Oxford Cambridge and RSA

# Thursday 26 May 2022 – Afternoon

## AS Level Further Mathematics B (MEI)

### Y413/01 Modelling with Algorithms

**Time allowed: 1 hour 15 minutes**

*9450076125*

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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Answer **all** the questions.

**1** **(a)** **(i)** State the number of arcs in the complete graph with 6 nodes. **[1]**

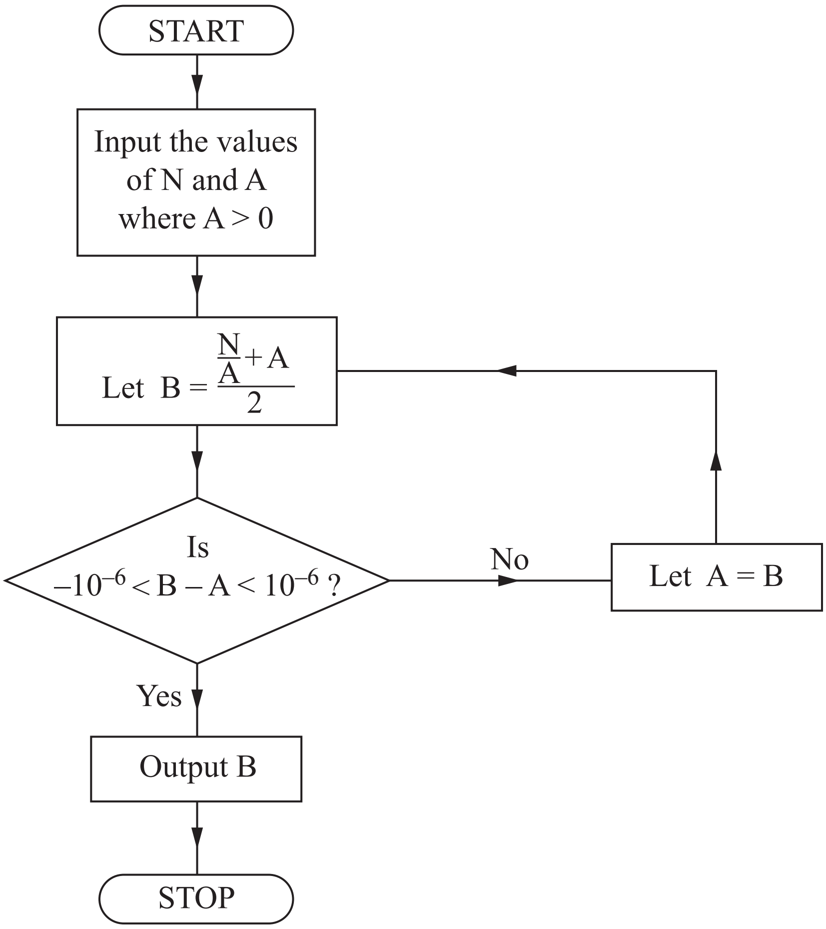
**(ii)** State the minimum number of arcs in a simply connected graph with 6 nodes. **[1]**

**(b)** **(i)** Using the nodes in the Printed Answer Booklet, draw the graph described by the incidence matrix below. **[1]**

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 1 & 2 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 3 & 1 & 0 \end{pmatrix} \end{array}$$

**(ii)** State the order of node C. **[1]**

**2** A process for finding a square root of the positive real number N is described by the flow chart below.




**(a)** Explain why the process described by the flow chart is an example of an algorithm. **[1]**

**(b)** Work through the algorithm using the inputs N = 73 and A = 8. Record the values of A and B, to at least **9** decimal places where necessary, every time they change.
Give the final output correct to **7** decimal places. **[3]**

**(c)** The inputs remain as N = 73 and A = 8. The box in the algorithm where B is defined needs adapting to ensure that the negative square root of 73 is the output.
Explain how to adapt the box. **[1]**

A student claims that if the statement $A > 0$ is removed from the algorithm, so that there is no longer a restriction on the value of A, the algorithm can still be used to find a square root of N.

**(d)** Explain whether the student's claim is correct. **[1]**

**3** In **Fig. 3** the weights of the arcs represent distances.

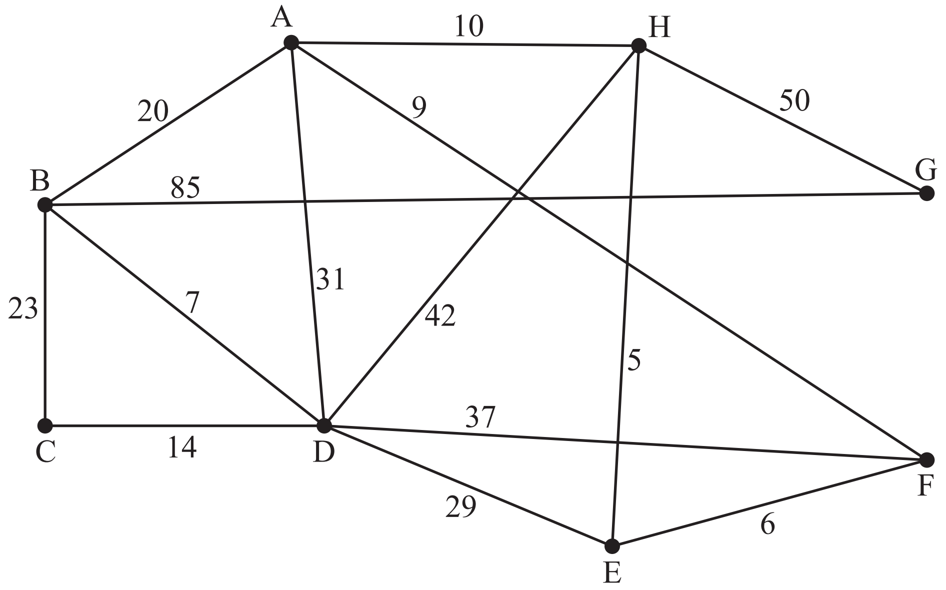



**Fig. 3**

Dijkstra's algorithm is to be used **once** to find both the shortest path from A to C **and** the shortest path from C to G.

**(a)** State which vertex should be chosen as the start vertex. **[1]**

**(b)** **(i)** On the copy of the network in the Printed Answer Booklet, apply Dijkstra's algorithm (with the starting vertex stated in part **(a)**) to find both the shortest path from A to C **and** the shortest path from C to G. **[5]**

**(ii)** State the weight of the shortest route from A to F via C. **[1]**

**(c)** Apply Prim's algorithm, starting at A, to find the minimum spanning tree for the network in **Fig. 3**.

- State the order in which the arcs were included in the tree.
- Draw the minimum spanning tree. **[3]**

**4** **Fig. 4.1** shows an activity network for a project. The arc weights show activity duration in hours.

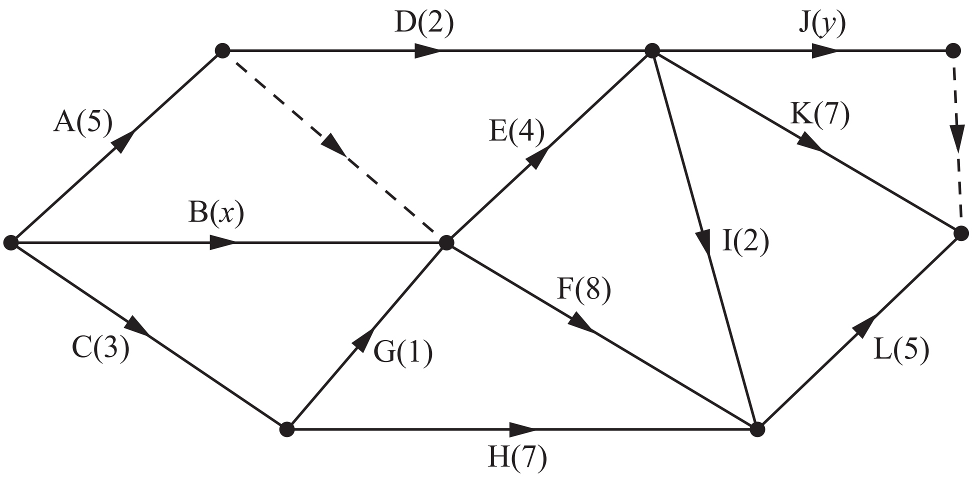



**Fig. 4.1**

**(a)** Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

It is given that the duration of activity B is $x$ hours, and the duration of activity J is $y$ hours where $x$ and $y$ are integers and

$0 < x < 5$ and $0 < y < 7$.

**(b)** Carry out a forward pass and a backward pass through the entire network to find the following.

- The minimum completion time for the project
- The critical activities **[5]**

It is given that the total float for activity J is 4 hours.

**(c)** Determine the value of $y$. **[1]**

Each activity requires one worker.

**Fig. 4.2** shows a partly completed resource histogram containing the eight activities A to H in which each of the eight activities begins at their earliest possible start time.

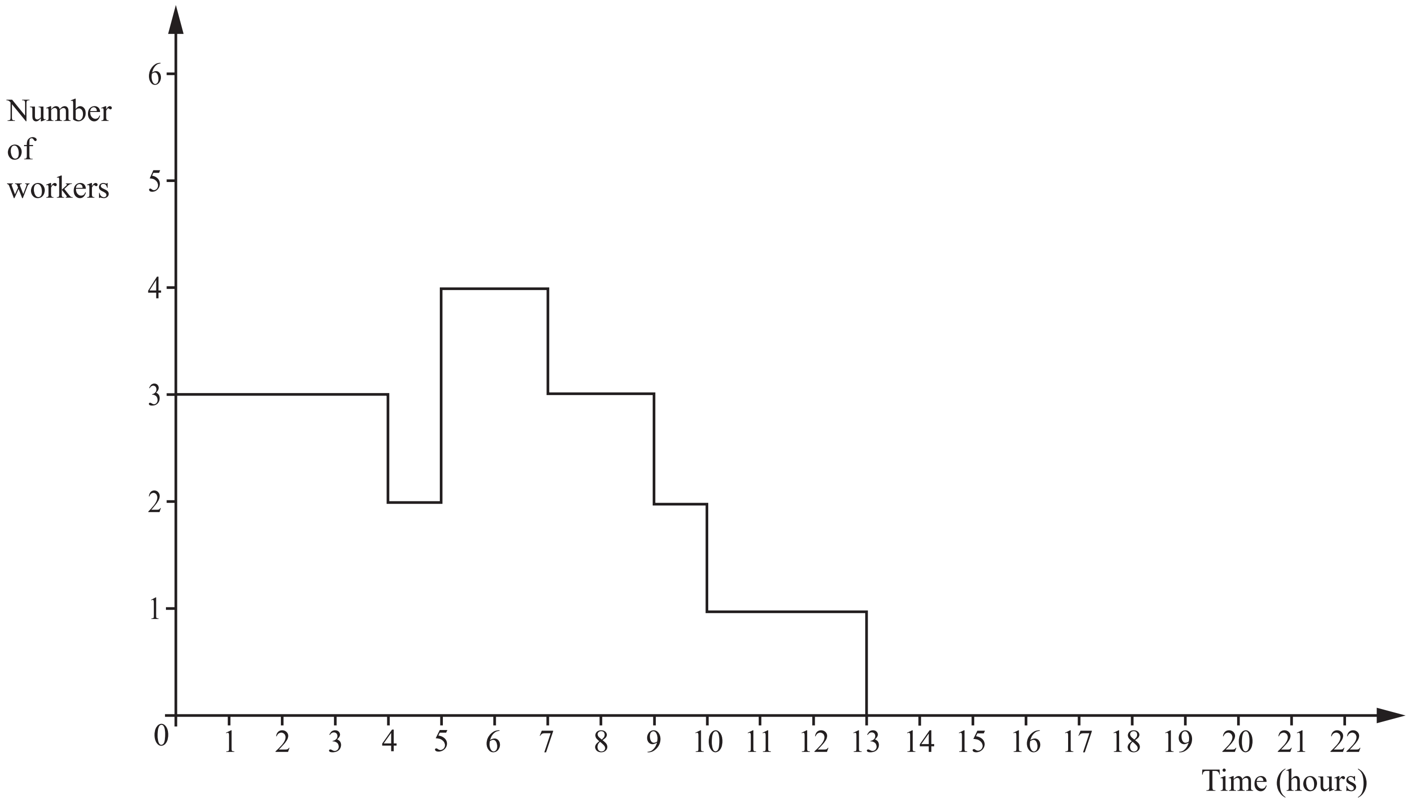



**Fig. 4.2**

**(d)** State the value of $x$. **[1]**

**(e)** Complete the resource histogram for the project by adding the remaining four activities I, J, K and L to the copy of **Fig. 4.2** in the Printed Answer Booklet.
Each of the four activities should begin at their earliest possible start time. **[2]**

**(f)** Draw a schedule to show how three workers can complete the project in the minimum completion time. Each box in the Printed Answer Booklet represents 1 hour.
For each worker, write the letter of the activity they are doing in each box, or leave the box blank if the worker is not required for that 1 hour. **[2]**

**5** **Fig. 5.1** represents a system of pipes through which a fluid flows continuously from a source S to a sink T. The weight on the arcs show the capacities of the pipes in litres per minute.

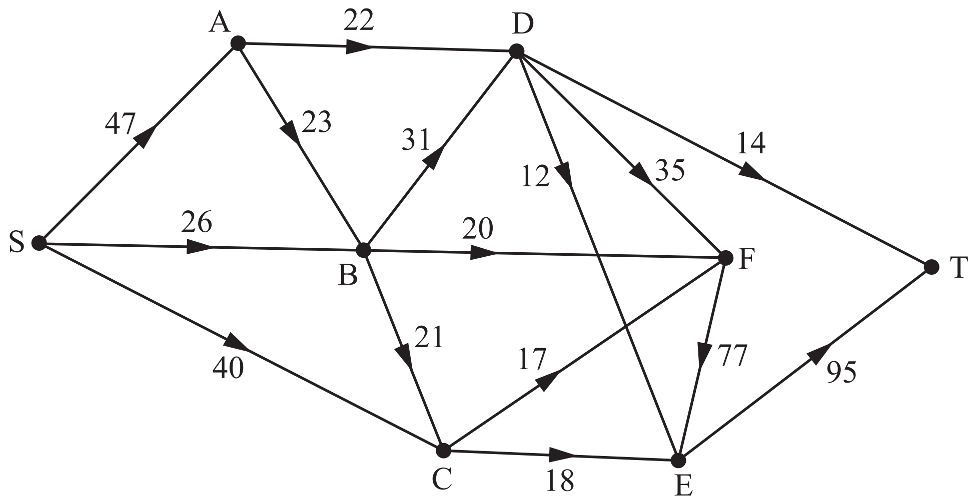



**Fig. 5.1**

**(a)** **(i)** The cut $\alpha$ partitions the vertices into the sets $\{S, A\}, \{B, C, D, E, F, T\}$.

Calculate the capacity of cut $\alpha$. **[1]**

**(ii)** The cut $\beta$ partitions the vertices into the sets $\{S, A, C, E\}, \{B, D, F, T\}$.

Calculate the capacity of cut $\beta$. **[1]**

**(b)** Using only the capacities of cuts $\alpha$ and $\beta$, explain what can be deduced about the maximum possible flow through the system. **[1]**

An LP formulation is set up to find the maximum flow through the network.

**(c)** Explain why a possible objective function for the LP formulation is to maximise SA + SB + CE + CF. **[1]**

**(d)** Write down the required constraint in the LP formulation regarding the flow through vertex F. **[1]**

The LP formulation for the network was run in a solver and some of the output is shown in **Fig. 5.2**.

| Variable | Value |
|---|---|
| SA | 45.000 000 |
| SC | 35.000 000 |
| BC | 0.000 000 |
| BD | 31.000 000 |
| DT | 11.000 000 |
| ET | 95.000 000 |
| FE | 70.000 000 |

**Fig. 5.2**

**(e)** Explain how the output in **Fig. 5.2** gives a flow of 106 litres per minute through the system of pipes. **[1]**

**(f)** Use the diagram in the Printed Answer Booklet to show how a flow of 106 litres per minute can be achieved. **[2]**

**(g)** Use a suitable cut to show that a flow of 106 litres per minute is the maximum possible flow through the system of pipes. **[2]**

**6** Each Monday morning a company has its weekly delivery of milk.
The milk comes in three types, whole, semi-skimmed and skimmed.

The company manager knows that each week she should order the following.

- At most 32 litres in total of semi-skimmed and skimmed milk.
- At least three times as much semi-skimmed milk as skimmed milk.

Furthermore, at least 10% of the milk should be skimmed milk.

The cost of one litre of whole milk is 55p, the cost of one litre of semi-skimmed milk is 50p, and the cost of one litre of skimmed milk is 40p.

In total the company has a budget of £50 to spend each week on milk.

Let $x$ represent the number of litres of whole milk.
Let $y$ represent the number of litres of semi-skimmed milk.
Let $z$ represent the number of litres of skimmed milk.

The company manager wants to maximise the total amount of milk ordered each week.

**(a)**

- Complete the initial tableau in the Printed Answer Booklet so that the simplex method may be used to solve this problem.
- Show how the constraints for the problem have been made into equations using slack variables. **[7]**

After two iterations of the simplex method a computer produces the tableau below.

| $P$ | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | RHS |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | $-\frac{10}{3}$ | 0 | 0 | $\frac{10}{3}$ | 1 | 0 | 0 |
| 0 | 0 | $\frac{4}{3}$ | 0 | 1 | $-\frac{1}{3}$ | 0 | 0 | 32 |
| 0 | 0 | $-\frac{1}{3}$ | 1 | 0 | $\frac{1}{3}$ | 0 | 0 | 0 |
| 0 | 1 | $-2$ | 0 | 0 | 3 | 1 | 0 | 0 |
| 0 | 0 | $\frac{104}{3}$ | 0 | 0 | $-\frac{107}{3}$ | $-11$ | 1 | 1000 |

**(b)** **(i)** Perform a third iteration of the simplex method. **[3]**

**(ii)** Explain how the answer to part **(b)(i)** shows that the solution obtained after the third iteration is optimal. **[1]**

**(c)** **(i)** State the number of litres of each type of milk the company manager should order each week. **[1]**

**(ii)** Calculate how much of the weekly milk budget will not be spent. **[1]**

Due to an increase in the amount of milk consumed, the manager believes that it may be possible, with a weekly budget of **at least** £50, to order exactly 40 litres in total of semi-skimmed and skimmed milk each week.

She still plans on ordering at least three times as much semi-skimmed milk as skimmed milk, and that at least 10% of the milk ordered should still be skimmed.

Furthermore, she still wishes to maximise the total amount of milk ordered each week.

**(d)** The two-stage simplex method is to be used to solve this modified problem.

- Formulate the modified constraints as equations.
- Define the new objective function.

In both cases, you are required to define the variables you use. Note that you do not need to re-state the original objective function or any constraints that are unchanged. **[4]**

**END OF QUESTION PAPER**