Oxford Cambridge and RSA

Monday 16 May 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 **(a)** **(i)** Write the following simultaneous equations as a matrix equation.

$$x + y + 2z = 7$$
$$2x - 4y - 3z = -5$$
$$-5x + 3y + 5z = 13$$

[1]

(ii) Hence solve the equations. **[2]**

(b) Determine the set of values of the constant k for which the matrix equation

$$\begin{pmatrix} k+1 & 1 \\ 2 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 23 \\ -17 \end{pmatrix}$$

has a unique solution. **[3]**

2 **(a)** Show that the vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ is parallel to the plane $2x + y - 3z = 10$. **[3]**

(b) Determine the acute angle between the planes $2x + y - 3z = 10$ and $x - y - 3z = 3$. **[4]**

3 The complex number z satisfies the equation $5(z - \mathrm{i}) = (-1 + 2\mathrm{i})z^*$.

Determine z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real. **[5]**

4 **In this question you must show detailed reasoning.**

The equation $z^3 + 2z^2 + kz + 3 = 0$, where k is a constant, has roots α, $\frac{1}{\alpha}$ and β.

Determine the roots in exact form. **[6]**

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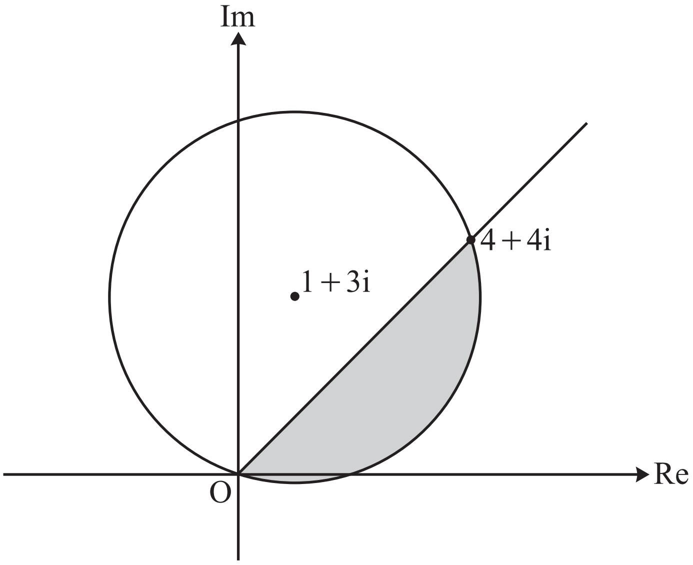
5 An Argand diagram is shown below. The circle has centre at the point representing $1+3\text{i}$, and the half line intersects the circle at the origin and at the point representing $4+4\text{i}$.



State the **two** conditions that define the set of complex numbers represented by points in the shaded segment, including its boundaries. **[5]**

6 **(a)** Using standard summation formulae, show that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$. **[4]**

(b) Use induction to prove the result in part **(a)**. **[6]**

7 On an Argand diagram, the point A represents the complex number z with modulus 2 and argument $\frac{1}{3}\pi$. The point B represents $\dfrac{1}{z}$.

(a) Sketch an Argand diagram showing the origin O and the points A and B. **[2]**

(b) The point C is such that OACB is a parallelogram. C represents the complex number w.

Determine each of the following.

- The modulus of w, giving your answer in exact form.
- The argument of w, giving your answer correct to **3** significant figures. **[7]**

8 A transformation T of the plane has matrix **M**, where $\mathbf{M} = \begin{pmatrix} \cos\theta & 2\cos\theta - \sin\theta \\ \sin\theta & 2\sin\theta + \cos\theta \end{pmatrix}$.

(a) Show that T leaves areas unchanged for all values of θ. **[2]**

(b) Find the value of θ, where $0 < \theta < \frac{1}{2}\pi$, for which the y-axis is an invariant line of T. **[4]**

The matrix **N** is $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(c) **(i)** Find $\mathbf{MN}^{-1}$. **[2]**

(ii) Hence describe fully a sequence of two transformations of the plane that is equivalent to T. **[4]**

END OF QUESTION PAPER

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