A Level Maths Formula Booklet

Pure

Arithmetic Series

$$S_n = \frac{1}{2}n(a + l)$$

$$S_n = \frac{1}{2}n(2a + (n - 1)d)$$

Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Binomial Expansion

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \cdots + {}^nC_r a^{n-r}b^r + \cdots + b^n$$

where $(n \in \mathbb{N})$ and ${}^nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \cdots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \cdots \times r} + \cdots$$

for $|x| < 1, n \in \mathbb{R}$

Curved Surface Area

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Exponentials and Logarithms

$$\log_a(x)\frac{\log_b(x)}{\log_b(a)}$$

$$e^{x\ln(a)} = a^x$$

Trigonometric Identities

$\sin(A \pm B) \equiv \sin(A)\cos(B) \pm \cos(A)\sin(B)$

$\cos(A \pm B) \equiv \cos(A)\cos(B) \mp \sin(A)\sin(B)$

$$\tan(A \pm B) \equiv \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)} \qquad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

$$\sin(A) \pm \sin(B) = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)$$

Small Angle Approximations

$\sin(\theta) \approx \theta$

$\cos(\theta) \approx 1 - \frac{1}{2}\theta^2$

$\tan(\theta) \approx \theta$

Differentiation

First principles: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Quotient rule: $\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$f(x)$	$f'(x)$
$\tan(kx)$	$k\sec^2(kx)$
$\sec(kx)$	$k\sec(kx)\tan(kx)$
$\cot(kx)$	$-k\operatorname{cosec}^2(kx)$
$\operatorname{cosec}(kx)$	$-k\operatorname{cosec}(kx)\cot(kx)$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts: $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

$\boldsymbol{f(x)}$	$\int \boldsymbol{f(x)\,dx}$
$\sec^2(kx)$	$\frac{1}{k}\tan(kx) + c$
$\tan(kx)$	$\frac{1}{k}\ln\lvert\sec(kx)\rvert + c$
$\cot(kx)$	$\frac{1}{k}\ln\lvert\sin(kx)\rvert + c$
$\mathrm{cosec}(kx)$	$-\frac{1}{k}\ln\lvert\mathrm{cosec}(kx) + \cot(kx)\rvert + c$ $\frac{1}{k}\ln\left\lvert\tan\left(\frac{1}{2}kx\right)\right\rvert + c$
$\sec(kx)$	$-\frac{1}{k}\ln\lvert\sec(kx) + \tan(kx)\rvert + c$ $\frac{1}{k}\ln\left\lvert\tan\left(\frac{1}{2}kx + \frac{\pi}{4}\right)\right\rvert + c$

Numerical Methods

Trapezium Rule: $\int_a^b y\,dx = \frac{1}{2}h(y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1}))$ where $h = \frac{b-a}{n}$

Newton-Raphson iteration for solving $f(x) = 0$ is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Statistics

Measures of Variation

Interquartile Range = IQR = $Q_3 - Q_1$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \left(\sum x_i^2\right) - \frac{\left(\sum x_i\right)^2}{n}$$

Standard deviation = $\sqrt{\text{variance}}$

$$\sigma = \sqrt{\frac{S_{xx}}{n}}$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$\sigma = \sqrt{\frac{\sum f x^2}{\sum f} - \bar{x}^2}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A') = 1 - P(A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

Independent Events

$$P(B|A) = P(B)$$

$$P(A|B) = P(A)$$

$$P(A \cap B) = P(A)P(B)$$

Binomial Distribution

If $X \sim B(n, p)$, then:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\bar{X} = np$$

$$\mathrm{Var}(X) = np(1-p)$$

Normal Distribution

If $X \sim N(\mu, \sigma^2)$, then:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Mechanics

Motion in a Straight Line

$v = u + at$

$s = \frac{1}{2}(u + v)t$

$s = ut + \frac{1}{2}at^2$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Motion in Two Dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$