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Oxford Cambridge and RSA

Wednesday 25 May 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Time allowed: 2 hours 40 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

Section A (37 marks)

1 **(a)** By considering $(r+1)^3 - r^3$, find $\sum_{r=1}^{n}(3r^2+3r+1)$. **[3]**

(b) Use this result to find $\sum_{r=1}^{n} r(r+1)$, expressing your answer in fully factorised form. **[4]**

2 **In this question you must show detailed reasoning.**

Find the exact value of $\int_3^{\infty} \frac{1}{x^2-4x+5}\,dx$. **[5]**

3 **In this question you must show detailed reasoning.**

Solve the equation $3\cosh x = 2\sinh^2 x$, giving your solutions in exact logarithmic form. **[6]**

4 **(a)** A transformation with associated matrix $\begin{pmatrix} m & 2 & 1 \\ 0 & 1 & -2 \\ 2 & 0 & 3 \end{pmatrix}$, where m is a constant, maps the vertices of a cube to points that all lie in a plane.

Find m. **[3]**

(b) The transformations S and T of the plane have associated matrices $\mathbf{M}$ and $\mathbf{N}$ respectively, where $\mathbf{M} = \begin{pmatrix} k & 1 \\ -3 & 4 \end{pmatrix}$ and the determinant of $\mathbf{N}$ is $3k+1$. The transformation U is equivalent to the combined transformation consisting of S followed by T.

Given that U preserves orientation and has an area scale factor 2, find the possible values of k. **[4]**

5 **(a)** Sketch the polar curve $r = a(1-\cos\theta)$, $0 \leqslant \theta < 2\pi$, where a is a positive constant. **[2]**

(b) Determine the exact area of the region enclosed by the curve. **[5]**

6 Prove by mathematical induction that $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}^n = \begin{pmatrix} 2^n & 0 \\ 1-2^n & 1 \end{pmatrix}$ for all positive integers n. **[5]**

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Answer **all** the questions.

Section B (107 marks)

7 **In this question you must show detailed reasoning.**

Show that $\int_{2}^{3} \frac{x+1}{(x-1)(x^2+1)}\,dx = \frac{1}{2}\ln 2$. **[9]**

8 Two sets of complex numbers are given by $\left\{z : \arg(z-10) = \frac{3}{4}\pi\right\}$ and $\{z : |z-3-6i| = k\}$, where k is a positive constant. In an Argand diagram, one of the points of intersection of the two loci representing these sets lies on the imaginary axis.

(a) Sketch the loci on an Argand diagram. **[4]**

(b) **In this question you must show detailed reasoning.**

Find the complex numbers represented by the points of intersection. **[7]**

9 The function $f(x)$ is defined by $f(x) = \ln(1+\sinh x)$.

(a) Given that k lies in the domain of this function, explain why k must be greater than $\ln(\sqrt{2}-1)$. **[2]**

(b) **(i)** Find $f'(x)$. **[2]**

(ii) Show that $f''(x) = \frac{a\sinh x + b}{(1+\sinh x)^2}$, where a and b are integers to be determined. **[3]**

(c) Hence find a quadratic approximation to $f(x)$ for small values of x. **[3]**

(d) Find the percentage error in this approximation when $x = 0.1$. **[2]**

10 The equation

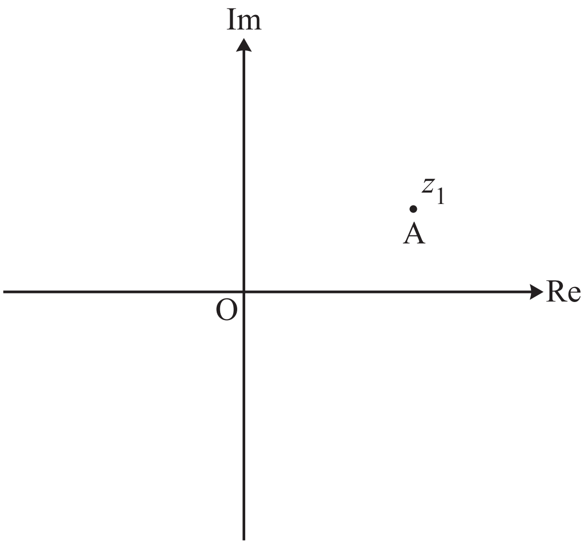
$$4x^4+16x^3+ax^2+bx+6=0,$$

where a and b are real, has roots α, $\frac{2}{\alpha}$, β and 3β.

(a) Given that $\beta<0$, determine all 4 roots. **[6]**

(b) Determine the values of a and b. **[4]**

11 An Argand diagram with the point A representing a complex number z_1 is shown below.



The complex numbers z_2 and z_3 are $z_1 e^{\frac{2}{3}i\pi}$ and $z_1 e^{\frac{4}{3}i\pi}$ respectively.

(a) **(i)** On the copy of the Argand diagram in the Printed Answer Booklet, mark the points B and C representing the complex numbers z_2 and z_3. **[2]**

(ii) Show that $z_1+z_2+z_3=0$. **[2]**

(b) Given now that z_1, z_2 and z_3 are roots of the equation $z^3=8i$, find these three roots, giving your answers in the form $a+ib$, where a and b are real and exact. **[4]**

12 Solve the differential equation $(4-x^2)\frac{\mathrm{d}y}{\mathrm{d}x}-xy=1$, given that $y=1$ when $x=0$, giving your answer in the form $y=\mathrm{f}(x)$. **[9]**

13 The points A and B have coordinates (4, 0, −1) and (10, 4, −3) respectively. The planes Π_1 and Π_2 have equations $x-2y=5$ and $2x+3y-z=-4$ respectively.

(a) Find the acute angle between the line AB and the plane Π_1. **[4]**

(b) Show that the line AB meets Π_1 and Π_2 at the same point, whose coordinates should be specified. **[5]**

(c) **(i)** Find $(\mathbf{i}-2\mathbf{j})\times(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$. **[1]**

(ii) Hence find the acute angle between the planes Π_1 and Π_2. **[3]**

(iii) Find the shortest distance between the point A and the line of intersection of the planes Π_1 and Π_2. **[4]**

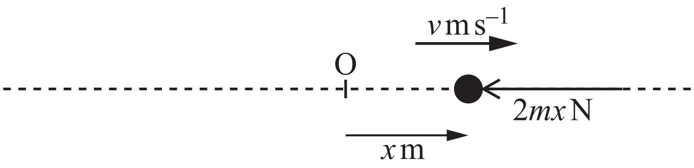
14 **(a)** Find $(3-\mathrm{e}^{2\mathrm{i}\theta})(3-\mathrm{e}^{-2\mathrm{i}\theta})$ in terms of $\cos 2\theta$. **[2]**

(b) Hence show that the sum of the infinite series

$$\sin\theta+\frac{1}{3}\sin 3\theta+\frac{1}{9}\sin 5\theta+\frac{1}{27}\sin 7\theta+\dots$$

can be expressed as $\frac{6\sin\theta}{5-3\cos 2\theta}$. **[6]**

15 In an oscillating system, a particle of mass m kg moves in a horizontal line. Its displacement from its equilibrium position O at time t seconds is x metres, its velocity is $v\,\text{m s}^{-1}$, and it is acted on by a force $2mx$ newtons acting towards O as shown in the diagram.



Initially, the particle is projected away from O with speed $1\,\text{m s}^{-1}$ from a point 2 m from O in the positive direction.

(a) **(i)** Show that the motion is modelled by the differential equation $\frac{d^2x}{dt^2} + 2x = 0$. **[1]**

(ii) State the type of motion. **[1]**

(iii) Write down the period of the motion. **[1]**

(iv) Find x in terms of t. **[4]**

(v) Find the amplitude of the motion. **[2]**

(b) The motion is now damped by a force $2mv$ newtons.

(i) Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0$. **[1]**

(ii) State, giving a reason, whether the system is under-damped, critically damped or over-damped. **[1]**

(iii) Determine the general solution of this differential equation. **[3]**

(c) Finally, a variable force $2m\cos 2t$ newtons is added, so that the motion is now modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 2\cos 2t.$$

(i) Find x in terms of t. **[7]**

In the long term, the particle is seen to perform simple harmonic motion with a period of just over 3 seconds.

(ii) Verify that this behaviour is consistent with the answer to part **(c)(i)**. **[2]**

END OF QUESTION PAPER

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