GCE

Further Mathematics B MEI

Y414/01: Numerical Methods

AS Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✔and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a   Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		$-\dfrac{0.1}{1.9} = -0.0526$ to $-0.05\dot{3}$	B1	1.1	may see $-\dfrac{0.0999...}{1.8999...}$
			$-\dfrac{0.1}{2.9} = -0.03448...$ to -0.035	B1	1.1	may see $-\dfrac{0.0999...}{2.8999...}$ allow **B1B1** for correct positive answers
				[2]		
1	(b)		one of $-90, -45, -22.5$ or 90 found from elimination of x	B1	3.1a	
			two correct values seen	B1	3.1a	
			either 90 or –90 seen in final answer	B1	1.1	in acceptable form
			$-90 < y < 90$	B1	3.2a	allow eg between –90 and 90 do not allow eg from –90 to 90
				[4]		
1	(c)		**eg** the solution involves the **division** by the **subtraction of nearly equal numbers**	B1	2.4	need to see subtraction of nearly numbers **oe and** division; allow eg subtraction of nearly equal numbers in the denominator
			eg the lines represented by the equations are almost parallel, so a small change in the coefficients leads to a big change in the point of intersection			
				[1]		
2	(a)		$\dfrac{1.11111695-1.10311648}{2\times0.02}$ **or** $\dfrac{1.10914075-1.10514069}{2\times0.01}$	M1	1.1	**NB** $\dfrac{0.00800047}{0.04}$ **or** $\dfrac{0.00400006}{0.02}$ with numerator correct to 2 or more sf implies **M1**
			0.20001175	A1	1.1	
			0.200003	A1	1.1	
				[3]		

	Question		Answer	Marks	AO	Guidance
2	(b)		0.2000 by comparison of the two estimates **or** 0.20000 because the second estimate will be more accurate (and the differences will decrease rapidly) [1]	B1FT	2.2b	allow 0.2000001 from extrapolation and this improves accuracy
2	(c)		$0.008 \times their\ 0.2000$ 0.0016 [2]	M1 A1	1.1a 1.1	may be embedded
3	(a)		$\frac{4 \times 0.9645336 - 0.9462734}{3}$ or $\frac{4 \times 0.9691932 - 0.9645336}{3}$ $0.970620(3333\dots)$ or $0.970746(4)$ seen $(S_2) = 0.970620$ **and** $(S_4) = 0.970746$ **cao** [3]	M1 A1 A1	1.1 1.1 1.1	may see $M_1 = 2T_2 - T_1 = 0.9827938$ and $M_2 = 2T_4 - T_2 = 0.9738528$ followed by eg $\frac{2M_1 + T_1}{3}$ either value correct to 6 or more dp ignore incorrect subscripts
3	(b)		0.9707 or 0.9708 is possible since eg the Simpson's rule estimates appear to be converging (rapidly) **oe** [1]	B1	2.2b	0.971 is certain since S_2 and S_4 agree to this precision
3	(c)		supports Ali's statement **oe** because the trapezium rule **estimates are increasing oe** (so they are (probably) underestimates) [1]	B1	2.2b	allow eg "yes", "true" for supports Ali's statement; accept diagram as part of explanation need confirmation **and** justification
4	(a)		f(1) and f(1.5) are both positive **and** a sign change is needed **oe**	B1	2.4	allow eg both y-values are both positive or f(a) and f(b) are both positive so no change of sign do not allow just eg because there's no sign change
4	(b)	(i)	=3*E4^5 – 13*E4^2 + 11 **oe** [1]	B1	1.1	formula to evaluate f(1.2)

	Question		Answer	Marks	AO	Guidance
4	(b)	(ii)	$= (D4*E4 - C4*F4)/(D4 - F4)$ **oe** or $= (C4*F4 - D4*E4)/(F4 - D4)$ **oe**	M1	1.1	evidence of correct formula for secant method; allow eg omission of * or =
				A1	1.1	all correct
				[2]		
4	(c)		=IF(H4<0,G4,E4) **or** =IF(H4>0,E4,G4)	B1 **[1]**	1.1	
4	(d)		1.1164 seems justified by comparison of 1.116438 and 1.116420	B1 **[2]**	2.2b	allow by comparison of E12 (or G11) and G12 **or** by comparison of last two estimates
4	(e)		$\frac{dy}{dx} = 15x^4 - 26x^2$ must see $x - \frac{3x^5 - 13x^2 + 11}{15x^4 - 26x^2}$ completion to $x_{n+1} = \frac{12x_n^5 - 13x_n^2 - 11}{15x_n^4 - 26x_n}$.	B1 B1 **[2]**	2.1 2.1	may be embedded in formula intermediate step must be seen, may or may not have some or all subscripts all subscripts
4	(f)		$\boxed{\begin{array}{l} -1 \\ -0.8780488 \\ -0.859516 \\ -0.8591461 \\ -0.8591459 \\ -0.8591459 \end{array}}$ -0.8591459	M1 A1 **[2]**	3.5a 1.1	x_1, x_2 to 3 or more dp seen **NB** $x_1 = -\frac{36}{41}$

	Question		Answer	Marks	AO	Guidance
4	(g)		the convergence is faster than 1$^{\text{st}}$ order **or** the order of convergence is higher than first order	B1	2.2a	do not allow eg 2$^{\text{nd}}$ order; do not allow 'greater' or 'higher' for faster
			because the ratios are decreasing	B1 [2]	2.4	if **B0B0** allow **SC1** for ratios not constant so not first order convergence **oe**
5	(a)		$-1 < \mathrm{g}^{/}(\alpha) < 1$ (so will converge) **oe**	B1	1.2	must refer to gradient of curve at the root do not allow eg because it converges, eg the gradient is less than 1, eg $\mathrm{g}'(x) < 1$
			$\mathrm{g}^{/}(\beta) \geq 1$ (so won't converge) **oe** **or** $\mathrm{g}^{/}(\beta) > 1$ (so diverges)	B1 [2]	3.5b	must refer to gradient of curve at the root do not allow just eg because it diverges, just eg the gradient is greater than 1, eg $\mathrm{g}'(x) > 1$
5	(b)		$\begin{array}{l} 1 \\ 0.6666667 \\ 0.3772291 \\ 0.3358796 \\ 0.3347583 \\ 0.3347346 \\ 0.3347342 \\ 0.3347341 \\ 0.3347341 \end{array}$	M1	1.1	x_1, x_2 to 3 or more dp seen ; may see $\frac{2}{3}, \frac{275}{729}, ..$
			0.334734	A1 [2]	1.1	
5	(c)		the calculation for x_6 cannot be performed	B1	1.2	allow too large to be stored **oe**
				[1]		do not allow eg too large to be displayed

	Question		Answer	Marks	AO	Guidance
5	(d)		1.5 1.049688 1.145902 1.196715 1.211642 1.214231 1.214592 1.214641 1.214647 1.214648 1.214648	M1	1.1	x_1, x_2 to 3 or more dp seen; may see $\frac{3359}{3200}$ for x_2
			1.214648	A1 [2]	1.1	
6	(a)		$M_1 = 0.2924813$ $M_2 = 0.5 \times (0.1609640 + 0.4036775)$ $= 0.2823208$ Difference $= -0.0101606$	B1 M1 A1 A1 [4]	1.1 1.1 1.1 1.1	allow -0.0101605
6	(b)		M_{32} and M_{64} are stored to a greater accuracy in the spreadsheet than the accuracy to which they are displayed, and the spreadsheet uses the stored values, (not the displayed values, to compute the difference)	B1 [1]	2.4	must refer to **stored** values being more accurate than **displayed** values **and** for the calculation allow eg because the stored values used for the calculation are more accurate than those displayed

Question			Answer	Marks	AO	Guidance
6	(c)		$M_n + d \times \frac{r}{1-r}$ **or** $M_n + d \times r$	M1	3.1a	any M_n from table, any d from table, $r = 0.25$ or any r from table
			$0.2786561 - 0.00001101 \times \frac{0.25}{1-0.25}$	A1	1.1	M_{64} and -0.00001101 used
			or $0.2786561 - 0.00001101 \times 0.25$	A1	1.1	$r = 0.25$ or 0.2500993
						NB $0.27865334\ldots$ from partial extrapolation
			$0.278652(4\ldots)$	A1	1.1	may see 0.278652428 to 0.27865243
			0.27865 or 0.278652 is possible since extrapolation improves accuracy oe	A1	3.2a	allow 0.2787 is certain by comparison with M_{64} **A0A0** if partial extrapolation used
				[5]		
7	(a)		because the t-values are not evenly spaced	B1	2.4	condone because the x-values are not evenly spaced
				[1]		
7	(b)		$\frac{(t-6)(t-10)}{(0-6)(0-10)} \times 22 + \frac{t(t-10)}{(6-0)(6-10)} \times 8.824 + \frac{t(t-6)}{(10-0)(10-6)} \times 3$	M1	3.3	must be three terms; basic structure correct; do not allow substitution of eg y values for t
			$0.074t^2 - 2.64t + 22$	A1	1.1	two coefficients correct; allow eg x instead of t
				A1	1.1	all three correct, allow eg x instead of t
			$y = 0.074t^2 - 2.64t + 22$	A1	1.1	allow eg f(t) instead of y, may be seen earlier; must be in terms of t **NB** $a = \frac{37}{500}$
				[4]		
7	(c)		$0.074 \times 20^2 - 2.64 \times 20 + 22$ evaluated	M1	3.4	**FT** their 3 term quadratic
			-1.2 **cao** (which is not close to -4) so not a good fit	A1	2.2a	
				[2]		

	Question		Answer	Marks	AO	Guidance
7	(d)		$a = \dfrac{22}{(-6)\times(-10)\times(-20)} + \dfrac{8.824}{6\times(-4)\times(-14)} +$ $\dfrac{3}{10\times4\times(-10)} + \dfrac{-4}{20\times14\times10}$	M1	3.1a	must see evidence of Lagrange's formula; ignore use of wrong variable; allow one sign error and/or one coefficient error in substitution
			$b = \dfrac{-36\times22}{(-6)\times(-10)\times(-20)} + \dfrac{-30\times8.824}{6\times(-4)\times(-14)} +$ $\dfrac{-26\times3}{10\times4\times(-10)} + \dfrac{-16\times-4}{20\times14\times10}$	M1	2.1	
			$a = -0.001$	A1	1.1	both coefficients may be embedded in cubic formula
			$b = 0.09$	A1	1.1	
			Alternatively use of Lagrange formula to find cubic	M1		**NB** $\dfrac{(t-6)(t-10)(t-20)}{(0-6)(0-10)(0-20)} \times 22 + \dfrac{t(t-10(t-20)}{6(6-10)(6-20)} \times 8.824 +$ $\dfrac{t(t-6)(t-20)}{10(10-6)(10-20)} \times 3 + \dfrac{t(t-6)(t-10)}{20(20-6)(20-10)} \times (-4)$; allow one sign error and/or one coefficient error in substitution
			value of a **or** b found	M1		
			$a = -0.001$	A1		
			$b = 0.09$	A1		
				[4]		
7	(e)		$-0.001 \times 30^3 + 0.09 \times 30^2 - 2.7 \times 30 + 22$ $= -27 + 81 - 81 + 22 = -5$	B1	3.4	need to see evidence of substitution
				[1]		
7	(f)		$y = -0.001t^3 + 0.09t^2 - 2.7t + 22$ for $(0 <) \, t \le 30$ or for $(0 <) \, t < 30$ $(y =) -5$ for $t > 30$ **or** $(y =) -5$ for $t \ge 30$	B1	3.5c	
				[1]		

APPENDIX

Response	Mark

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