12y = 8x$ (or $18y = 12x$) => $y=\frac{2}{3}x$ or $y=\frac{2}{3}x+0$	A1	1.1	Need to check that both equations give same straight line.	
			Alternative method: Line passes through O => $c = 0$	B1		Used in the solution	
			$\frac{1}{13}\begin{pmatrix}5 & 12\\12 & -5\end{pmatrix}\begin{pmatrix}x\\mx\end{pmatrix}$	M1		Considering the matrix acting on a general point on the line $y = mx$ $(+ c)$	
			$=\frac{1}{13}\begin{pmatrix}5x+12mx\\12x-5mx\end{pmatrix}=\begin{pmatrix}x\\mx\end{pmatrix}$	M1		Multiplying and equating	
			$5x + 12mx = 13x$ and $12x - 5mx = 13mx$ => $m = 2/3$ so $y=\frac{2}{3}x$ or $y=\frac{2}{3}x+0$	A1		Need to check that both equations are satisfied by m=2/3.	

			Alternative method 2: $\frac{1}{13}\begin{pmatrix}5 & 12\\12 & -5\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}$ $=\frac{1}{13}\begin{pmatrix}5x+12mx+12c\\12x-5mx-5c\end{pmatrix}=\begin{pmatrix}x'\\y'\end{pmatrix}$ $y'=mx'+c\Rightarrow$ $\frac{1}{13}(12x-5mx-5c)=\frac{m}{13}(5x+12mx+12c)+c$ $x(12m^2+10m-12)+c(12m+18)=0$ $2x(2m+3)(3m-2)+6c(2m+3)=0$ Either we have $m=\frac{-3}{2}$, and c can be anything or $m=\frac{2}{3}$ and we have $c=0$. This gives a single line and infinitely many which are perpendicular to it. Therefore the reflection line is the single line (and the perpendicular ones are invariant lines). Hence we have $m=\frac{2}{3}$ $and\ so$ $y=\frac{2}{3}x$	**M1** **M1** **A1** **A1**		Considering the matrix acting on a general point on the line $y=mx+c$ Multiplying and substituting into $y=mx+c$ Finding two correct values of m and no others (linking to c not necessary here) Convincing reason why m=-3/2 is rejected as a possibility.	
				[4]			

Question			Answer	Marks	AO	Guidance	
6	(c)		$\frac{1}{13}\begin{pmatrix}5 & 12\\12 & -5\end{pmatrix}\begin{pmatrix}1\\5\end{pmatrix}=\begin{pmatrix}5\\-1\end{pmatrix}$ [so P' is (5, –1)]	M1	1.1		**SC**: If answer given as a vector then maximum mark is **M1 A1 A0** **M1** could be awarded for solving **Ax**=(5, 1) to get **x=**(5, -1)
			So required *x*-coord is ½(5 + 1) = 3... ...and required *y*-coord is ½ (5 + –1) = 2	A1 A1	2.2a 1.1		
			Alternative method: Gradient of line *PP′* is –3/2	B1		Could be from –1/(2/3) or (–1 – 5)/(5 – 1)	**SC:** if using this method but gain 0 marks, can get **B1** for sight of *P'*=(5, -1)
			Equation of *PP′* is $y - 5 = -3/2\ (x - 1)$	M1		Using their gradient and (1, 5) or their (5, –1) to form the equation of the line	$y - -1 = -3/2\ (x - 5)$ $y = -3/2\ x + 13/2$
			$2/3\ x = -3/2\ x + 13/2 \Rightarrow x = 3 \Rightarrow y = 2$	A1			
				[3]			
	(d)		–3/2 (Since T is a reflection) the invariant lines are the lines perpendicular to the mirror line (which reflect onto themselves)	B1FT B1	2.2a 2.4	FT their –1/*m* from **(b)**	**B0** for 2/3.
			Alternative method: $\frac{1}{13}\begin{pmatrix}5 & 12\\12 & -5\end{pmatrix}\begin{pmatrix}x\\ax+2\end{pmatrix}=\frac{1}{13}\begin{pmatrix}5x+12ax+24\\12x-5ax-10\end{pmatrix}$ **and** $(12x - 5ax - 10)/13 = a(5x + 12ax + 24)/13 + 2$	M1		Multiplying any point on the line $y = ax + 2$ into **A** and specifying that the image point lies on the same straight line.	
			$\Rightarrow 12x - 5ax - 10 = 5ax + 12a^2x + 24a + 26$ $\Rightarrow (12a^2 + 10a - 12)x + 36 + 24a = 0$ But true for any $x \Rightarrow 36 + 24a = 0 \Rightarrow a = -3/2$	A1		If $12a^2 + 10a - 12 = 0$ leading to $(3a - 2)(2a + 3) = 0$ then $a = 2/3$ must be properly rejected for **A1**.	
				[2]			

Question	Answer	Marks	AO	Guidance	
7	**DR** $m_1 = [\tan(\pi/4)] = 1$ and $m_2 = [\tan(2\pi/3)] = -\sqrt{3}$	**B1**	3.1a	soi in solution	Must be connect to the gradients of the lines (soi)
	C_1: $z + 2 - \mathrm{i} = z - (-2 + \mathrm{i})$ so equation of (half)-line is $y - 1 = x - -2$ or $y = x + 3$	**M1**	3.1a	Identifying a point on the extended line (condone sign errors) and using it and their gradient to form the equation of a line	Don't need to see equation in z first Gradient must have come from considering angle of $(\pi/4)$
	C_2: $z - 2 - \sqrt{3} - 2\,\mathrm{i} = z - (2 + \sqrt{3} + 2\mathrm{i})$ so equation is $y - 2 = -\sqrt{3}(x - (2 + \sqrt{3}))$ or $y = -\sqrt{3}x + 2\sqrt{3} + 5$	**M1**	1.1	Identifying a point on the extended line (condone sign errors) and using it and their gradient to form the equation of a line	Don't need to see equation in z first Gradient must have come from considering angle of $(2\pi/3)$ or $(\pi/3)$
	$x + 3 = -\sqrt{3}x + 2\sqrt{3} + 5$ $\Rightarrow (1+ \sqrt{3})x = 2(1 + \sqrt{3}) \Rightarrow x = 2$	**M1**	2.1	Eliminating one unknown and solving for the other	
	$\Rightarrow y = 2 + 3 = 5$	**A1**	1.1		
		**B1**	2.3	A sketch to show C_1 and C_2 as half lines with correct start points clearly indicated. PoI lies on both half lines in approximately the right positions. Angles should look approximately correct. Lines need to intersect.	Or : C_1: Need $x > -2$. C_2: Need $x < 2 + \sqrt{3}$, therefore solution with $x = 2$ is valid Or: C_1: Need $y > 1$. C_2 Need $y > 2$, so solution with $y = 5$ is valid
	$C_1 \cap C_2 = \{2 + 5\mathrm{i}\}$ (or in words: "so the required locus contains only the number $2 + 5\mathrm{i}$")	**A1** **[7]**	3.2a	A1 can be awarded if answer represented unambiguously on an Argand Diagram either as $2+5\mathrm{i}$, or 2 and $5\mathrm{i}$ marked on axes.	Not just (2, 5) or (2, 5i) Needs to be an indication that the locus is a single point, expressed as a complex number. Cannot be expressed as coordinates.

Question			Answer	Marks	AO	Guidance	
8	(a)		(If C is not at A or B) whatever plane A, B and C lie in, AB is the diameter of a circle and C is on the perimeter so angle ACB is a right angle because it is an angle in a semi-circle.	**B1**	3.1a	Angle ACB is "an angle in a semi-circle" so 90° oe.	Could choose to prove the circle theorem.
			So $\overrightarrow{AC}.\overrightarrow{BC}=0$ since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ are perpendicular and the dot product of perpendicular vectors is zero (because cos90° = 0)	**B1**	2.4	A reason must be given. Just "So $\overrightarrow{AC}.\overrightarrow{BC}=0$" is insufficient.	"The dot product of perpendicular vectors is zero." Implication must be the correct way around.
			In the case where $C = A$ or $C = B$ then $\overrightarrow{AC}=\mathbf{0}$ or $\overrightarrow{BC}=\mathbf{0}$ and so $\overrightarrow{AC}.\overrightarrow{BC}=0$ since the zero vector dotted with any vector is zero.	**B1**	2.1	Must mention both cases but second case can just be covered by "Similarly for B" or "eg consider A" oe.	
			Let the centre of the sphere be at O. Let the coordinates of $A=(\alpha,\beta,\gamma)$ where $\alpha^2+\beta^2+\gamma^2=r^2$ Since AB is a diameter, B has coordinates $(-\alpha,-\beta,-\gamma)$ Let C have coordinates (x,y,z), where $x^2+y^2+z^2=r^2$ $\overrightarrow{AC}=\begin{pmatrix}x-\alpha\\y-\beta\\z-\gamma\end{pmatrix}$ $\overrightarrow{BC}=\begin{pmatrix}x+\alpha\\y+\beta\\z+\gamma\end{pmatrix}$	**B1**		Centre at O and attempting $\overrightarrow{AC}$ or $\overrightarrow{BC}$	
			Then $\overrightarrow{AC}\cdot\overrightarrow{BC}=(x-\alpha)(x+\alpha)+(y-\beta)(y+\beta)+(z-\gamma)(z+\gamma)$	**B1**		$\overrightarrow{AC}$ and $\overrightarrow{BC}$ both correct and dot product formed	
			$\overrightarrow{AC}\cdot\overrightarrow{BC}=(x^2+y^2+z^2)-(\alpha^2+\beta^2+\gamma^2)$ $\overrightarrow{AC}\cdot\overrightarrow{BC}=r^2-r^2=0$	**B1**		Correctly showing that dot product is 0.	

			Let the centre of the sphere be at *O*. Let the position vectors of *A, B, C* be **a, b, c**. We have |**a**| = |**b**| = |**c**| (= *r*) Then: $\overrightarrow{AC}\cdot\overrightarrow{BC} = (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{b})$ $\overrightarrow{AC}\cdot\overrightarrow{BC} = \lvert\mathbf{c}\rvert^2 - \mathbf{a}\cdot\mathbf{c} - \mathbf{b}\cdot\mathbf{c} + \mathbf{a}\cdot\mathbf{b}$ $= \lvert\mathbf{c}\rvert^2 - (\mathbf{a}+\mathbf{b})\cdot\mathbf{c} + \mathbf{a}\cdot\mathbf{b}$ But **a = -b** (as A,B ends of diameter) $\overrightarrow{AC}\cdot\overrightarrow{BC} = \lvert\mathbf{c}\rvert^2 - (\mathbf{a}-\mathbf{a})\cdot\mathbf{c} - \mathbf{a}\cdot\mathbf{a}$ $= \lvert\mathbf{c}\rvert^2 - \lvert\mathbf{a}\rvert^2$ $= \mathbf{r}^2 - \mathbf{r}^2 = \mathbf{0}$	**B1** **B1** **B1**		Centre at *O* and attempting $\overrightarrow{AC}$ or $\overrightarrow{BC}$ $\overrightarrow{AC}$ and $\overrightarrow{BC}$ both correct and dot product formed Stating and using **a = -b** to show that dot product is 0.	
				[3]			
	(b)		$\overrightarrow{AC} = \begin{pmatrix} 2p-11 \\ p-12 \\ 1--14 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} 2p-9 \\ p-13 \\ 1-6 \end{pmatrix}$	**M1**	3.1a	Attempt to subtract (in either order)	
			$\overrightarrow{AC} = \begin{pmatrix} 2p-11 \\ p-12 \\ 15 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2p-9 \\ p-13 \\ -5 \end{pmatrix}$	**A1**	1.1	Both completely correct (could be $\overrightarrow{CA} = \begin{pmatrix} 11-2p \\ 12-p \\ -15 \end{pmatrix}$ etc)	
			$\overrightarrow{AC}.\overrightarrow{BC} = 0 \Rightarrow \begin{pmatrix} 2p-11 \\ p-12 \\ 15 \end{pmatrix}.\begin{pmatrix} 2p-9 \\ p-13 \\ -5 \end{pmatrix} =$ $(2p-11)(2p-9)+(p-12)(p-13)+15(-5)=0$	**M1**	3.1a	Attempt to dot their $\overrightarrow{AC}$ and $\overrightarrow{BC}$ Dot product must result in a scalar quantity.	"=0" not necessary for M1 here
			$5p^2-65p+180=0$ or $p^2-13p+36=0$	**A1**	1.1	Correct three term quadratic	(must have "= 0" appearing somewhere Might be near start).
			$p=4$ or $p=9$	**A1**	1.1		
			So the possible locations are (8, 4, 1) or (18, 9, 1)	**A1FT**	1.1	Solution must be given as coordinates	For FT must have solved a quadratic coming from attempt at dot product

			Alternative method: $(\|AB\|^2 = 2^2 + 1^2 + 20^2 = 405$ $\|AC\|^2 = (11-2p)^2 + (12-p)^2 + (-14-1)^2$ $\|BC\|^2 = (9-2p)^2 + (13-p)^2 + (6-1)^2$ $\|AB\|^2 = \|AC\|^2 + \|BC\|^2$ $10p^2 - 130p + 765 = 405$ $p^2 - 13p + 36 = 0$ $p = 4$ or $p = 9$ So the possible locations are (8, 4, 1) or (18, 9, 1)	**M1** **A1** **M1** **A1** **A1** **A1 FT**		Attempting to find $\|AC\|^2$ or $\|BC\|^2$ Correct expressions for $\|AC\|^2$ and $\|BC\|^2$ Using Pythagoras Correct 3 term quadratic Solution must be given as coordinates	For FT must have solved a quadratic coming from attempt at dot product
			Alternative method 2: (Centre of sphere is at (10, 12.5, -4)) $r^2 = (11-10)^2 + (12-12.5)^2 + (-14+4)^2$ $r^2 = \frac{405}{4}$ Distance OC is: $\|OC\|^2 = (2p-10)^2 + (p-12.5)^2 + (1+4)^2$ $= \frac{405}{4}$ $p^2 - 13p + 36 = 0$ $p = 4$ or $p = 9$ So the possible locations are (8, 4, 1) or (18, 9, 1)	**M1** **M1** **A1** **A1** **A1** **A1 FT**		Attempting to find radius^2 (could be via diameter) Attempting to find $\|OC\|^2$ Correct expressions for $\|OC\|^2$ Correct 3 term quadratic Solution must be given as coordinates	For FT must have solved a quadratic coming from attempt at dot product
				[6]			

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