Oxford Cambridge and RSA

Wednesday 15 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Newton's gravitational constant, G, is approximately $6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$.

(a) Find the dimensions of G. **[2]**

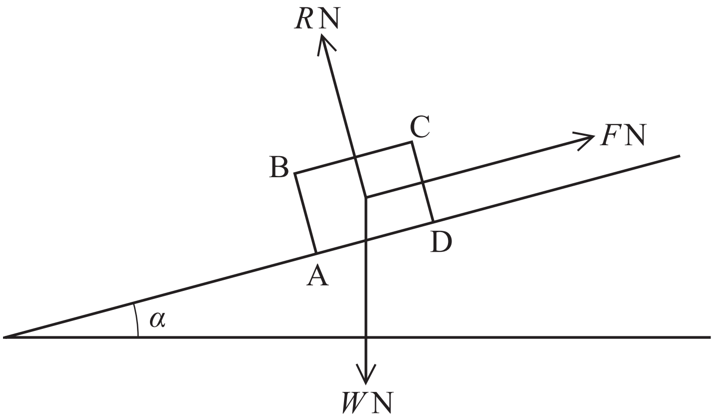
The escape velocity, v, of a body from a planet's surface, is given by the formula

$v = kG^{\alpha}M^{\beta}r^{\gamma}$,

where M is the planet's mass, r is the planet's radius and k is a dimensionless constant.

(b) Use dimensional analysis to find α, β and γ. **[4]**

2 The diagram below shows the cross-section through the centre of mass of a uniform block of weight W N, resting on a slope inclined at an angle α to the horizontal. The cross-section is a rectangle ABCD. The slope exerts a frictional force of magnitude F N and a normal contact force of magnitude R N.



(a) Explain why a triangle of forces may be used to model the scenario. **[2]**

(b) In the space provided in the Printed Answer Booklet, draw such a triangle, fully annotated, including the angle α in the correct position. **[2]**

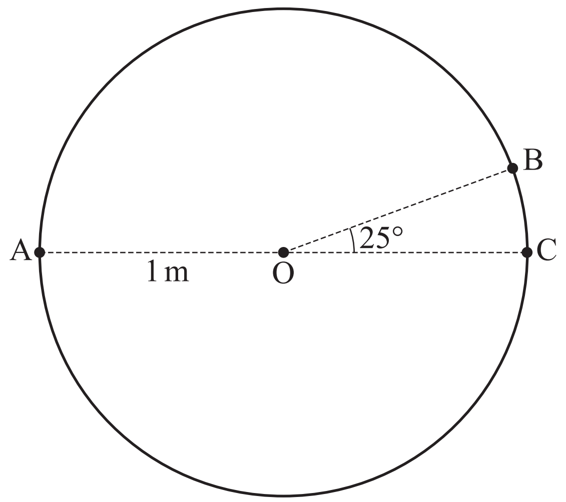
The coefficient of friction between the block and the slope is μ.

(c) Given that the block is in limiting equilibrium, use your diagram in part **(b)** to show that $\mu = \tan\alpha$. **[2]**

It is given that $AB = 8.9$ cm and $AD = 11.6$ cm. The coefficient of friction between the slope and the block is 1.35. The slope is slowly tilted so that α increases.

(d) Determine whether the block topples first without sliding or slides first without toppling. **[2]**

3 A rough circular hoop, with centre O and radius 1 m, is fixed in a vertical plane. A, B and C are points on the hoop such that A and C are at the same horizontal level as O, and OB makes an angle of 25° above the horizontal, as shown in the diagram.



A bead P of mass 0.3 kg is threaded onto the hoop. P is projected vertically downwards from A on two separate occasions.

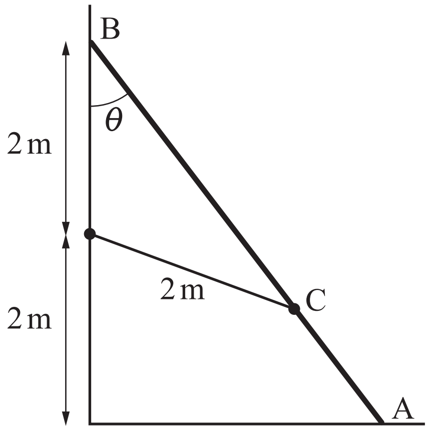
- The first time, when P is projected with a speed of $4\,\mathrm{m\,s^{-1}}$, it first comes to rest at B.
- The second time, when P is projected with a speed of $v\,\mathrm{m\,s^{-1}}$, it first comes to rest at C.

The situation is modelled by assuming that during the motion of P the magnitude of the frictional force exerted by the hoop on P is constant.

(a) Determine the value of v. **[5]**

(b) Comment on the validity of the modelling assumption used in this question. **[1]**

4 A uniform beam AB of mass 6 kg and length 5 m rests with its end A on smooth horizontal ground and its end B against a smooth vertical wall. The vertical distance between the ground and B is 4 m, and the angle between the beam and the downward vertical is θ. To prevent the beam from sliding, one end of a light taut rope of length 2 m is attached to the beam at C and the other end of the rope is attached to a point on the wall 2 m above the ground, as shown in the diagram.



(a) By considering the value of $\cos\theta$, determine the distance BC. **[2]**

An object of mass 75 kg is placed on the beam at a point which is x m from A.

It is given that the tension in the rope is T N and the magnitude of the normal contact force between the ground and the beam is R N.

(b) By taking moments about B for the beam, show that $25R + 3675x - 16T = 19110$. **[4]**

(c) Given that the rope can withstand a maximum tension of 720 N, determine the largest possible value of x. **[4]**

5 Point A lies 20 m vertically below a point B. A particle P of mass $4m$ kg is projected upwards from A, at a speed of $17.5\,\mathrm{m\,s^{-1}}$. At the same time, a particle Q of mass m kg is released from rest at point B. The particles collide directly, and it is given that the coefficient of restitution in the collision between P and Q is 0.6.

(a) Show that, immediately after the collision, P continues to travel upwards at $0.7\,\mathrm{m\,s^{-1}}$ and determine, at this time, the corresponding velocity of Q. **[8]**

In another situation, a particle of mass $3m$ kg is released from rest and falls vertically. After it has fallen 10 m, it explodes into two fragments. Immediately after the explosion, the lower fragment, of mass $2m$ kg, moves vertically downwards with speed $v_1\,\mathrm{m\,s^{-1}}$, and the upper fragment, of mass m kg, moves vertically upwards with speed $v_2\,\mathrm{m\,s^{-1}}$.

(b) Given that, in the explosion, the kinetic energy of the system increases by 72%, show that $2v_1^2 + v_2^2 = 1011.36$. **[3]**

(c) By finding another equation connecting v_1 and v_2, determine the speeds of the fragments immediately after the explosion. **[6]**

6 **Fig. 6.1** shows a light rod ABC, of length 60 cm, where B is the midpoint of AC. Particles of masses 3.5 kg, 1.4 kg and 2.1 kg are attached to A, B and C respectively.

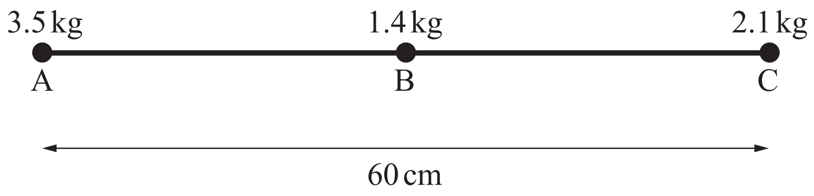


Fig. 6.1

The centre of mass is located at a point G along the rod.

(a) Determine the distance AG. **[2]**

Two light inextensible strings, each of length 40 cm, are attached to the rod, one at A, the other at C. The other ends of these strings are attached to a fixed point D. The rod is allowed to hang in equilibrium.

(b) Determine the angle AD makes with the vertical. **[3]**

The two strings are now replaced by a single light inextensible string of length 80 cm. One end of the string is attached to A and the other end of the string is attached to C. The string passes over a smooth peg fixed at D. The rod hangs in equilibrium, but is not vertical, as shown in **Fig. 6.2**.

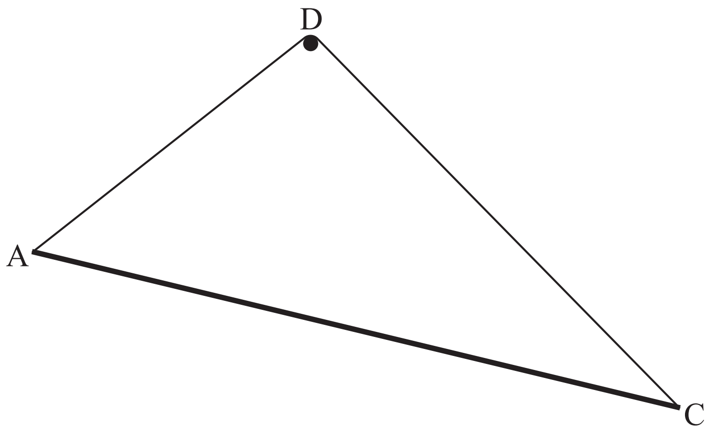


Fig. 6.2

(c) Explain why angle ADG and angle CDG must be equal. **[2]**

(d) Determine the tension in the string. **[6]**

END OF QUESTION PAPER