Oxford Cambridge and RSA

Wednesday 15 June 2022 – Afternoon

AS Level Further Mathematics A

Y533/01 Mechanics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Two stones, A and B, are sliding along the same straight line on a horizontal sheet of ice. Stone A, of mass 50 kg, is moving with a constant velocity of $2.1\,\mathrm{m\,s^{-1}}$ towards stone B. Stone B, of mass 70 kg, is moving with a constant velocity of $0.8\,\mathrm{m\,s^{-1}}$ towards stone A.

A and B collide directly. Immediately after their collision stone A's velocity is $0.35\,\mathrm{m\,s^{-1}}$ in the same direction as its velocity before the collision.

(a) Find the speed of stone B immediately after the collision. **[2]**

(b) Find the coefficient of restitution for the collision. **[2]**

(c) Find the total loss of kinetic energy caused by the collision. **[3]**

(d) Explain whether the collision was perfectly elastic. **[1]**

2 A hockey puck of mass 0.2 kg is sliding down a rough slope which is inclined at 10° to the horizontal. At the instant that its velocity is $14\,\mathrm{m\,s^{-1}}$ directly down the slope it is hit by a hockey stick. Immediately after it is hit its velocity is $24\,\mathrm{m\,s^{-1}}$ directly up the slope.

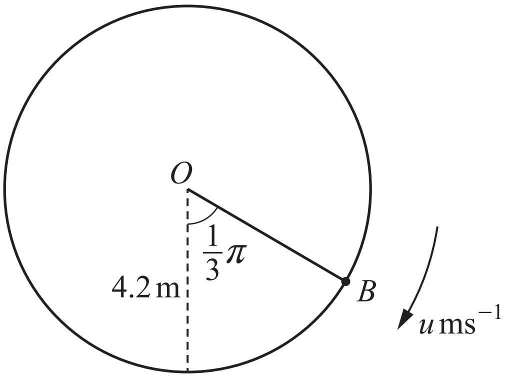
(a) Find the magnitude of the impulse exerted by the hockey stick on the puck. **[2]**

After it has been hit, the puck first comes to instantaneous rest when it has travelled 15 m up the slope. While the puck is moving up the slope, the resistance to its motion has constant magnitude R N.

(b) Use an energy method to determine the value of R. **[5]**

3 A smooth wire is shaped into a circle of radius 4.2 m which is fixed in a vertical plane with its centre at a point *O*. A small bead *B* is threaded onto the wire. *B* is held so that *OB* makes an angle of $\frac{1}{3}\pi$ radians with the downwards vertical through *O*.

B is projected downwards along the wire with initial speed $u\,\text{ms}^{-1}$ (see diagram). In its subsequent motion *B* describes complete circles about *O*.



Given that the lowest speed of *B* in its motion is $4\,\text{ms}^{-1}$ determine the value of *u*. **[5]**

4 A cyclist is riding a bicycle along a straight road which is inclined at an angle of 4° to the horizontal. The cyclist is working at a constant rate of 250 W. The combined mass of the cyclist and bicycle is 80 kg and the resistance to their motion is a constant 70 N.

Determine the maximum constant speed at which the cyclist can ride the bicycle

- up the hill, and
- down the hill. **[5]**

5 One end of a light inextensible string of length 3.5 m is attached to a fixed point *O* on a smooth horizontal plane. The other end of the string is attached to a particle *P* of mass 0.45 kg. *P* moves with constant speed in a circular path on the plane with the string taut.

The string will break if the tension in it exceeds 70 N.

Determine the minimum possible time in which *P* can describe a complete circle about *O*. **[4]**

6 A particle moves in a straight line with constant acceleration a. Its initial velocity is u and at time t its velocity is v.

It is assumed that v depends only on u, a and t.

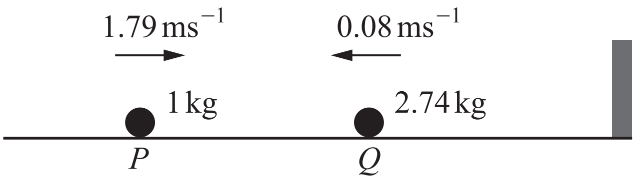
(a) Assuming that this dependency is of the form $u^{\alpha}a^{\beta}t^{\gamma}$, use dimensional analysis to find α and γ in terms of β. **[5]**

(b) By noting that the graph of v against t must be a straight line, determine the possible values of β. **[2]**

You may assume that the units of the given quantities are the corresponding SI units.

(c) By considering v when $t = 0$ seconds and when $t = 1$ second, derive the equation of motion $v = u + at$, explaining your reasoning. **[3]**

7 Two particles, P and Q, are on a smooth horizontal floor. P, of mass 1 kg, is moving with speed $1.79\,\text{ms}^{-1}$ directly towards a vertical wall. Q, of mass 2.74 kg, is between P and the wall, moving directly towards P with speed $0.08\,\text{ms}^{-1}$ (see diagram).



P and Q collide directly and the coefficient of restitution for this collision is denoted by e.

(a) Show that after this collision the speed of Q is given by $0.42 + 0.5e\,\text{ms}^{-1}$. **[5]**

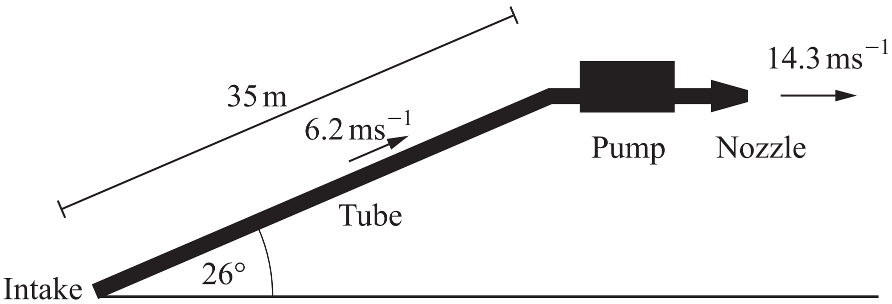
After this collision, Q then goes on to collide directly with the wall. The coefficient of restitution for the collision between Q and the wall is also e. There is then no subsequent collision between P and Q.

(b) Determine the range of possible values of e. **[7]**

8 As part of an industrial process a single pump causes the intake of a liquid chemical to the bottom end of a tube, draws it up the tube and then discharges it through a nozzle at the top end of the tube.

The tube is straight and narrow, 35 m long and inclined at an angle of 26° to the horizontal. The chemical arrives at the intake at the bottom end of the tube with a speed of $6.2\,\mathrm{m\,s^{-1}}$. At the top end of the tube the chemical is discharged horizontally with a speed of $14.3\,\mathrm{m\,s^{-1}}$ (see diagram).

In total, the pump discharges 1500 kg of chemical through the nozzle each hour.



In order to model the changes to the mechanical energy of the chemical during the entire process of intake, drawing and discharge, the following modelling assumptions are made.

- At any instant the total resistance to the motion of all the liquid in the tube is 40 N.
- All other resistances to motion are ignored.
- The liquid in the tube moves at a constant speed of $6.2\,\mathrm{m\,s^{-1}}$.

(a) State **one** other modelling assumption which is required to model the changes to the mechanical energy of the liquid with the given information. **[1]**

(b) Determine the power at which the pump is working, according to the model. **[5]**

When the power at which the pump is working is measured it is in fact found to be 450 W.

(c) (i) Find the difference between the total amount of energy output by the pump each hour and the total amount of mechanical energy gained by the chemical each hour. **[2]**

(ii) Give **one** reason why the model underestimates the power of the engine. **[1]**

END OF QUESTION PAPER

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