GCE

Further Mathematics A

Y540/01: Pure Core 1

A Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

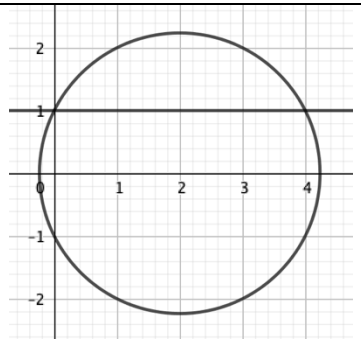
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

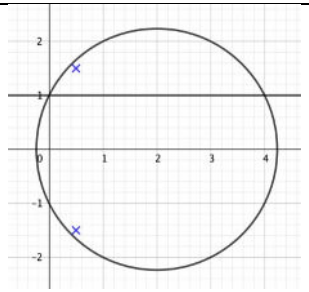
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		**DR** $\cosh(2\ln 3) = \dfrac{e^{2\ln 3} + e^{-2\ln 3}}{2}$ $= \dfrac{1}{2}\left(9 + \dfrac{1}{9}\right) = \dfrac{41}{9}$	**M1** **A1** **[2]**	**1.1** **2.1**	Correct use of definition of $\cosh x$ must be seen **AG**, must see either $e^{\ln 9}$ and $e^{\ln\frac{1}{9}}$ or 3^2 and 3^{-2} or $\dfrac{1}{2}\left(9 + \dfrac{1}{9}\right)$
	(b)		**DR** $V = \pi \int_0^{2\ln 3} \left(\sqrt{\sinh x}\right)^2 \mathrm{d}x$ $= \pi\left[\cosh x\right]_0^{2\ln 3}$ $= \pi\left(\cosh(2\ln 3) - \cosh 0\right)$ $= \pi\left(\dfrac{41}{9} - 1\right)$ $= \dfrac{32}{9}\pi \ \left(\text{cm}^3\right)$ **oe**	**M1** **A1** **M1** **A1** **[4]**	**3.3** **1.1** **3.4** **1.1**	oe, intention to integrate y^2. Condone missing π, ignore limits. For + $\cosh x$. Ignore any reference to c Substituting correct limits and subtracting Ignore units

Question			Answer	Marks	AO	Guidance
2	(a)		$\text{Det}A = 3\times 2 - -2\times 1 = 8$	**B1** **[1]**	**1.1**	
	(b)		$\frac{1}{8}\begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix}$	**B1** **[1]**	**1.1**	**ft** *their* Det A
	(c)		$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\Rightarrow x = \frac{1}{8},\ y = \frac{5}{8}$	**M1** **A1** **[2]**	**1.1** **1.1**	Sight of *their* A^{-1} multiplied by $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **ft** *their* $\mathbf{A^{-1}}$. Could be given as a vector.
	(d)		$\frac{1}{2}\begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix}$ **oe**	**B1** **[1]**	**2.2a**	**ft** their $\mathbf{A^{-1}}$. Accept $4\mathbf{A^{-1}}$.
	(e)		$DC = \begin{pmatrix} p \end{pmatrix}$ $CD = \begin{pmatrix} 0 & 4 & 2p \\ 0 & 0 & 0 \\ 0 & 2 & p \end{pmatrix}$	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	Must be a matrix; do not award for just *p*. for 3 × 3 matrix with at least one correct row or (non-0) column. **SC B2** for correct matrices, but **CD** and **DC** interchanged or unspecified **SC B1** for one correct matrix.
	(f)		Commutativity	**B1** **[1]**	**1.2**	Or the "commutative property". Accept "commutative". Allow also "non commutative"

Question			Answer	Marks	AO	Guidance
3	(a)		**DR** $z = \frac{2 \pm \sqrt{-36}}{4} \Rightarrow z = \frac{1}{2} \pm \frac{3}{2}\text{i}$ or $2\left(z^2 - z\right) = -5 \Rightarrow 2\left(z^2 - z + \frac{1}{4}\right) = -\frac{9}{2}$ $\left(z - \frac{1}{2}\right)^2 = -\frac{9}{4} \Rightarrow z - \frac{1}{2} = \pm\frac{3}{2}\text{i} \Rightarrow z = \frac{1}{2} \pm \frac{3}{2}\text{i}$	**M1** **A1** **[2]**	**1.1** **1.1**	Using correct quadratic formula with correct substitutions Or completing the square (Root of negative number must be seen) Final answer
	(b)	**(i)**	 	**B1** **B1** **B1** **[3]**	**1.1** **1.1** **2.2a**	Correct line $\text{Im}(z) = 1$ Circle centre 2 Circle intersect on imaginary axis at ±i
		(ii)	Major segment (area below line and inside circle) shaded.	**B1** **[1]**	**1.1**	**ft** *their* circle and *their* horizontal line from **(ii)**

Question			Answer	Marks	AO	Guidance
3	(c)	(i)	**DR** $\|z-2\| = \left\|-\frac{3}{2} \pm \frac{3}{2}\mathrm{i}\right\| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\pm\frac{3}{2}\right)^2}$ $= \sqrt{\frac{9}{2}} < \left(\sqrt{\frac{10}{2}}\right) = \sqrt{5}$	**M1** **A1** **[2]**	**2.1** **2.2a**	**ft** For calculating modulus of their $\|z-2\|$ for at least one of *their* values of z. **AG**. Must see clear statement of inequality (eg as shown or by values). Both roots If shown that $\|z-2\|^2 < 5$, must explain that $\|z-2\| < \sqrt{5}$ follows from $\|z-2\| \geq 0$.
		(ii)	$\frac{1}{2} - \frac{3}{2}\mathrm{i}$ because $\mathrm{Im}\left(\frac{1}{2} - \frac{3}{2}\mathrm{i}\right) = -\frac{3}{2} < 1$	**B1** **[1]**	**2.2a**	Or $\left\|\frac{1}{2} - \frac{3}{2}\mathrm{i}\right\| = \sqrt{\frac{5}{2}} < \left\|\frac{1}{2} - \frac{3}{2}\mathrm{i} - 2\mathrm{i}\right\| = \frac{5\sqrt{2}}{2}$ **ft** *their* roots as long as their roots are conjugate pairs with imaginary part of magnitude greater than one. Could be explained in terms of loci (ie "because (the point representing) $\frac{1}{2} - \frac{3}{2}\mathrm{i}$ is closer to O than it is to (the point representing) 2i") but must be consistent with their diagram after **(d)**. Or "below the perpendicular bisector" or "below axis" isw after acceptable answer
	(d)		 	**M1** **A1** **[2]**	**3.1a** **1.1**	For a complex conjugate pair. Approximate positions **inside** their circle above and below the line

Question			Answer	Marks	AO	Guidance
4			Direction of y-axis is $\begin{pmatrix}0\\1\\0\end{pmatrix}$	**B1**	**3.1a**	Correct direction vector representation of the y-axis.
			$\begin{pmatrix}1\\2\sqrt{3}\\-\sqrt{3}\end{pmatrix}.\begin{pmatrix}0\\1\\0\end{pmatrix}=2\sqrt{3}$	**M1**	**1.1**	Correct use of dot product with $\begin{pmatrix}1\\2\sqrt{3}\\-\sqrt{3}\end{pmatrix}$ and *their* direction vector for y-axis.**soi**
			$\cos\theta=\dfrac{2\sqrt{3}}{1\times\sqrt{1+\left(2\sqrt{3}\right)^2+\left(-\sqrt{3}\right)^2}}$ $=\dfrac{2\sqrt{3}}{4}$	**M1**	**1.1**	Correct use of dot product with *their* vectors to find cosine of angle **soi** . Condone eg.$\left(\sqrt{3}\right)^2$ in place of $\left(-\sqrt{3}\right)^2$.
			$\Rightarrow\theta=\dfrac{\pi}{6}$ or 30^0	**A1**	**1.1**	Or $\cos\varphi=-\dfrac{\sqrt{3}}{2}\Rightarrow\theta=\pi-\varphi=\dfrac{\pi}{6}$ or 30^0 Accept 0.524^c Mark the final answer SC B2 right answer only www
				[4]		

Question			Answer	Marks	AO	Guidance
5	(a)		**AG** $r = 3\left(1 - \frac{2xy}{r^2}\right)$ $\Rightarrow r^3 = 3\left(r^2 - 2xy\right) = 3\left(x-y\right)^2$ $\Rightarrow r^6 = 9\left(x-y\right)^4$ $\Rightarrow \left(x^2+y^2\right)^3 = 9\left(x-y\right)^4$	**M1** **A1** **A1** **[3]**	**1.1** **1.1** 	Fully shown (AG). Use of double angle formula and $y = r\sin\theta$, $x = r\cos\theta$ oe Any correct form of the equation with θ eliminated. Fully shown
	(b)		$\Rightarrow \left(y^2+x^2\right)^3 = 9\left(y-x\right)^4$ $\Rightarrow \left(x^2+y^2\right)^3 = 9\left(-\left(x-y\right)\right)^4$ $= 9\left(-1^4\right)\left(x-y\right)^4$ $\Rightarrow \left(x^2+y^2\right)^3 = 9\left(x-y\right)^4$	**M1** **A1**	**3.1a** **2.4**	Interchange of x and y Showing/explaining clearly that interchange of x and y leaves the equation unchanged.
			Alternative method $\sin 2\left(\frac{1}{4}\pi + \alpha\right) = \sin 2\left(\frac{1}{4}\pi - \alpha\right)$ or $\sin 2\left(\frac{5}{4}\pi + \alpha\right) = \sin 2\left(\frac{5}{4}\pi - \alpha\right)$ So $\theta = \frac{1}{4}\pi$ (and $\theta = \frac{5}{4}\pi$ are lines of symmetry). So $y = x$ is a line of symmetry.	**M1** **A1**		Showing that $\sin 2\theta$ is symmetrical about $\theta = \frac{1}{4}\pi$ (or $\theta = \frac{5}{4}\pi$). May be done graphically or using trig identities. so $r = 3(1 - \sin 2\theta)$ has lines of symmetry at $\theta = \frac{1}{4}\pi$ and $\theta = \frac{5}{4}\pi$ and the line $y = x$ has polar equation $\theta = \frac{1}{4}\pi$ and $\theta = \frac{5}{4}\pi$. Any mention of $\theta = \frac{5}{4}\pi$ is unnecessary since it is implied by the $\theta = \frac{1}{4}\pi$ case.
				[2]		

Question	Answer	Marks	AO	Guidance
(c)	**DR**	**B1**	**2.2a**	For identifying appropriate limits correctly
	$A=\frac{1}{2}\int_{\pi/4}^{5\pi/4} 9(1-\sin 2\theta)^2\,\mathrm{d}\theta$	**M1**	**1.1**	Use of $A=\frac{1}{2}\int \mathrm{r}^2\mathrm{d}\theta$
	$=\frac{1}{2}\int_{\pi/4}^{5\pi/4} 9(1-2\sin 2\theta+\sin^2 2\theta)\,\mathrm{d}\theta$	**M1***	**1.1**	Expand correctly and use of $\sin^2 2\theta=\frac{1}{2}(1-\cos 4\theta)$ Ignore limits
	$=\frac{1}{2}\int_{\pi/4}^{5\pi/4} 9\left(1-2\sin 2\theta+\frac{1}{2}(1-\cos 4\theta)\right)\mathrm{d}\theta$			
	$=\frac{9}{2}\left[\frac{3}{2}\theta+\cos 2\theta-\frac{1}{8}\sin 4\theta\right]_{\pi/4}^{5\pi/4}$	**M1dep**	**1.1**	Integrate
	$=\frac{9}{2}\left(\frac{3}{2}\left(\frac{5\pi}{4}-\frac{\pi}{4}\right)+0+0\right) \quad =\frac{27\pi}{4}$	**M1**	**3.1a**	Substituting correct limits and subtracting
	Other loop is the same	**A1**	**2.2a**	Answer for each loop which must be stated
	Alternative method for last 2 marks			
	Using limits 0 to 2π	**M1**		Substituting correct limits and subtracting
	$=\frac{9}{2}\left[\frac{3}{2}\theta+\cos 2\theta-\frac{1}{8}\sin 4\theta\right]_{0}^{2\pi}$			
	$=\frac{9}{2}(3\pi+0+0) \quad =\frac{27\pi}{2}$ for both loops	**A1**		Answer for each loop which must be stated
	$\Rightarrow \frac{27\pi}{4}$ for one loop by symmetry in $y=x$			
		[6]		

Question			Answer	Marks	AO	Guidance
6			Base case: $y = x\cosh x \Rightarrow \frac{dy}{dx} = \cosh x + x\sinh x$ $= (2\times 1 - 1)\cosh x + x\sinh x$ So true for $n = 1$	**B1**	**2.5**	Evidence of correct differentiation using product rule and a substitution $n = 1$ into RHS
			Assume result holds for $n = k$ $\frac{d^{2k-1}y}{dx^{2k-1}} = x\sinh x + (2k-1)\cosh x$	**M1***	**2.1**	
			$\Rightarrow \frac{d^{2k}y}{dx^{2k}} = \sinh x + x\cosh x + (2k-1)\sinh x$	**M1dep**	**1.1**	Differentiate using the product rule
			$\Rightarrow \frac{d^{2k+1}y}{dx^{2k+1}} = \cosh x + \cosh x + x\sinh x + (2k-1)\cosh x$ $= x\sinh x + (2k+1)\cosh x$	**M1dep**	**3.1a**	Differentiate second time using the product rule
			i.e. $\frac{d^{2(k+1)-1}y}{dx^{2(k+1)-1}} = x\sinh x + (2(k+1)-1)\cosh x$	**A1***	**2.2a**	Must be equated to correct derivative form
			So if true for $n = k$ then also true for $n = k + 1$ But it is true for n= 1 and so is true generally	**A1dep** **[6]**	**2.4**	Dependent on previous A1 and the B1

Question			Answer	Marks	AO	Guidance
7	(a)		$x^2+18 \equiv Ax\left(x^2+9\right)+B\left(x^2+9\right)+\left(Cx+D\right)x^2$	**B1**	**1.1**	Correct multiplying out of fractions
			e.g. $x=0 \Rightarrow 9B=18 \Rightarrow B=2$ $x=1 \Rightarrow 10A+10B+C+D=19$ $x=-1 \Rightarrow -10A+10B-C+D=19$ $\Rightarrow 10B+D=19 \Rightarrow D=-1$ $x=3\text{i} \Rightarrow -9\left(D+3C\text{i}\right)=9 \Rightarrow C=0$ $\Rightarrow 10A+20-1=19 \Rightarrow A=0$	**M1** **A1**	**1.1** **1.1**	Any substitutions to get a set of (at least) four simultaneous equations solvable for A, B, C and D. Or equating coefficients which gives $A+C=0, B+D=1$, $9A=0, 9B=18$. Any two coefficients correct.
			i.e. $A=0$, $B=2$, $C=0$, $D=-1$	**A1** **[4]**	**1.1**	All four coefficients correct. SC B1 after M0 if one or more coefficients are correct.
	(b)		**DR** $\int\left(\frac{2}{x^2}-\frac{1}{x^2+9}\right)\text{d}x=-\frac{2}{x}-\frac{1}{3}\tan^{-1}\frac{x}{3}\ (+c)$	**M1** **A1**	**1.1** **1.1**	integration including a $\tan^{-1}$ term **ft** *their* part (a)
			$\Rightarrow \int_3^{\infty}\left(\frac{2}{x^2}-\frac{1}{x^2+9}\right)\text{d}x$	**M1**	**1.1**	Use of limiting process on *their* integrated function**.** Ignore notation for limits
			$=\lim_{k\to\infty}\left(-\left(\frac{2}{k}-\frac{2}{3}\right)-\frac{1}{3}\left(\tan^{-1}\frac{k}{3}-\tan^{-1}1\right)\right)$	**A1**	**2.1**	for $\lim_{k\to\infty}\left(\frac{1}{k}\right)=0$, or as $k\to\infty$, $\frac{1}{k}\to 0$, **A0** for eg. $\frac{1}{\infty}=0$.
			$=\frac{2}{3}-\lim_{k\to\infty}\frac{2}{k}+\frac{\pi}{12}-\frac{1}{3}\lim_{k\to\infty}\left(\tan^{-1}\frac{k}{3}\right)$ $=\frac{2}{3}-0+\frac{\pi}{12}-\frac{1}{3}\times\frac{\pi}{2}$	**A1**	**2.1**	$\lim_{k\to\infty}\left(\tan^{-1}\frac{k}{c}\right)=\frac{1}{2}\pi$, or as $k\to\infty$, $\tan^{-1}\frac{k}{c}\to\frac{1}{2}\pi$, **A0** for eg.$\tan^{-1}\infty=\frac{1}{2}\pi$. In both cases must see some evidence of the limiting process.
			$=\frac{2}{3}-\frac{\pi}{12}$	**A1** **[6]**	**1.1**	

Question			Answer	Marks	AO	Guidance
8	(a)	(i)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -2\frac{\mathrm{d}x}{\mathrm{d}t} + 78(2x - 78y)$ $= -2\frac{\mathrm{d}x}{\mathrm{d}t} + 156x - 78\left(\frac{\mathrm{d}x}{\mathrm{d}t} + 2x - k\right)$ $\Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -80\frac{\mathrm{d}x}{\mathrm{d}t} + 78k$ $\Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 80\frac{\mathrm{d}x}{\mathrm{d}t} = 78k$	**M1** **A1** **[2]**	**3.3** **1.1**	Differentiate $\frac{\mathrm{d}x}{\mathrm{d}t}$ and substitute $\frac{\mathrm{d}y}{\mathrm{d}t}$ **AG** convincingly shown after second substitution.
		(ii)	$\lambda^2 + 80\lambda = 0$ $\Rightarrow \lambda = 0, -80$ Complementary function is $x = A + B\mathrm{e}^{-80t}$ Trial function is $x = at$ (+ b) $\frac{\mathrm{d}x}{\mathrm{d}t} = a,\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 0$ $\Rightarrow 80a = 78k \Rightarrow a = \frac{39}{40}k$ $\Rightarrow$ GS is $x = A + B\mathrm{e}^{-80t} + \frac{39}{40}kt$	**M1** **A1** **M1** **A1**	**1.1** **2.2a** **3.1a** **1.1**	Attempt to solve auxiliary equation for their DE Correct (or recovered) trial function for their CF. For GS

Question	Answer	Marks	AO	Guidance
	Alternative method for first 4 marks:			
	$\dot{x} + 80x = 78kt + c$	**M1**		Integrating $\ddot{x} + 80\dot{x} = 78k$ wrt t and then using integrating factor
	$e^{\int 80dt}(\dot{x} + 80x) = (78kt + c)e^{\int 80dt}$			
	$\frac{d}{dt}(e^{80t}x) = (78kt + c)e^{80t}$			
	$e^{80t}x = \int (78kt + c)e^{80t}dt + d$	**M1**		Integrating wrt t
	$e^{80t}x = \frac{(78kt + c)e^{80t}}{80} - \int \frac{78k}{80}e^{80t}dt + d$	**M1**		Integrating by parts ($78kt + c$ may be separated)
	$e^{80t}x = \frac{(78kt + c)e^{80t}}{80} - \frac{39k}{3200}e^{80t} + d$			
	$x = \frac{39}{40}kt + \frac{c}{80} - \frac{39k}{3200} + de^{-80t}$ $= A + Be^{-80t} + \frac{39}{40}kt$	**A1**		GS of form $x = A + Be^{-80t} + \frac{39}{40}kt$

Question			Answer	Marks	AO	Guidance
			When $t = 0, x = 0 \Rightarrow A + B = 0$	**M1**	**3.3**	Using $t = 0$ and $x = 0$ in *their* GS to find an equation in A and B.
			$\frac{dx}{dt} = -80Be^{-80t} + \frac{39}{40}k$ When $t = 0, x = 0, y = 0 \Rightarrow \frac{dx}{dt} = 0 + 0 + k$ $\Rightarrow k = -80B + \frac{39}{40}k$ $\Rightarrow B = -\frac{1}{3200}k, A = \frac{1}{3200}k$	**M1**	**3.1a**	Differentiating and using $t = 0, x = 0, (y = 0), \dot{x} = k$ to find another equation in (A and) B. Could also use $\dot{x} + \dot{y} = k$ and $x + y = kt$.
			Particular solution: $x = \frac{k}{3200}\left(1 - e^{-80t} + 3120t\right)$	**A1**	**3.3**	N.B. $\frac{3120}{3200} = \frac{39}{40}$
			Alternative method for this M mark: $78y = \dot{x} + 2x - k$ $= -78Be^{-80t} - \frac{1}{40}k + 2A + \frac{78}{40}kt$ When $t = 0, x = 0, y = 0$ $0 = 2A - 78B - \frac{1}{40}k$	**M1**		Finding GS for y and using $x = 0, y = 0$ to find another equation for A and B
				[7]		
		(iii)	When $t = 50$, $x > \frac{156000k}{3200}$ $\left(\text{because } e^{-4000} < 1 \text{ so } 1 - e^{-4000} > 0\right)$ i.e. $x > 48.75k > 292.5$ for $k > 6$ i.e. $x > 250$ so fails safety food standards	**B1** **[1]**	**3.4**	Use of $t = 50$ must be seen to give a term in k Must reference $k > 6$. Condone the idea of ignoring exponential term as negligible Sight of 292.5 >250 earns the mark

Question	Answer	Marks	AO	Guidance
(b)	$\sqrt{1677} = 40.95... < 41$ $\Rightarrow e^{-41t}\left(a\cosh\left(\sqrt{1677}t\right)+b\sinh\left(\sqrt{1677}t\right)\right)$ contains only negative exponentials when expanded so as $t \to \infty$, the exponential/hyperbolic parts $\to 0$ so $x \to 20k < 240 < 250$ since $k < 12$ so yes, food safety standards are met in the long run.	**M1** **A1** **[2]**	**3.4** **3.2a**	Turning function into exponentials. Could see eg $e^{-0.0488t}$ and $e^{-81.95t}$. Condone error(s) in coefficients. Allow argument such as e^{-41t} dominates Argument must be complete and correct but could be based on sufficiently large values of t rather than formal limiting process. SC B1 argument that $20k = 240 < 250$ if 2nd term is assumed to tend to 0
(c)	Pesticide is likely to be added periodically, eg, during the day; or depending on the weather/time of year; or only when it's needed; or the amount of pesticide added changes as the amount of crop changes/grows or if it is subject to pest attack	**B1** **[1]**	**3.5a**	The idea that regularly is not the same as continuously (and may not even mean "at a constant average rate"). Other sensible answers possible, but must be in context.

Question			Answer	Marks	AO	Guidance
9			Vertices of ABC satisfy $z^3 - 1$ (= 0)	**B1**	**1.1**	Or $(z-1)\left(z-e^{\frac{2}{3}\pi i}\right)\left(z-e^{\frac{4}{3}\pi i}\right)$(= 0) Or all three values stated
			(Complex number represented by) $M = -\frac{1}{2}$	**B1**	**3.1a**	Or one of $M=\frac{1}{2}e^{\pi i}, L=\frac{1}{2}e^{\frac{1}{3}\pi i}, N=\frac{1}{2}e^{-\frac{1}{3}\pi i}$
			Vertices of LMN satisfy $8z^3+1\ (=0)$	**M1**	**3.1a**	Attempt at polynomial relating to LMN. $\left(z-\frac{1}{2}e^{\pi i}\right)\left(z-\frac{1}{2}e^{\frac{1}{3}\pi i}\right)\left(z-\frac{1}{2}e^{-\frac{1}{3}\pi i}\right)$ suffices for this mark.
			$(z^3 - 1)(8z^3 + 1)$ (=0)	**M1**	**2.1**	Attempt product of their two cubic factors.
			$8z^6 - 7z^3 - 1 = 0$	**A1**	**2.2a**	**A0** without justification of $8z^3 + 1 = 0$. Must see = 0.
				[5]		
			Alternative method			
			$A(1), B\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right), C\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$	**B1**		Finding A, B, C
			$L\left(\frac{1}{4}+\frac{\sqrt{3}}{4}i\right), N\left(\frac{1}{4}-\frac{\sqrt{3}}{4}i\right), M\left(-\frac{1}{2}\right)$	**B1**		Finding L, M, N
			Quadratic satisfying B, C $z^2+z+1=0$ Quadratic satisfying L,N $4z^2-2z+1=0$ Quadratic satisfying A, M $2z^2-z-1=0$	**M1**		Combining to give a 6th degree polynomial Multiplying out some terms to give quadratics or cubics
			Eqn satisfying all 6 points $(z^2+z+1)(4z^2-2z+1)(2z^2-z-1)=0$	**M1**		Must see = 0
			$\Rightarrow 8z^6-7z^3-1=0$	**A1**		

Question			Answer	Marks	AO	Guidance
			Alternative methods By rotational symmetry Because $\left(e^{\frac{2}{3}\pi i}\right)^3 = 1$ Calculation of vertices of L, M and N. Use of $1 + \omega + \omega^2 = 0$ (and $\omega^3 = 1$) to simplify eg $(z - ½(\omega + 1))(z - ½(\omega^2 + 1))\ ))(z - ½(\omega + \omega^2))$ $= (z + ½\omega^2))(z + ½\omega)\ ))(z + ½)$ etc or to find sum/product of roots.			

APPENDIX

Mark scheme for Q8(a)(ii) if candidate solves for y, rather than x.

8	**(a)**	**(ii)**	$\ddot{y} = 2\dot{x} - 78\dot{y} = 2(-2x + 78y + k) - 78\dot{y}$ $\ddot{y} = -2(\dot{y} + 78y) + 156y + 2k - 78\dot{y}$ $\ddot{y} + 80\dot{y} = 2k$			
			$\lambda^2 + 80\lambda = 0$	**M1**	**3.1a**	Auxiliary equation for their DE
			$\lambda = 0, -80$			
			Complementary function is $y = A + B\mathrm{e}^{-80t}$	**M1**	**3.3**	
			Trial function is y= at	**M1**	**2.2a**	Correct trial function for their CF
			$\dot{y} = a, \ddot{y} = 0$ So $80a = 2k \Leftrightarrow a = \frac{1}{40}k$			Note the GS for x in the main mark scheme achieves **A1** but here **A1** is not awarded until GS for x found.
			General solution is $y = A + B\mathrm{e}^{-80t} + \frac{1}{40}kt$			
			Alternative method			
			$\dot{y} + 80y = 2kt + c$	**M1**		Integrating $\ddot{y} + 78\dot{y} = 2k$ wrt t and then using integrating factor
			$\mathrm{e}^{\int 80\mathrm{d}t}(\dot{y} + 80y) = (2kt + c)\mathrm{e}^{\int 80\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^{80t}y) = (2kt + c)\mathrm{e}^{80t}$			
			$\mathrm{e}^{80t}y = \int (2kt + c)\mathrm{e}^{80t}\mathrm{d}t + d$	**M1**		Integrating wrt t
			$\mathrm{e}^{80t}y = \frac{(2kt + c)\mathrm{e}^{80t}}{80} - \int \frac{2}{80}k\mathrm{e}^{80t}\mathrm{d}t + d$	**M1**		Integrating by parts
			$\mathrm{e}^{80t}y = \frac{(2kt + c)\mathrm{e}^{80t}}{80} - \frac{1}{3200}k\mathrm{e}^{80t} + d$ $y = \frac{1}{40}kt + \frac{1}{80}c - \frac{1}{3200}k + d\mathrm{e}^{-80t}$ $= A + B\mathrm{e}^{-80t} + \frac{1}{40}kt$			

			$\dot{y} = -80B\mathrm{e}^{-80t} + \frac{1}{40}k$ $x = \frac{1}{2}(\dot{y} + 78y)$ $= 39A - B\mathrm{e}^{-80t} + \frac{39}{40}kt + \frac{1}{80}k$	**A1**	**1.1**	GS of form $x = C + D\mathrm{e}^{-80t} + \frac{39}{40}kt$
			When $t = 0, x = 0, y = 0,\ \dot{y} = 0 + 0 = 0$ $0 = A + B$ $0 = 39A - B + \frac{1}{80}k$ $0 = -80B + \frac{1}{40}k$	**M1** **M1**	**3.1a** **1.1**	Using two of $x = y = \dot{y} = 0$ to find simultaneous equations for A and B (**M1** for each equation).
			$B = \frac{1}{3200}k\ , A = -\frac{1}{3200}k$ Particular solution for x is $x = -\frac{39}{3200}k - \frac{1}{3200}k\mathrm{e}^{-80t} + \frac{39}{40}kt + \frac{1}{80}k$ $= \frac{1}{3200}k(1 - \mathrm{e}^{-80t} + 3120t)$	**A1** **[7]**	**3.3**	

Another alternative method for Q8(a)(ii) for first 4 marks

8	**(a)**	**(i)**	D.E. is $\frac{\mathrm{d}y}{\mathrm{d}t}+80y=78k$ where $y=\frac{\mathrm{d}x}{\mathrm{d}t}$ I.F. $\mathrm{e}^{80t} \Rightarrow \mathrm{e}^{80t}\frac{\mathrm{d}y}{\mathrm{d}t}+80\mathrm{e}^{80t}y=78k\mathrm{e}^{80t}$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{80t}y\right)=78k\mathrm{e}^{80t}$ $\Rightarrow \mathrm{e}^{80t}y=\frac{39k}{40}\mathrm{e}^{80t}+A \Rightarrow y=\frac{39k}{40}+A\mathrm{e}^{-80t}$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{39k}{40}+A\mathrm{e}^{-80t} \Rightarrow x=\frac{39kt}{40}-\frac{A}{80}\mathrm{e}^{-80t}+B$	**M1** **A1** **M1** **A1**		

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