Oxford Cambridge and RSA

Wednesday 15 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y421/01 Mechanics Major

Duration: 2 hour 15 minutes

MAXIMUM MARK 120

Post Standardisation

This document consists of 24 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

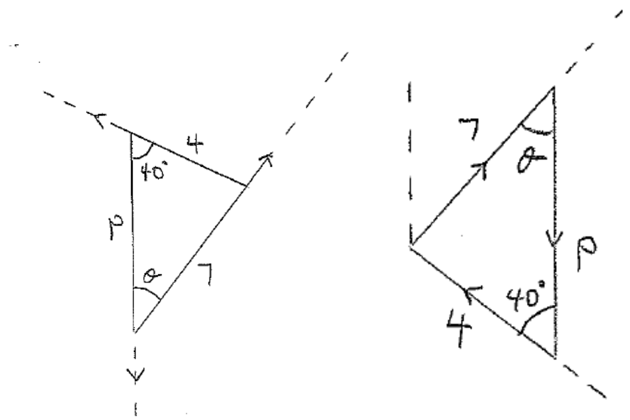
g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		e.g. 	B1 [1]	1.1	All three lengths and both angles must be correctly placed (arrows not required)
	(b)	(i)	$\frac{\sin\theta}{4} = \frac{\sin 40}{7}$ $\theta = 21.5^\circ$	M1 A1 [2]	1.1 1.1	Correct application of sine rule (oe) for their closed figure **or** resolving horizontally $7\sin\theta = 4\sin 40$ 21.54964123…
	(b)	(ii)	$P^2 = 4^2 + 7^2 - 2(4)(7)\cos(180 - 40 - \theta)$ $P = 9.57$	M1 A1 [2]	1.1 1.1	Correct application of cosine rule (oe) with their θ (or just θ) **or** correctly resolving vertically $P = 4\cos 40 + 7\cos\theta$ oe for example, $7^2 = 4^2 + P^2 - 2(4)(P)\cos 40$ $\frac{\sin(140-\theta)}{P} = \frac{\sin 40}{7}$ 9.57487554… - accept 9.58 (from using 21.5 for θ)

Question			Answer	Marks	AO	Guidance
2			$[v] = \mathrm{LT}^{-1}$ or $[g] = \mathrm{LT}^{-2}$	**B1**	1.2	Correctly stating the dimensions of either v or g
			$\mathrm{L}^2 = \dfrac{(\mathrm{LT}^{-1})^{\alpha}}{(\mathrm{LT}^{-2})^{\beta}}$ or $\dfrac{(\mathrm{LT}^{-1})^{\alpha}}{(\mathrm{LT}^{-2})^{\beta}} = \dfrac{\mathrm{L}(\mathrm{LT}^{-1})^2}{\mathrm{LT}^{-2}}$	**M1***	2.1	Setting up an equation in L and T using either $\left[d^2\right] = \left[\dfrac{v^{\alpha}\sin^2 2\theta}{g^{\beta}}\right]$ or $\left[\dfrac{v^{\alpha}\sin^2 2\theta}{g^{\beta}}\right] = \left[\dfrac{8hv^2\cos^2\theta}{g}\right]$
			$\alpha - \beta = 2$ $-\alpha + 2\beta = 0$	**M1dep***	1.1a	Setting up two consistent equations in α and β
			$\alpha = 4, \beta = 2$	**A1** **[4]**	1.1	

Question			Answer	Marks	AO	Guidance
3	(a)		$-0.1(2g)=2a$	M1*	3.3	Apply N2L with correct number of terms (must use mass and not weight on RHS) – condone sign error $-0.1g$
			$a=-0.98$	A1	1.1	
			$v^2=4.2^2+2(-0.98)(5)$	M1dep*	3.4	Use of $v^2=u^2+2as$ (oe complete method to find v) with their a and $u=4.2, s=5$
			$v=2.8$ (m s^{-1})	A1	1.1	
				[4]		
			Alternative method (energy)			
				M1		Apply work-energy principle – correct number of terms – condone sign errors
			$\frac{1}{2}(2)(4.2)^2-\frac{1}{2}(2)v^2=5F$	A1		Where F is the friction acting on the particle
			$\frac{1}{2}(2)(4.2)^2-\frac{1}{2}(2)v^2=5(0.1)(2g)$	M1		Using $F=0.1(2g)$ in their work-energy principle
			$v=2.8$ (m s^{-1})	A1		
				[4]		
	(b)		$9.8=2(w-(-2.8))$	M1	3.3	Use of Impulse = change in momentum – with their $v\neq 4.2$ and correct values for the mass and impulse. Condone sign error, w is the speed after impact oe e.g. $-9.8=2(w-2.8)$
			$w=2.1$ (m s^{-1})	A1	1.1	Allow –2.1 being changed to 2.1 for both marks
				[2]		

Question			Answer	Marks	AO	Guidance
4	(a)		$r = 0.5\sin\theta$	B1	3.1b	Where r is the radius of the horizontal circle
			$T\sin\theta = 0.1r(5)^2$	M1	3.3	Use of N2L with T resolved (allow sin/cos confusion) and $a = mr\omega^2$ with m, ω correct and any form for r (allow just r)
			$T = 1.25$ (N)	A1 [3]	1.1	
	(b)		$T\cos\theta = 0.1g$	M1	3.3	Resolve vertically (allow sin/cos confusion) – correct number of terms (allow T for the tension) and dimensionally consistent
			$\theta = 38.4$	A1 [2]	1.1	$\theta = 38.371715...$ or 0.66971276… (in radians)
	(c)		KE = $\frac{1}{2}(0.1)r^2(5)^2$	M1	1.1	Use of KE = $\frac{1}{2}mv^2$ with correct m and $v = 5r$
			0.120 (J)	A1 [2]	1.1	0.12042 – accept 0.121 from using $\theta = 38.4$ or 0.118 from using $\theta = 38$

Question			Answer	Marks	AO	Guidance
5	(a)		$\mathbf{F}=(-2\mathbf{i}+6\mathbf{j})+(2\cos(2t)\mathbf{i}+4\sin t\,\mathbf{j})$	M1	1.1	Find the resultant of the two forces by vector addition
			$\mathbf{a}=(\cos(2t)-1)\mathbf{i}+(2\sin t+3)\mathbf{j}\ (\text{m s}^{-2})$	A1 [2]	1.1	Using $\mathbf{F}=m\mathbf{a}$ to correctly find **a** Must be written as a vector but ISW if magnitude considered. Brackets must be present (oe) for this mark
	(b)			M1*	2.1	Attempt to integrate - at least two (of their four) terms correct – must be a vector expression for **a**
			$\mathbf{v}=\left(\frac{1}{2}\sin(2t)-t\right)\mathbf{i}+(-2\cos t+3t)\mathbf{j}\ (+\mathbf{c})$	A1	1.1	Correct integration +**c** not required for this mark
			$t=0, \mathbf{v}=\mathbf{0}\Rightarrow\mathbf{c}=\ldots$	M1dep*	3.4	Uses correct initial conditions to determine **c** (if correct $\mathbf{c}=2\mathbf{j}$) – just stating **c** = **0** because **v** = **0** when t = 0 is **M0**
			$\lvert\mathbf{v}\rvert=\sqrt{\left(\frac{1}{2}\sin 4-2\right)^2+(8-2\cos 2)^2}$	M1	1.1	Substitute t = 2 and attempt to calculate the speed (dependent on first **M** mark only)
			9.15 (m s^{-1})	A1 [5]	1.1	9.1469232… (an answer of 6.31477… implies M1 (working in degrees))

Question			Answer	Marks	AO	Guidance
6	(a)		$R = mg\cos\alpha, F = mg\sin\alpha$	**M1**	**2.1**	Attempt to resolve perpendicular and parallel to the plane. Allow sin/cos confusion only. Missing g is **M0**. Allow W for mg. Stating $mg\sin\alpha = mg\mu\cos\alpha$ can imply this mark
			Point of slipping $\Rightarrow mg\sin\alpha = mg\mu\cos\alpha$ So $\mu = \tan\alpha$	**A1** **[2]**	**2.2a**	**AG –** must state or clearly imply that $F = \mu R$
	(b)		Change in KE: $\pm\frac{1}{2}(5)(5^2 - 2^2)$	**B1**	**1.1**	52.5
			Change in GPE: $\pm 5g(10\sin 15)$	**B1**	**1.1**	126.8213321…
			Work done by pulling force is $10(P\cos 25)$	**B1**	**1.1**	Correct expression for the work done by P (9.063077…)P
			$\frac{1}{2}(5)(5)^2 - \frac{1}{2}(5)(2)^2 = 10(P\cos 25) - 10(3) - 5g(10\sin 15)$	**M1**	**3.3**	Use of work-energy principle – correct number of terms – allow sign errors (and sin/cos confusion) but must be dimensionally consistent
			$P = 23.1$	**A1** **[5]**	**1.1**	23.0960535…

Question			Answer	Marks	AO	Guidance
7	(a)		$2(2)+3(-1)=2v_A+3v_B$ and $v_B - v_A = 3e$ $v_A = \frac{1}{5}(1-9e)$ $\quad v_B = \frac{1}{5}(1+6e)$	M1 M1 A1 M1 A1 [5]	3.3 3.3 1.1 1.1 2.2a	Use of CLM – correct number of terms Use of NEL (e on correct side) – correct number of terms Use of NEL must be consistent with CLM oe NEL $v_A - v_B = -e(2-(-1))$ Solve simultaneously to find v_A or v_B **AG** for v_B - sufficient working must be shown
7	(b)	(i)	0.2 ($\mathrm{m\,s^{-1}}$)	B1 [1]	3.4	
		(ii)	$v_A < 0 \Rightarrow \frac{1}{5}(1-9e) < 0$ $\frac{1}{9} < e \leqslant 1$	M1 A1 [2]	3.1b 2.5	Setting their expression for $v_A < 0$ (condone $\leqslant 0$ or =) – or for $v_A > 0$ if $v_A = \frac{1}{5}(9e-1)$ from reversing the motion of A in **(a)** www
		(iii)	KE before collision: $\frac{1}{2}(2)(2)^2 + \frac{1}{2}(3)(-1)^2 \quad (=5.5)$ KE after collision: $\frac{1}{2}(2)\left(\frac{1-9e}{5}\right)^2 + \frac{1}{2}(3)\left(\frac{1+6e}{5}\right)^2$ $\frac{11}{2} - \frac{1}{25}(1-18e+81e^2) - \frac{3}{50}(1+12e+36e^2) = 3$ $27-27e^2 = 15 \Rightarrow e = \frac{2}{3}$	B1 B1ft M1 A1 [4]	1.1 1.1 3.3 1.1	 Correct expression for the KE after collision with correct v_B and their v_A Setting up an equation in e using KE before – KE after = 3 with correct number of terms – dependent on one previous **B** mark

Question			Answer	Marks	AO	Guidance
8	(a)		$\ddot{\mathbf{r}} = -g\mathbf{j}$	B1	1.2	
			$\dot{\mathbf{r}} = -gt\mathbf{j} + \mathbf{c}_1$ $t = 0, \dot{\mathbf{r}} = u\mathbf{i} + ku\mathbf{j} \Rightarrow \mathbf{c}_1 = \ldots$	M1	3.4	Correctly integrates their $\ddot{\mathbf{r}}$ and attempt to find $\mathbf{c}_1$ using correct initial conditions – correct expression for $\dot{\mathbf{r}}$ can imply this mark
			$\dot{\mathbf{r}} = u\mathbf{i} + (ku - gt)\mathbf{j} \Rightarrow \mathbf{r} = \ldots$	M1	1.1	Integrates their expression for $\dot{\mathbf{r}}$ (all powers increased by one)
			$\mathbf{r} = ut\mathbf{i} + \left(kut - \frac{1}{2}gt^2\right)\mathbf{j} + \mathbf{c}_2$ and showing $\mathbf{c}_2 = \mathbf{0}$ leading to $\mathbf{r} = ut\mathbf{i} + \left(kut - \frac{1}{2}gt^2\right)\mathbf{j}$	A1 **[4]**	2.2a	**AG** – must show (or explicitly) state that second constant of integration is **0** **Deriving the given result from constant acceleration formulae scores no marks**
	(b)		$x = ut, y = kut - \frac{1}{2}gt^2$	B1	1.1	Correctly remove vectors and states the correct equations for x and y
			$y = ku\left(\frac{x}{u}\right) - \frac{1}{2}g\left(\frac{x}{u}\right)^2$	M1	1.1	Eliminates t
			$y = kx - \frac{gx^2}{2u^2} \Rightarrow gx^2 - 2ku^2x + 2u^2y = 0$	A1 **[3]**	1.1	**AG** - sufficient working must be shown

Question			Answer	Marks	AO	Guidance
	(c)		$\Delta = \left(-2ku^2\right)^2 - 4g\left(2u^2 y\right)$	M1*	3.1a	Considers equation of the path as a quadratic in x and calculates its discriminant
			$4k^2u^4 - 8gu^2 y = 0 \Rightarrow y = \ldots$	M1dep*	2.1	Sets discriminant equal to zero and solves for y
			$y_{\max} = \dfrac{4k^2u^4}{8gu^2} = \dfrac{k^2u^2}{2g}$	A1 [3]	2.2a	
			Alternative method			
			$2gx - 2ku^2 + 2u^2\dfrac{dy}{dx} = 0$ and $\dfrac{dy}{dx} = 0$	M1*		Differentiates (powers decreased by one) and sets the derivative equal to zero
			$x = \dfrac{ku^2}{g} \Rightarrow g\left(\dfrac{ku^2}{g}\right)^2 - 2ku^2\left(\dfrac{ku^2}{g}\right) + 2u^2 y = 0$	M1dep*		Solves for x and substitutes into original equation
			$y_{\max} = \dfrac{1}{2u^2}\left(\dfrac{2k^2u^4}{g} - \dfrac{k^2u^4}{g}\right) = \dfrac{k^2u^2}{2g}$	A1 [3]		
	(d)		$y = 0 \Rightarrow \text{OA} = \dfrac{2ku^2}{g}$	B1	3.4	Correct expression for OA or implied e.g. $kx = \dfrac{gx^2}{2u^2}$ (where x = OA) **or** finding the time of flight T as $\dfrac{2ku}{g}$
			$\dfrac{k^2u^2}{2g} = \dfrac{2ku^2}{g} \Rightarrow k = \ldots$	M1	3.4	Sets answer to **(c)** equal to their expression for OA and solving for k **or** for equating the time of flight from $y = 0$ with the time of flight from OA = uT and solving for k
			$k^2 - 4k = 0 \Rightarrow k = 4$	A1	1.1	
				[3]		

Question			Answer	Marks	AO	Guidance
9	(a)		$V_1 = \pi\int_0^2 \left(e^{\frac{1}{2}x}\right)^2 dx$	M1*	2.1	Correct integral representation for the volume V_1 of the curve $y = e^{\frac{1}{2}x}$ between 0 and 2
			$= \pi\left[e^x\right]_0^2 = \pi\left(e^2 - 1\right)$	A1	1.1	
			$V_1\bar{x} = \pi\int_0^2 x\left(e^{\frac{1}{2}x}\right)^2 dx = ...$	M1*	1.1	Correct integral representation for $V_1\bar{x}$ and attempt to integrate by parts
			$V_1\bar{x} = \pi\left[xe^x\right]_0^2 - \pi\int_0^2 e^x dx = \pi\left[xe^x - e^x\right]_0^2$	A1	1.1	Correct integration – limits not required for this mark
			$V_1\bar{x} = \pi\left(e^2 + 1\right)$	A1	1.1	Implied by $\bar{x} = \dfrac{e^2+1}{e^2-1}$
				M1dep*	3.1a	Table of values idea – correct number of terms
			$V_1\bar{x} + 3\left(\frac{1}{3}\pi(e)^2(4)\right) = \bar{x}_T\left(\frac{1}{3}\pi(e)^2(4) + \pi\left(e^2 - 1\right)\right)$	A1ft	1.1	Correct equation for the centre of mass of T with their $V_1\bar{x}$ and correct V_1
			$\left(e^2+1\right)\pi + 4\pi e^2 = \bar{x}_T\left(\frac{4}{3}\pi e^2 + \pi e^2 - \pi\right)$ $\bar{x}_T\left(\frac{7}{3}e^2 - 1\right) = 5e^2 + 1 \Rightarrow \bar{x}_T = \dfrac{3\left(5e^2+1\right)}{7e^2-3}$	A1	2.2a	**AG** – sufficient working must be shown – condone absence of π throughout for full marks
				[8]		
	(b)		$\tan\theta_1 = \dfrac{\left(\dfrac{3\left(5e^2+1\right)}{7e^2-3} - 2\right)}{e}$ $\left(\tan\theta_1 = 0.1237... \Rightarrow \theta_1 = 7.0541...\right)$	B1	3.1b	Correct expression for either θ_1 or $\tan\theta_1$ where θ_1 is the angle between the line through (2, 0) and A and the line through A and $\left(\bar{x}_T, 0\right)$
			$\tan\theta_2 = \dfrac{2}{e}$ $\left(\tan\theta_2 = 0.7357... \Rightarrow \theta_1 = 36.3441...\right)$	B1	1.1	Correct expression for either θ_2 or $\tan\theta_2$ where θ_2 is the angle between the line through (2, 0) and A and the line OA
			$\theta = \theta_1 + \theta_2 \Rightarrow \theta = 43.4^\circ$	B1	2.2a	43.3982198… or 0.75744182… (in radians) – where θ is the angle between OA and the vertical
				[3]		
			Alternative method 1 (cosine rule)			

Question			Answer	Marks	AO	Guidance
			$\cos\theta = \dfrac{\left(4+e^2\right)+\left(e^2+\left(2-\bar{x}\right)^2\right)-\bar{x}^2}{2\sqrt{4+e^2}\sqrt{e^2+\left(2-\bar{x}\right)^2}}$	**B2**		**B2** for correct application of cosine rule with correct $\bar{x}$ – for **B1** allow one incorrect length $\cos\theta = \dfrac{(3.37476...)^2+(2.7390...)^2-(2.33636...)^2}{2(3.37476...)(2.7390...)}$
			$\theta = 43.4^\circ$	**B1**		43.3982198… or 0.75744182… (in radians) – where θ is the angle between OA and the vertical
			Alternative method 2 (scalar product)	**[3]**		
			$-2(\bar{x}-2)+e^2 = \sqrt{4+e^2}\sqrt{(\bar{x}-2)^2+e^2}\cos\theta$	**B2**		Or **B1** for $\begin{pmatrix}-2\\-e\end{pmatrix}\cdot\begin{pmatrix}\bar{x}-2\\-e\end{pmatrix} = \left\lvert\begin{pmatrix}-2\\-e\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}\bar{x}-2\\-e\end{pmatrix}\right\rvert\cos\theta$
			$\theta = 43.4^\circ$	**B1**		If **B0** then **SC B2** for 136.601780…
				[3]		

Question			Answer	Marks	AO	Guidance
10	(a)		$mgh = \frac{1}{2}mu_C^{\ 2}$	M1	3.3	Use of conservation of energy between A and C (or B) loss in PE = gain in KE (where u_C is the speed at C) – this mark may be applied by later working
			At angle θ, PE $= mgr(1-\cos\theta)$ and KE = $\frac{1}{2}mv^2$	B1	1.1	Correct PE and KE expressions for P at angle θ
			$\frac{1}{2}mu_C^{\ 2} = mgr(1-\cos\theta) + \frac{1}{2}mv^2$	M1	3.3	Use of conservation of energy between C (or A) and when P is at angle θ
			$v^2 = 2gh - 2gr(1-\cos\theta)$	A1	1.1	Correct expression for the speed or speed-squared when P is at an angle θ
			$R - mg\cos\theta = \frac{mv^2}{r}$	M1*	3.3	N2L radially with correct number of terms and weight resolved
			$R - mg\cos\theta = \frac{m}{r}\left(2gh - 2gr(1-\cos\theta)\right)$	M1dep*	3.4	Substitute expression for v^2 (with correct number of terms)
			$R = mg\cos\theta + \frac{2mgh}{r} - 2mg + 2mg\cos\theta$ $\Rightarrow R = mg\left(3\cos\theta - 2 + \frac{2h}{r}\right)$	A1 [7]	2.2a	**AG** – sufficient working must be shown
	(b)		$mg\left(3\cos\pi - 2 + \frac{2h}{r}\right) \ldots 0$	M1	3.1b	Set $\theta = \pi$ and $R = 0, > 0$ or $\ldots 0$
			$h \ldots \frac{5}{2}r \therefore$ least value of h is $\frac{5}{2}r$	A1 [2]	3.2a	Allow $h > \frac{5}{2}r$ or $h \ldots \frac{5}{2}r$
	(c)		Include air resistance in the model	B1 [1]	3.5c	Any correct refinement e.g. model the track as rough

Question			Answer	Marks	AO	Guidance
11	(a)		$\sin\theta = \frac{r}{2r} \Rightarrow \theta = 30^\circ$	B1 [1]	3.1b	**AG** – where r is the common radius of the spheres – as a minimum accept $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30$
	(b)			M1*	3.3	Use of conservation of linear momentum (parallel to the line of centres) – correct number of terms (condone no masses present for this mark)
			$v_A + v_B = U\cos 30$	A1	1.1	If no masses present, then do not award this mark (but all later marks can be awarded)
				M1*	3.3	Use of Newton's experimental law (parallel to the line of centres) – correct number of terms
			$v_A - v_B = -\frac{1}{2}U\cos 30$	A1	1.1	Use of NEL must be consistent with CLM
			$v_B = \frac{3\sqrt{3}}{8}U$	A1	1.1	Correct expression for the speed of B after impact (accept equivalent in terms of sine or cosine)
			$0 = (v_B\cos 30)t - \frac{1}{2}gt^2$	M1dep*	3.1b	Apply $s = ut + \frac{1}{2}at^2$ vertically with $s = 0$, $u = v_B\cos 30$ with their v_B, $a = \pm g$ or other complete method for finding t (e.g. $-v_B\cos 30 = v_B\cos 30 - gt$)
			$t = \frac{9U}{8g}$	A1 [7]	1.1	oe exact expression

Question			Answer	Marks	AO	Guidance
12	(a)			M1*	3.1b	Resolve horizontally – correct number of terms, allow sin/cos confusion (and sign errors) but no θ's present where R_A and R_B are the normal contact forces at A and B respectively
			$R_A \sin 30 = R_B \cos 30$	A1	1.1	
				M1*	3.3	Resolve vertically – correct number of terms, allow sin/cos confusion (and sign errors) but no θ's present
			$R_A \cos 30 + R_B \sin 30 = W$	A1	1.1	
			$R_A = R_B\sqrt{3} \Rightarrow \sqrt{3}\left(R_B\sqrt{3}\right) + R_B = 2W$	M1dep*	3.4	Eliminate R_A to form an equation in R_B (or equivalent to find an expression for R_B)
			$R_B = \frac{1}{2}W$	A1	1.1	
				[6]		

Question			Answer	Marks	AO	Guidance
	(b)			M1*	3.3	Taking moments about A – one couple term, at least one weight term and at least one term for the contact force at B – dimensionally correct – condone sin/cos confusion and allow R_B for $\frac{1}{2}W$
			$\frac{1}{8}aW + \frac{1}{2}a(W\cos\theta) = \frac{1}{2}a(W\sin\theta) + a\left(\frac{1}{2}W\cos(60-\theta)\right)$	A2	1.1 1.1	**A1** for two terms correct $\frac{1}{8}aW + \frac{1}{2}a(W\cos\theta)(1-\tan\theta) = a\left(\frac{1}{2}W\cos(60-\theta)\right)$ $\frac{1}{8}aW + \frac{1}{2}a\left(W\sqrt{2}\cos(\theta+45)\right)$ $= \frac{1}{2}a(W\sin 30\cos\theta + \cos 30\sin\theta)$
			$\frac{1}{8} + \frac{1}{2}\cos\theta = \frac{1}{2}\sin\theta + \frac{1}{2}\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$	M1dep*	3.1a	Obtain an equation in $\sin\theta$ and $\cos\theta$ only – weight component must be equivalent to a term in $\sin\theta$ and a term in $\cos\theta$ and the angle of the contact force at B must be of the form $\sin(\pm\alpha\pm\theta)$ or $\cos(\pm\alpha\pm\theta)$ where $\alpha = 30$ or 60
			$1 + 4\cos\theta = 4\sin\theta + 2\cos\theta + 2\sqrt{3}\sin\theta$ $\Rightarrow 2\left(\sqrt{3}+2\right)\sin\theta - 2\cos\theta = 1$	A1 **[5]**	2.2a	**AG** – sufficient working must be shown
	(c)		Considers both $\text{f}(22.35)$ and $\text{f}(22.45)$ where $\text{f}(\theta) = \pm\left[2\left(\sqrt{3}+2\right)\sin\theta - 2\cos\theta - 1\right]$	M1	1.1	Working or correct answer for one value is sufficient evidence of correct method but both 22.35 and 22.45 must be seen
			$\text{f}(22.35) = -0.0114\ldots < 0$ and $\text{f}(22.45) = 0.00194\ldots > 0$ change of sign indicates that $\theta = 22.4$ correct to 1 decimal place	A1 **[2]**	2.4	Correct values (to at least 1 sf rot) together with explanation (change of sign) and conclusion (as a minimum 'root') oe (e.g., comparing 1.00194… and 0.98856… to 1)

Question			Answer	Marks	AO	Guidance
13	**(a)**		$T=0.15g$	**M1**	**1.1**	Resolving vertically for P at A
			$\frac{13.5e}{0.45}=0.15g \Rightarrow e=...$	**M1**	**3.3**	Apply Hooke's law and solve for e – where e is the extension of the string from its natural length to A
			$\text{OA}=0.45+e=0.5\text{ (m)}$	**A1**	**1.1**	**AG** - if $g = 9.8$ used then **A0**
				[3]		
	(b)		$0.15\frac{d^2x}{dt^2}=0.15g-T_x-0.6\frac{dx}{dt}$	**M1***	**2.1**	Apply N2L vertically – correct number of terms
			$0.15\frac{d^2x}{dt^2}=0.15g-\frac{13.5}{0.45}(0.05+x)-0.6\frac{dx}{dt}$	**M1dep***	**1.1**	Substitute correct expression for the tension when the extension is $0.05 + x$
			$0.15\frac{d^2x}{dt^2}=-0.6\frac{dx}{dt}-30x$	**A1**	**2.2a**	**AG** if $g = 9.8$ used then **A0**
			$\Rightarrow \frac{d^2x}{dt^2}+4\frac{dx}{dt}+200x=0$			
				[3]		

Question			Answer	Marks	AO	Guidance
	(c)		$x=\frac{5}{56}\mathrm{e}^{-2t}\sin(14t)$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+4\frac{\mathrm{d}x}{\mathrm{d}t}+200x=0$			
				M1*	1.1	Differentiating x using the product rule (two terms of the form $uv' + vu'$ with $\sin(14t) \to \pm\cos(14t)$)
			$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{4}\mathrm{e}^{-2t}\cos(14t)-\frac{5}{28}\mathrm{e}^{-2t}\sin(14t)$	A1	1.1	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{56}\mathrm{e}^{-2t}\left(14\cos(14t)-2\sin(14t)\right)$
			$t=0, \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{4}\mathrm{e}^{-0}\cos(0)-\frac{5}{28}\mathrm{e}^{-0}\sin(0)=1.25$ $t=0, x=\frac{5}{56}\mathrm{e}^{-0}\sin(0)=0$	B1	3.4	Verifying that when $t = 0$, $\frac{\mathrm{d}x}{\mathrm{d}t}=1.25$ and $x = 0$ – from a correct first derivative
			$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-\frac{120}{7}\mathrm{e}^{-2t}\sin(14t)-5\mathrm{e}^{-2t}\cos(14t)$	A1	1.1	Correct second derivative (need not be simplified), for example $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{5}{56}\left(-2\mathrm{e}^{-2t}\right)\left(14\cos(14t)-2\sin(14t)\right)$ $+\frac{5}{56}\mathrm{e}^{-2t}\left(-196\sin(14t)-28\cos(14t)\right)$
			$\frac{d^2x}{dt^2}+4\frac{dx}{dt}+200x$ $=\mathrm{e}^{-2t}\left(-\frac{120}{7}sin(14t)-5\,cos(14t)\right)$ $+4e^{-2t}\left(\frac{5}{4}cos(14t)-\frac{5}{28}sin(14t)\right)$ $+200e^{-2t}\left(\frac{5}{56}sin(14t)\right)$	M1dep*	3.4	Substitute x, and its first and second derivatives into the correct differential equation
			$=\mathrm{e}^{-2t}\sin(14t)\left(-\frac{120}{7}-\frac{20}{28}+\frac{1000}{56}\right)+\mathrm{e}^{-2t}\cos(14t)(-5+5)=0$	A1	2.2a	**AG** – sufficient working must be shown
				[6]		

Question			Answer	Marks	AO	Guidance
			Alternative method (solving the given differential equation)			Solving the differential equation $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+4\frac{\mathrm{d}x}{\mathrm{d}t}+200x=0$
			$m^2+4m+200=0 \Rightarrow m=-2\pm 14\mathrm{i}$	**B1**		Correct roots of auxiliary equation – allow $-2+14\mathrm{i}$ if correct general solution seen
			$x=\mathrm{e}^{-2t}\left(A\cos(14t)+B\sin(14t)\right)$	**M1***		Correct general solution from their roots of the auxiliary equation
			$t=0, x=0 \Rightarrow A=0$	**A1**		Must follow from correct general solution
			$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{-2t}\left(14B\cos(14t)\right)-2\mathrm{e}^{-2t}\left(B\sin(14t)\right)$	**M1dep***		Differentiating using the product rule (ignore terms in A)
			$t=0, \frac{\mathrm{d}x}{\mathrm{d}t}=1.25 \Rightarrow B=\frac{5}{56}$	**A1**		Dependent on correctly differentiated terms in B
			$x=\frac{5}{56}\mathrm{e}^{-2t}\sin(14t)$	**A1**		Dependent on completely correct working
				[6]		
	(d)		$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{56}\left(\mathrm{e}^{-2t}\left(14\cos(14t)\right)-2\mathrm{e}^{-2t}\left(\sin(14t)\right)\right)=0$	**M1***	**3.1b**	Setting first derivative (two terms from the product rule) equal to zero (to find when P is at rest) Or equivalent $\frac{5}{28}\mathrm{e}^{-2t}\left(7\cos 14t-\sin 14t\right)=0$
			$\tan(14t)=7$	**A1**	**1.1**	
			$t=0.10206...,0.32646...$	**M1dep***	**3.1a**	Either finding the correct first two times from their $\tan(14t)=k$ for any non-zero k or just considers the second time (stating 0.326… is sufficient for this mark)
			when $t=0.32646\ldots$, $x=-0.046007...$	**M1**	**3.4**	Finding the displacement of P when P is at rest for the second time – dependent on both previous M marks
			$\lvert -0.046007...\rvert<0.05 \Rightarrow$ string does not go slack	**A1**	**2.2b**	Comparison and conclusion that the string does not go slack
				[5]		