

# **GCE**

**Further Mathematics B (MEI)** 

Y413/01: MEI Modelling with algorithms

**AS Level** 

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## **Text Instructions**

# **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and <b>≭</b>	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
Е	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

# 2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

#### M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### В

Mark for a correct result or statement independent of Method marks.

### Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
  - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
  - We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
    - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
    - When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
      - NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g Rules for replaced work and multiple attempts:
  - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
  - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
  - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- i If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question	Answer	Marks	AO	Guidance
1 (a) (i)	15	<b>B1</b>	1.1	cao

Questi	on	Answer	Marks	AO	Guidance
			[1]		
	(ii)	5	B1	1.1	cao
			[1]		
(b)	(i)	E D C	B1	1.1	cao
			[1]		
	(ii)	8	B1	1.1	cao
			[1]		

	Questi	ion			Answer		Marks	AO	Guidance
2	(a)				_	of operations for solving the and so is an algorithm	B1	1.2	Correct definition of an algorithm – must include 'finite sequence of operations', 'finite set of instruction' oe but not just 'finite'
							[1]		
	(b)	$\begin{array}{ c c c c c c }\hline N & A & B & Is \\ & -10^{-6} < B - A < 10^{-6} \\ \hline 73 & 8 & 8.5625 \ (or \frac{137}{16}) & No \\ \hline & 8.5625 \ (or \frac{137}{16}) & 8.544023723 & No \\ \hline & 8.544023723 & 8.544003745 & No \\ \hline & 8.544003745 & 8.544003745 & Yes \\ \hline \end{array}$					M1	1.1	Initial values for A and B correct and each updated at least once (with B to at least 7 d.p.)  cao – table completely correct with values correct to at least 8 d.p. (where appropriate) oe. Ignore rows if completed after 'Yes' statement
			Outpu	at = 8.5440037			<b>A1</b>	1.1	Correct output (to exactly 7 d.p.) – independent of first A mark
							[3]		
	(c)		Chan	ge A to -A in the	e line 'Let $B = \frac{N}{A}$	+ A ,	B1	3.5c	oe e.g. change to '(Let B =) $\frac{-\frac{N}{A} - A}{2}$
							[1]		
	( <b>d</b> )		The s	tudent's claim is	incorrect as if A =	0 then B will be undefined	B1	2.3	Mention that A cannot take the value of zero
							[1]		

(	Questi	on	Answer	Marks	AO	Guidance
3	(a)		С	B1	1.1	
				[1]		
3	(b)	(i)	3     21       23     21       23     21       23     21       3     21       4     41       4     41       4     41       4     41       4     56       51     48       49     51       50     8       8     98       106     98    F  T  49  51  50  49	M1 A1 A1	1.2 1.1a 1.1a 1.1	Correct working values at B  Working values  Labels  Order of labelling  SC for starting at A or G only can score M1 A1 A0 A0 max. For example, starting at A, M1 for correct working values at D (31 27) and A1 for correct working values at all other nodes.  Starting at any other node is M0
			Shortest path from A to C: ABDC	<b>B</b> 1	2.2a	Both correct but not for CDBA
			Shortest path from C to G: CDEHG			
				[5]		
	<b>(b)</b>	(ii)	Weight is $41 + 49 = 90$	B1ft	3.1b	90 or follow through final values at A and F but
						no follow through if not starting at C in (b)(i)
				[1]		

Question	Answer	Marks	AO	Guidance
(c)	Prim's: AF, EF, EH, AB, BD, CD, GH	M1	1.1	First four arcs (or five nodes) chosen correctly in the correct order
		A1	1.1	cao (must be stated as arcs in the correct order) – M1 only if starting at a different node (than A) – explicit rejection of arcs scores M1 only
	$\bigcap_{E}^{A}$	B1	1.1	cao
		[3]		

	Question	n	Answer		Marks	AO	Guidance
4	(a)						
		Activity	Immediate Predecessor(s)				
		A	-				
		В	-				
		C	-				
		D	A		D1	1.1	A 5 A D C
		E	A, B, G		B1	1.1	Any 5 rows correct (not including rows A, B, C)
		F	A, B, G				
		G	C				
		Н	C		<b>B</b> 1	1.1	cao
		I	D, E		<b>D1</b>	1.1	Cuo
		J	D, E				
		K	D, E				
		L	F, H, I				
					[2]		
	<b>(b)</b>	5	5 D(2) 9 11 J(y)		3.54		
				9+y 18	M1	1.1	Forward pass - numbers increasing from
		A(5)	E(4) K(7)	¥			source to sink (allow one slip) – all complete
			B(x)				(condone crossed out values not replaced
		0 0	<b>↓</b> I(2)	18 18			especially at the event at the end of J)
		C(3	G(1) L(5	)	M1	1.1	Backward pass – numbers decreasing from
			3 4 H(7)		1,11	111	sink to source (allow one slip) – all complete
			13 13				- condone missing 0
					<b>A1</b>	1.1	cao for backward and forward pass – bod if
							9 + y seen and replaced with their value of $y$
			pletion time = 18 (hours)		B1	1.1	cao
		Critical activitie	es are A, F and L		B1	1.1	cao
					[5]		

Question	Answer	Marks	AO	Guidance
(c)	$18 - 9 - y = 4 \Rightarrow y = 5$	B1	3.1a	
		[1]		
(d)	x = 3	B1	2.2a	cao
		[1]		
(e)	Number of workers 5-  4- 3 2- 1- 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 Time (hours)	M1	1.1	Five workers in time interval 9 – 10 and correct number of workers from 14 – 18
		[2]		

(	Questi	ion	Answer	Marks	AO	Guidance
	<b>(f)</b>		e.g.			
			Worker 1 A A A A A F F F F F F			Three workers used at most, at least 10
			Worker 2 B B B H H H H H H H	M1	3.1a	activities placed, no activity finishing after
			Worker 3 C C C G E E E D D			the completion time for the entire project and at least 5 activities dealt with correctly
			11 12 12 14 15 16 17 18 18 20 21 22			
			11 12 13 14 15 16 17 18 19 20 21 22 Worker 1 F F L L L L L L	<b>A1</b>	1.1	Cao (oe e.g. D and E can be swapped)
			Worker 2 K K K K K K K			
			Worker 3 I I J J J J J			
				[2]		
5	(a)	(i)	$\alpha (= 22 + 23 + 26 + 40) = 111 (litres/min)$	B1	1.1	cao
				[1]		
		(ii)	$\beta$ (= 22 + 23 + 26 + 17 + 95) = 183 (litres/min)	B1	1.1	cao
				[1]		
	(b)		The maximum possible flow is at most 111 (litres/min)	B1ft	2.2a	Follow through least value from part (a) – just the value is sufficient
				[1]		
	(c)		All the flow must come from the source S and so therefore one	B1	2.4	Indication that all the flow (from S to T) must
			objective would be to maximise SA + SB + SC. However, as all the			pass through these four arcs – allow as a
			flow from SC must pass through arcs CE and CF then an equivalent			minimum 'the arcs SA, SB, CE, CF ({S, C},
			objective function would be to maximise SA + SB + CE + CF			{A, B, D, E, F, T}) form a cut' or a clear
						reason for using CE & CF rather than SC
				[1]		
	(d)		DF + BF + CF - FE = 0	B1	3.3	cao ( <b>B0</b> for DF + BF + CF – EF = 0)
				[1]		

Question	Answer	Marks	AO	Guidance
(e)	There are two arcs, DT and ET, that flow into the sink T and so the flow through the pipes (according to the output) is 11 + 95 = 106 (litres/min)	B1	2.4	Must mention that the 106 comes from DT (11) and ET (95)
		[1]		
(f)	A 22 D 45 23 31 7 35 11  S 26 B 18 F 70 95	M1	1.1	Consistent flow pattern (flow in = flow out at each node) – one number on each arc. SA = 45, SC = 35, BC = 0, BD = 31, DT = 11, ET = 95, FE = 70 and SB = 26 – condone lack of 0 on BC for M1 only  Condone incorrect flow across one vertex for the M mark  cao
		[2]		
(g)	The (capacity of the) cut which partitions the vertices into the sets {S, A, C}, {B, D, E, F, T} (is 22 + 23 + 26 + 17 + 18 = 106)	B1*	3.1b	Condone without S and/or T - allow listing of cut arcs (AD, AB, SB, BC, CF, CE)
	$\therefore$ minimum cut $\le 106$ so by the maximum flow-minimum cut theorem the maximum flow is equal to the minimum cut and so therefore the maximum flow through the system is 106 (litres per minute)	B1dep*	2.1	Requires correct cut, 106 stated and mention of max flow-min cut theorem
		[2]		

	Question Answer Ma									Marks	AO	Guidance		
6	(a)		<i>y</i> + <i>z</i> ≤									M1	3.1b	Any one correct equation or two correct
			$y \ge 3z$	$\Rightarrow -y +$	$3z \leq 0$ :	-y+3z	$z + s_2 =$	0					inequality constraints	
			$\frac{1}{10}(x+$							A1	1.1	Any two correct equations		
			$\frac{10}{10}(x+$	$y+z)\leq$	$\leq z \Rightarrow x$	+y-9z	$+ s_3 = 0$	)				A1	1.1	Any three correct equations
			0.55x +	-0.5y + 0.00	$0.4z \le 5$	i0⇒11∞	x+10y	$+8z+s_{4}$	=1000	)		A1	1.1	All four correct equations
			P = x +	y+z=	$\Rightarrow P - x$	-y-z=	= O					B1	3.1a	Can be implied by correct corresponding row in
														tableau
				1	1	1	Т	T	1	1		1		
			P	X	У	Z	$s_1$	$s_2$	$s_3$	$s_4$	RHS			
			1	-1	-1	-1	0	0	0	0	0	M1	3.3	Two constraint rows correct
			0	0	1	1	1	0	0	0	32			
			0	0	-1	3	0	1	0	0	0	A1	1.1	cao oe
			0	1	1	<b>-9</b>	0	0	1	0	0	111		
			0	11	10	8	0	0	0	1	1000			
												[7]		

Quest	ion					Answer	•				Marks	AO	Guidance
(b)	(i)	P	х	у	z	$s_1$	$s_2$	<i>s</i> <sub>3</sub>	$s_4$	RHS	M1	3.4	Pivot row and column correct (condone pivoting
		1	0	0	0	2.5	2.5	1	0	80	1411	3.4	on the 104/3 for this mark)
		0	0	1	0	0.75	-0.25	0	0	24			2 2 1
		0	0	0	1	0.25	0.25	0	0	8	<b>A1</b>	1.1	One of columns $s_1, s_2, s_3$ or RHS correct
		0	1	0	0	1.5	2.5	1	0	48	<b>A1</b>	1.1	cao
		0	0	0	0	-26	-27	-11	1	168			
											[3]		
<b>(b)</b>	(ii)			_		v	ective (to on is optin		nd so t	he	B1	2.2a	Must mention objective $\underline{row} - \mathbf{B0}$ for 'there are only positive values in the objective row'
											[1]		
(c)	(i)	Should of skim			f whole	e, 24 litro	es of semi	-skimm	ed and	8 litres	B1	3.2a	Must be in context (but litres not required)
											[1]		
(c)	(ii)	50-0.5	55(48) –	0.5(24)	-0.4(8	)=(£)8.	40				B1	3.4	Condone 8.4
											[1]		
(d)			$40 \Rightarrow y$ $s_5 = 40,$		-	$z + z \ge 40$ $= 40$	)				B1	3.1a	oe allow any alternative subscript (but the two $s$ 's must have different subscripts) – must be using $s$ and $a$ (any other letter must be defined)
		11x + 10	$0y+8z \ge$	≥1000=	$\Rightarrow 11x +$	10y + 8z	$-s_7 + a_2$	=1000			B1	1.1	oe
		$Q = a_1 + a_2 \Rightarrow Q + 11x + 11y + 9z - s_6 - s_7 = 1040$							B1	3.5c			
	Where $Q$ is the new objective to be minimised, $s_5$ is a slack variable,							B1	2.5	$s_i$ , $a_i$ must be consistent with earlier constraints –			
	$s_6, s_7$ are surplus variables, and $a_1, a_2$ are artificial variables							dependent on using $y + z = 40$ and their					
	(with no change to the constraints $x + y - 9z + s_3 = 0$ ,							$11x + 10y + 8z \ge 1000$					
	$-y+3z+s_2=0$ or the objective $P-x-y-z=0$ )												
											[4]		

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