Oxford Cambridge and RSA

Friday 17 June 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes

* 9 4 5 0 4 7 3 6 7 1 *

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 You are given the following simultaneous equations.

$2x + 1.8y = 5$

$3x + 2.8y = 3$

The constants and the coefficients of x are exact, but the coefficients of y have been **chopped** to **1** decimal place.

(a) Calculate the maximum possible relative error in each of the coefficients of y. **[2]**

(b) Determine the range of possible values of y. **[4]**

(c) Explain why this range is so large. **[1]**

2 The table shows some values of x and the associated values of $y = \mathrm{f}(x)$.

x	1.98	1.99	2	2.01	2.02
$\mathrm{f}(x)$	1.10311648	1.10514069	1.10714872	1.10914075	1.11111695

(a) Use the central difference method to calculate **two** approximations to the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$. **[3]**

(b) State the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$ as accurately as you can, justifying the precision quoted. **[1]**

(c) Calculate an approximation of the **error** in using $\mathrm{f}(2)$ to approximate $\mathrm{f}(2.008)$. **[2]**

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3 Ali uses the trapezium rule with $h = 1$, $h = 0.5$ and $h = 0.25$ to calculate three approximations to $\int_1^2 f(x)\,dx$. Ali's results are shown in the table.

h	n	T_n
1	1	0.9462734
0.5	2	0.9645336
0.25	4	0.9691932

(a) Use the information in the table to calculate **two** approximations to $\int_1^2 f(x)\,dx$ using Simpson's rule, giving your answers correct to **6** decimal places. **[3]**

(b) **Without** doing any further calculation, state the value of $\int_1^2 f(x)\,dx$ as accurately as you can, justifying the precision quoted. **[1]**

Ali states that the graph of $y = f(x)$ is concave downwards between $x = 1$ and $x = 2$.

(c) Explain whether the information in the table supports Ali's statement. **[1]**

4 The equation $3x^5 - 13x^2 + 11 = 0$ has three roots, α, β and γ such that $\alpha < \beta < \gamma$.
Fig. 4.1 shows part of the graph of $y = 3x^5 - 13x^2 + 11$.

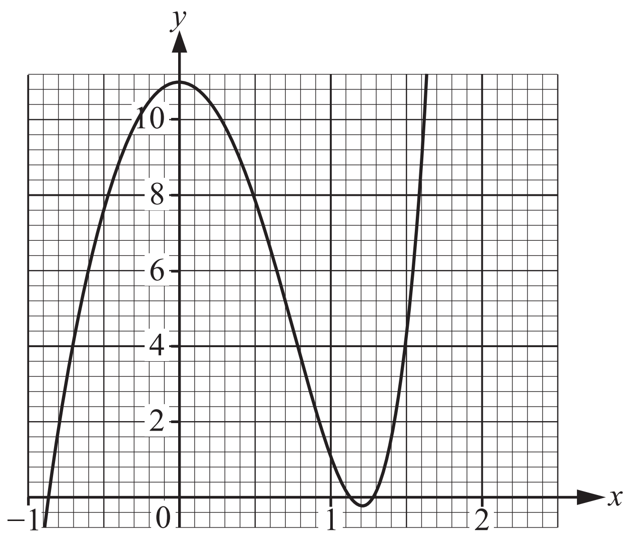

Fig. 4.1

(a) Explain why it is not possible to use the method of false position with initial values of $a = 1$ and $b = 1.5$ to find β. **[1]**

Taylor uses the method of false position to find β using initial values of $a = 1$ and $b = 1.2$. The associated spreadsheet output is shown in **Fig. 4.2**.

	C	D	E	F	G	H
3	a	f(a)	b	f(b)	x_{new}	f(x_{new})
4	1	1	1.2	−0.255040	1.159357	−0.189833
5	1	1	1.159357	−0.189833	1.133933	−0.091284
6	1	1	1.133933	−0.091284	1.122729	−0.035016
7	1	1	1.122729	−0.035016	1.118577	−0.012267
8	1	1	1.118577	−0.012267	1.11714	−0.004160
9	1	1	1.117140	−0.004160	1.116655	−0.001395
10	1	1	1.116655	−0.001395	1.116492	−0.000466
11	1	1	1.116492	−0.000466	1.116438	−0.000156
12	1	1	1.116438	−0.000156	1.116420	−5.19E-05

Fig. 4.2

(b) Write down a suitable spreadsheet formula for the following.

(i) cell F4 **[1]**

(ii) cell G4 **[2]**

The spreadsheet formula in cell C5 is

=IF(H4>0,G4,C4) .

(c) Write down a suitable formula for cell E5. **[1]**

(d) **Without** doing any further calculation, state the value of β as accurately as you can, justifying your answer. **[1]**

(e) Show that the Newton-Raphson iteration is

$$x_{n+1} = \frac{12x_n^5 - 13x_n^2 - 11}{15x_n^4 - 26x_n}.$$ **[2]**

(f) Use the Newton-Raphson iteration with $x_0 = -1$ to find α correct to **7** decimal places. **[2]**

Taylor uses the Newton-Raphson iteration to find γ correct to **7** decimal places. The associated spreadsheet output, together with some further analysis, is shown in **Fig. 4.3**.

r	x_r	difference	ratio
0	1.5		
1	1.3773266	−0.1226734	
2	1.3108192	−0.0665073	0.5421493
3	1.2840785	−0.0267407	0.4020718
4	1.2789338	−0.0051447	0.1923936
5	1.2787404	−0.0001934	0.0375917
6	1.2787401	−2.713E-07	0.0014026
7	1.2787401	−5.329E-13	1.965E-06

Fig. 4.3

(g) Explain what the values in the ratio column in **Fig. 4.3** tell you about the convergence of this sequence of estimates to γ. **[2]**

5 The equation $x^5 - 3x + 1 = 0$ has two positive roots, α and β, where $\alpha < \beta$.

Charlie is using the iterative formula $x_{n+1} = \mathrm{g}(x_n)$ where $\mathrm{g}(x_n) = \dfrac{x_n^5 + 1}{3}$ to try to find α and β.

Fig. 5.1 shows part of the graphs of $y = x$ and $y = \mathrm{g}(x)$.

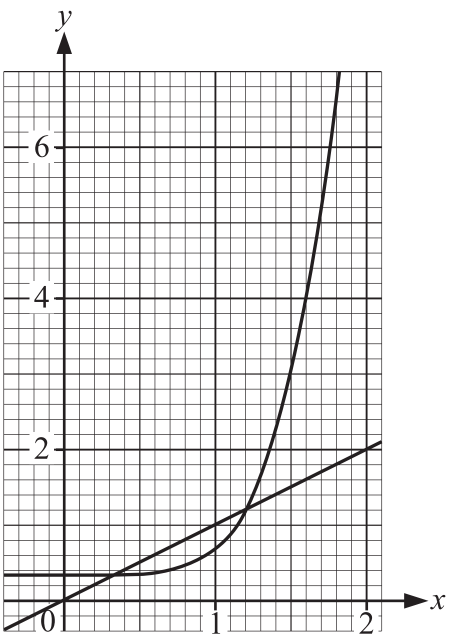

Fig. 5.1

(a) With reference to **Fig. 5.1**, explain why Charlie's iterative formula may be successfully used to find α, but will not find β. **[2]**

(b) Use the iterative formula with a starting value of $x_0 = 1$ to find α correct to **6** decimal places. **[2]**

Charlie uses the iterative formula to try to find β. Her spreadsheet output is shown in **Fig. 5.2**.

r	x_r
0	1.5
1	2.8645833
2	64.629643
3	375869894
4	2.501E+42
5	3.26E+211
6	#NUM!

Fig. 5.2

(c) Explain why the spreadsheet displays #NUM! for the value of x_6. **[1]**

(d) Use the relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n)$, with $\lambda = -0.33$ and $x_0 = 1.5$ to find β correct to **6** decimal places. **[2]**

6 **Fig. 6.1** shows some values of x and the associated values of $f(x)$.

x	1	1.25	1.5	1.75	2
$f(x)$	0	0.1609640	0.2924813	0.4036775	0.5

Fig. 6.1

Sam is using the midpoint rule to find a sequence of estimates to $\int_1^2 f(x)\,dx$. Some of Sam's spreadsheet output is shown in **Fig. 6.2**.

n	M_n	difference	ratio
1			
2			
4	0.2795859	−0.0027349	0.2691687
8	0.2788869	−0.0006989	0.2555667
16	0.2787112	−0.0001758	0.2514659
32	0.2786672	−4.401E-05	0.2503719
64	0.2786561	−1.101E-05	0.2500933

Fig. 6.2

(a) Use the information in **Fig. 6.1** to fill in the **three** missing values on the copy of **Fig. 6.2** in the Printed Answer Booklet, giving your answers correct to **7** decimal places. **[4]**

(b) Explain why the last difference is displayed as −1.101E-05 **not** −1.1E-05. **[1]**

(c) Use extrapolation to determine the value of $\int_1^2 f(x)\,dx$ as accurately as you can, justifying the precision quoted. **[5]**

7 Azmi placed a container of liquid in a refrigerator unit in a laboratory. Initially the temperature of the liquid was 22 °C.

Azmi recorded the temperature, y °C, of the liquid t minutes after putting the liquid in the refrigerator unit.

The results are shown in the table.

t	0	6	10
y	22	8.824	3

Azmi believes that the data may be modelled by a polynomial.

(a) Explain why it is not possible to use Newton's forward difference interpolation method for these data. **[1]**

(b) Use Lagrange's form of the interpolating polynomial to construct a quadratic model for these data. **[4]**

Later Azmi finds that when $t = 20$, $y = -4$.

(c) Determine whether the quadratic model is a good fit for these values. **[2]**

Azmi uses all the data available to construct the cubic model

$y = at^3 + bt^2 - 2.7t + 22$, where a and b are constants.

(d) Use Lagrange's method to determine the values of a and b. **[4]**

The background temperature in the refrigerator unit is −5 °C.

(e) Verify that, according to the cubic model, the liquid has reached the background temperature after 30 minutes. **[1]**

(f) Adapt this model so that it would be suitable for large values of t. **[1]**

END OF QUESTION PAPER

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