OCR
Oxford Cambridge and RSA

Friday 17 June 2022 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The points A, B and C have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix}$ respectively, relative to the origin O.

(a) Calculate, in its simplest exact form, the area of triangle OAB. **[3]**

(b) Show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$. **[3]**

2 The surface S has equation $z = x^3 + y^3 - 2x^2 - 5y^2 + 3xy$.

It is given that S has two stationary points; one at the origin, O, and the other at the point A.

Determine the coordinates of A. **[6]**

3 The sequence $\{U_n\}$ is given by $U_1 = 0$, $U_2 = -1$ and $U_{n+2} = U_{n+1} + U_n + n - 1$ for $n \geqslant 1$.

(a) List the first seven terms of this sequence. **[2]**

The Fibonacci sequence $\{F_n\}$ is given by $F_1 = 1$, $F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n \geqslant 1$.

(b) (i) By comparing the two sequences, give the relationship between U_n and F_n. **[2]**

(ii) Show that the relationship found in part **(b)(i)** holds for all $n \geqslant 1$. **[3]**

4 Let $N = 10a + b$ and $M = a + 3b$, where a and b are integers such that $a \geqslant 1$ and $0 \leqslant b \leqslant 9$.

(a) Prove that $29 \mid N$ if and only if $29 \mid M$. **[5]**

(b) Use an iterative method based on the result of part **(a)** to show that $899\,364\,472$ is a multiple of 29. **[3]**

5 A research student is using 3-D graph-plotting software to model a chain of volcanic islands in the Pacific Ocean. These islands appear above sea-level at regular intervals, (approximately) distributed along a straight line. Each island takes the form of a single peak; also, along the line of islands, the heights of these peaks decrease in size in an (approximately) regular fashion (see **Fig. 1.1**).

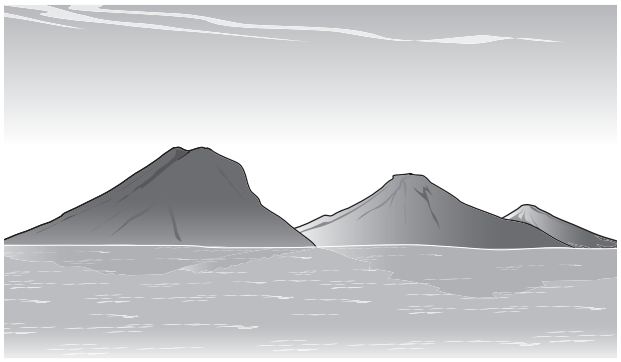


Fig. 1.1

The student's model uses the surface with equation $z = \sin x + \sin y$, a part of which is shown in **Fig. 1.2** below. The surface of the sea is taken to be the plane $z = 0$.

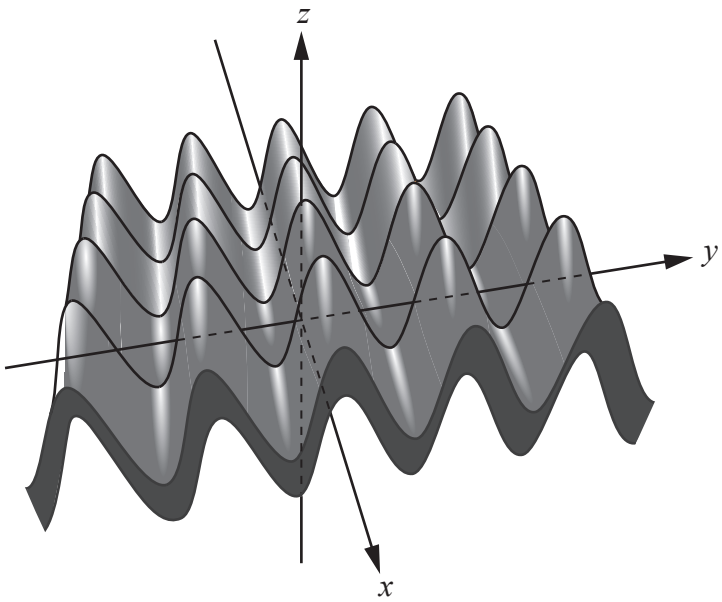


Fig. 1.2

(a)
- Describe **two** problems with this model.
- Suggest revisions to this model so that each of these problems is addressed. **[4]**

(b) Still using their original model, the student examines the contour $z = 2$ for their surface only to find that the software shows what appears to be an empty graph.

Explain what has happened. **[2]**

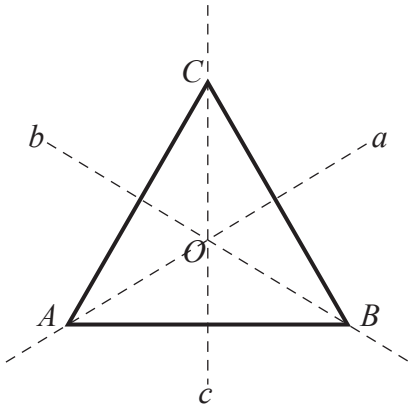
6 The sequence $\{u_n\}$ is such that $u_1 = 7$, $u_2 = 37$, $u_3 = 337$, $u_4 = 3337$, … .

(a) Write down a first-order recurrence system for $\{u_n\}$. **[2]**

(b) By solving the recurrence system of part **(a)**, show that $u_n = \frac{1}{3}(10^n + 11)$. **[5]**

(c) Prove that $\{u_n\}$ contains infinitely many terms which are multiples of 37. **[7]**

7 The diagram below shows an equilateral triangle *ABC*. The three lines of reflection symmetry of *ABC* (the lines *a*, *b* and *c*) are shown as broken lines. The point of intersection of these three lines, *O*, is the centre of rotational symmetry of the triangle.



The group D_3 is defined as the set of symmetries of *ABC* under the composition of the following transformations.

i : the identity transformation

a : reflection in line *a*

b : reflection in line *b*

c : reflection in line *c*

p : an anticlockwise rotation about *O* through 120°

q : a clockwise rotation about *O* through 120°

Note that the lines *a*, *b* and *c* are unaffected by the transformations and remain fixed.

(a) On the diagrams provided in the Printed Answer Booklet, show each of the six elements of D_3 obtained when the above transformations are applied to triangle *ABC*. **[3]**

(b) Complete the Cayley table given in the Printed Answer Booklet. **[3]**

(c) List all the proper subgroups of D_3. **[3]**

(d) State, with justification, whether D_3 is

(i) cyclic, **[1]**

(ii) abelian. **[1]**

(e) The group H, also of order 6, is the set of rotational symmetries of the regular hexagon. Describe **two structural** differences between D_3 and H. **[2]**

END OF QUESTION PAPER

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