1 Types of Numbers

Question 1: $1 \times 45 = 45$, $3 \times 15 = 45$, $5 \times 9 = 45$
There are no more factor pairs, so the complete list of factors is

$$1,\ 3,\ 5,\ 9,\ 15,\ 45$$

Question 2:
a) $1^3 = 1$, so 1 is a cube number.
b) $3^3 = 27$, so 27 is a cube number.
c) $4^3 = 64$, so 64 is a cube number.
d) The next cube number is $5^3 = 125$. So, 100 is not a cube number.

Question 3: 0.89 is a rational number, as can be written as a fraction:

$$0.89 = \frac{89}{100}$$

Question 4: b) $2\sqrt{4}$ is an integer ($2\sqrt{4} = 2 \times \sqrt{4} = 2 \times 2 = 4$)

Question 5: c) $0.\dot{3}$ is the only rational number ($0.\dot{3} = \frac{1}{3}$)

2 BIDMAS/ BODMAS

Question 1: There are two brackets (B), (2×3^3) and $(15-9)$. Inside the first bracket, there is a power or index number (I or O), $2 \times 3^3 = 2 \times 27$. Carry out any divisions or multiplications (DM) then additions or subtractions (AS) inside the brackets, $(2 \times 27 = 54)$ and $(15 - 9 = 6)$. Completing the calculation,$(2 \times 3^3) \div (15 - 9) = 54 \div 6 = 9$

Question 2: (B), $12 \div 4 = 3$, (I),$(3)^2 = 9$, (M), $16 \times 9 = 144$

Question 3: Numerator: (B) substitute in the given value of x and apply the power (I), before the addition (A).$((-3)^2 + 3) = (9 + 3) = 12$. Denominator: (B) there is a subtraction inside the bracket (S), $(10 - 6) = 4$. Finally simplify the fraction (D), so,$\frac{12}{4} = 3$

Question 4: (B), $y^2 + 5y^2 = 6y^2$.(M), $3y \times 7y = 21y^2$.(S), and we get, $6y^2 - 21y^2 = -15y^2$

3 Place Value

Question 1: 800 or eight hundred

Question 2: $\frac{1}{10}$ or one tenth

Question 3: $\frac{6}{1000}$ or six thousandths

Question 4: $500,000$ or five hundred thousand

Question 5: 3 thousands $= 3,000$ 5 tens $= 50$ 1 hundredth $= 0.01$
Adding these all together, we get Beckys number to be

$$3,000 + 50 + 0.01 = 3,050.01$$

4 Long Division

Question 1: When the result is not an integer the remaining bit left over can be written either as a decimal or as a remainder.

$$\begin{array}{r} 0\,2\,2\,.\,5 \\ 14\overline{)3^31^35.^70} \end{array}$$

$$\begin{array}{r} 0\,2\,2\,r\,7 \\ 14\overline{)3^31^35.^70} \end{array}$$

Question 2: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 1\ 5 \\ 15\overline{)2^22^75} \end{array}$$

Question 3: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 2\ 3 \\ 13\overline{)2^29^39} \end{array}$$

Question 4: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 2\ 6 \\ 9\overline{)2^23^54} \end{array}$$

5 Long Multiplication

Question 1: Using the long multiplication method, multiplying 619 first by 5 then by 40 and summing the results,

$$\begin{array}{r} 619 \\ \times 45 \\ \hline 3095 \\ +24760 \\ \hline 27855 \end{array}$$

Question 2: Using the long multiplication or the grid method

$$\begin{array}{r} 52 \\ \times 31 \\ \hline 52 \\ +1560 \\ \hline 1612 \end{array}$$

Question 3: Using the long multiplication or the grid method

$$\begin{array}{r} 760 \\ \times 24 \\ \hline 3040 \\ +15200 \\ \hline 18240 \end{array}$$

Question 4: Using the long multiplication or the grid method

$$\begin{array}{r} 364 \\ \times 52 \\ \hline 728 \\ +18200 \\ \hline 18928 \end{array}$$

Question 5: Using the long multiplication or the grid method

$$\begin{array}{r} 473 \\ \times 326 \\ \hline 2838 \\ +9460 \\ +141900 \\ \hline 154198 \end{array}$$

6 Decimals

Question 1: By means of column addition or otherwise,

$$\begin{array}{r} 82.070 \\ +31.865 \\ \hline 113.935 \end{array}$$

Question 2: We can make the calculation easier by converting the divisor to a whole number by multiplying both 2.3 and 18.63 by 10, so,

$$\begin{array}{r} 8\,.\,1 \\ 23\overline{)\cancel{1}{}^{1}\cancel{8}{}^{18}6\,.{}^{2}3} \end{array}$$

Question 3: To make the first number whole: $3.566 \times 1,000 = 3,566$. Thus the column multiplication is,

$$\begin{array}{r} 3566 \\ \times 14 \\ \hline 14264 \\ 35660 \end{array}$$

Adding the the two parts,

$$\begin{array}{r} 14264 \\ +35660 \\ \hline 49924 \end{array}$$

We multiplied one of our numbers by 1000, which means our result is 1000 times too big. Therefore, the final answer is, $3.566 \times 14 = 49924 \div 1000 = 49.924$

Question 4:

$$\begin{array}{r} 0.113 \\ +0.890 \\ \hline 1.003 \end{array}$$

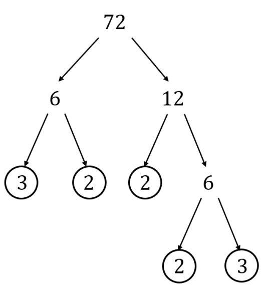
Question 5: Converting the decimals to a whole numbers. If we multiply 0.002 and 0.043 by 1000, we have a simple integer multiplication,

$$\begin{array}{r} 2 \\ \times 43 \\ \hline 86 \end{array}$$

However this value is $1000 \times 1000 = 1000000$ times too big, so we have to divide the result by this, $86 \div 1000000 = 0.000086$

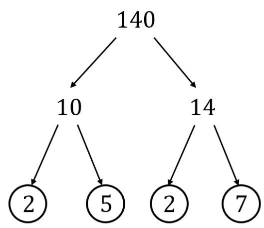
7 Prime Factors, LCM and HCF

Question 1: Using a prime factor tree:



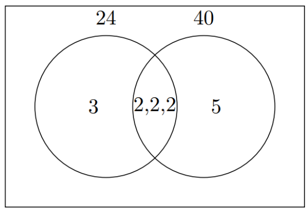
The prime factorisation of 72 is, $72 = 2 \times 2 \times 2 \times 3 \times 3$
Index notation: $72 = 2^3 \times 3^2$

Question 2: Using a prime factor tree:



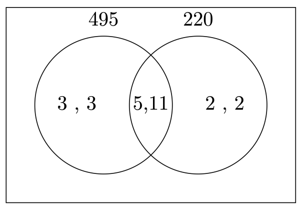
The prime factorisation of 140 is, $140 = 2 \times 2 \times 5 \times 7$
Index notation: $140 = 2^2 \times 5 \times 7$

Question 3:
Prime factors of 24: $2 \times 2 \times 2 \times 3$.
Prime factors of 40: $2 \times 2 \times 2 \times 5$



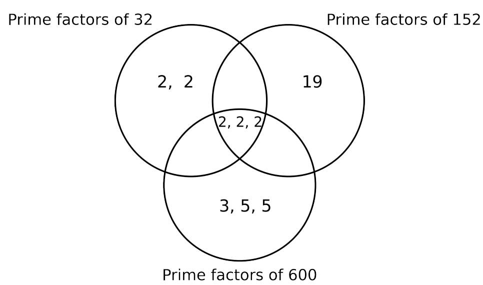
HCF $= 2 \times 2 \times 2 = 8$.
LCM $= 8 \times 3 \times 5 = 120$

Question 4:
Prime factors of $495 = 3 \times 3 \times 5 \times 11$
Prime factors of $220 = 2 \times 2 \times 5 \times 11$



HCF $= 5 \times 11 = 55$
LCM $= 2 \times 2 \times 3 \times 3 \times 5 \times 11 = 1980$

Question 5: Prime factors of $32 = 2 \times 2 \times 2 \times 2 \times 2$
Prime factors of $152 = 2 \times 2 \times 2 \times 19$
Prime factors of $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$



HCF $= 2 \times 2 \times 2 = 8$
LCM $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 19 = 45600$

8 Fractions

Question 1:

$$\frac{6}{13}\times\frac{4}{3}=\frac{6\times 4}{3\times 13}=\frac{24}{39}=\frac{8}{13}$$

Question 2:

$$\frac{7}{10}-\frac{8}{3}=\left(\frac{7}{10}\times\frac{3}{3}\right)-\left(\frac{8}{3}\times\frac{10}{10}\right)=\frac{21}{30}-\frac{80}{30}=-\frac{59}{30}$$

Question 3:

$$\frac{9}{11}\div\frac{6}{7}=\frac{9}{11}\times\frac{7}{6}=\frac{9\times 7}{11\times 6}=\frac{63}{66}=\frac{21}{22}$$

Question 4:

$$\frac{5}{4}\times\frac{2}{3}=\frac{5\times 2}{4\times 3}=\frac{10}{12}=\frac{5}{6}$$

Question 5: First, convert the mixed fraction to an improper fraction,

$$12\frac{1}{2}=\frac{25}{2},\quad \frac{25}{2}\div\frac{5}{8}=\frac{25}{2}\times\frac{8}{5}=\frac{25\times 8}{2\times 5}=\frac{200}{10}=20$$

9 Fractions, Decimals, and Percentages

Question 1: $54.4\%\div 100=0.544$

Question 2: $16.4\%=\frac{164}{1000}=\frac{41}{250}$

Question 3: $\frac{17}{40}=\frac{42.5}{100}=0.425$

Question 4: $0.256=\frac{256}{1000}=\frac{32}{125}$

Question 5: $\frac{13}{20}=\frac{65}{10}=0.65$

10 Rounding Numbers

Question 1: $560,180$ rounded to the nearest thousand is $560,000$

Question 2: 97.96 rounded to 1 decimal place is 98.0

Question 3: 0.02345 rounded to 3 significant figures is 0.0235

Question 4: 1.0093 rounded to 3 significant figures is 1.01

Question 5: 55.099 rounded to 2 decimal places is 55.10

11 Ordering Numbers

Question 1: Descending order means from largest to smallest. Hence,

$$23,\ 4,\ 1,\ -23.5,\ -42$$

Question 2: Ascending order means from smallest to largest, hence,

$$2.04,\ 2.5,\ 2.58,\ 2.8,\ 3.5$$

Question 3:

$$4.092,\ 4.87,\ 5.01,\ 5.12,\ 5.23$$

Question 4: Convert all numbers to the same form. $64\%=0.64$ and, $64.4\%=0.644$. $\frac{5}{8}=0.625$. Hence, $0.625,\ 0.633,\ 0.64,\ 0.644$
Finally, putting them in order in the original forms,

$$\frac{5}{8},\ 0.633,\ 64\%,\ 64.4\%$$

12 Estimating

Question 1: Round each number to 1 significant figure:

$$\frac{9.02+6.65}{0.042\times 11}\approx\frac{9+7}{0.04\times 10}=\frac{16}{0.4}=\frac{160}{4}=40$$

Question 2: Round each number to 1 significant figure:

$$\frac{57.33-29.88}{8.66-5.55}\approx\frac{60-30}{9-6}=\frac{30}{3}=10$$

Question 3: Round each number to 1 significant figure:
45p = £0.45, 1.89 rounds to 2 and 0.45 rounds to 0.5
(Pens) £2 × 5 = £10, (Pencils) £0.50 × 3 = £1.50
(Total) £10 + £1.50 = £11.50

Question 4: Round each number to 1 significant figure:
32.60 rounds to 30, 17.50 rounds to 20.
(Children) £20 × 3 = £60, (Adults) £30 × 2 = £60
(Total) £60 + £60 = £120

13 Upper and Lower Bounds

Question 1:
Lower bound: $5.43-0.005=5.425$
Upper bound: $5.43+0.005=5.435$
The interval is therefore $5.425\le C<5.435$

Question 2:
Lower bound: $175-0.5=174.5$
Upper bound: $175+0.5=175.5$
The interval is therefore $174.5\le h<175.5$

Question 3:
Lower bound: $5.45+0.005=5.455$
Upper bound: $5.45-0.005=5.445$
The interval is therefore £5.445 B $\le C<$ £5.455 B

14 Standard Form

Question 1: $1.15\times 10^{-6}=0.00000115$.

Question 2: $5,980,000=5.98\times 10^{6}$

Question 3: $0.0068=6.8\times 10^{-3}$

Question 4: 5.6×10^{6} and 8×10^{2}

$$(5.6\times 10^{6})\div(8\times 10^{2})=(5.6\div 8)\times(10^{6}\div 10^{2})$$

Using the formula $10^{a}\div 10^{b}=10^{a-b}$ we can rewrite the equation as,

$$(5.6\div 8)\times 10^{6-2}=0.7\times 10^{4}$$

Standard form requires the number be between 1 and 10, thus

$$0.7\times 10^{4}=7\times 10^{-1}\times 10^{4}=7\times 10^{3}$$

Question 5:

$$(2.5\times 10^{4})\times(6\times 10^{-9})=2.5\times 6\times 10^{4}\times 10^{-9}=15\times 10^{-5}$$

Standard form requires the number be between 1 and 10, thus

$$15\times 10^{-5}=1.5\times 10\times 10^{-5}=1.5\times 10^{-4}$$

1 Collecting Like Terms

Question 1:

$$5x+5-2x+3-4-x=(5x-2x-x)+(5+3-4)=2x+4$$

Question 2:

$$ab+bc+2ab-bc+a=a+(ab+2ab)+(bc-bc)=a+3ab$$

Question 3:

$$11x+7y-2x-13y=(11x-2x)+(7y-13y)=9x-6y$$

Question 4:

$$2m+6n-3+8n+5m=(2m+5m)+(6n+8n)-3=7m+14n-3$$

Question 5:

$$2a^2+5b-2a-3b+5a^2=(2a^2+5a^2)+(5b-3b)-2a=7a^2+2b-2a$$

2 Powers and Roots

Question 1: $a^b \times a^c = a^{b+c}$, so, $a^2 \times a^3 = a^{2+3}$, $a^2 \times a^3 = a^5$

Question 2: We can recognise, $12^2 = 144$ and $14^2 = 196$
So, $\sqrt{144}+\sqrt{196} = 12+14 = 26$

Question 3: Using the laws of indices, $(3^2)^3 = 3^{2\times3} = 3^6. Hence, the expression now looks like, 3^6 \div 3^4$.
Then, $3^6 \div 3^4 = 3^{6-4} = 3^2 = 9$

Question 4: First considering the numerator, the laws of indices tell us, $7^5 \times 7^3 = 7^{5+3} = 7^8$. Thus the expression now is, $\frac{7^8}{7^6}$. This can be simplified to, $\frac{7^8}{7^6} = 7^{8-6} = 7^2 = 49$.

Question 5: We know that, $20^1 = 20$ and $100^0 = 1$. So $20+1 = 21$

3 Expanding Brackets

Question 1:

$$\begin{aligned} 3xy(x^2+2x-8) &= 3xy \times x^2 + 3xy \times 2x + 3xy \times (-8) \\ &= 3x^3y + 6x^2y - 24xy \end{aligned}$$

Question 2:

$$\begin{aligned} 9pq(2-pq^2-7p^4) &= 9pq \times 2 - 9pq \times pq^2 - 9pq \times 7p^4 \\ &= 18pq - 9p^2q^3 - 63p^5q \end{aligned}$$

Question 3:

$$\begin{aligned} (y-3)(y-10) &= y \times y + y \times (-10) + (-3) \times y + (-3) \times (-10) \\ &= y^2 - 10y - 3y + 30 \\ &= y^2 - 13y + 30 \end{aligned}$$

Question 4:

$$\begin{aligned} (m+2n)(m-n) &= m \times m + m \times (-n) + 2n \times m + 2n \times (-n) \\ &= m^2 - mn + 2mn - 2n^2 \\ &= m^2 + mn - 2n^2 \end{aligned}$$

Question 5: We can write this as two sets of brackets,

$$\begin{aligned} (2y^2+3x)(2y^2+3x) &= 2y^2 \times 2y^2 + 2y^2 \times 3x + 3x \times 2y^2 + 3x \times 3x \\ &= 4y^4 + 6xy^2 + 6xy^2 + 9x^2 \\ &= 4y^4 + 12xy^2 + 9x^2 \end{aligned}$$

4 Factorising

Question 1: $5pq(2+3r)$

Question 2: $u(u^2+3v^3+2)$.

Question 3: $y^5(4x+1+12y^2)$

Question 4: $5xy(y-x-xy)$

Question 5: $7abc(1+2a+3b+7c^2)$

5 Solving Equations

Question 1:

$$\begin{aligned} 2x+1 &= 2 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Question 2:

$$\begin{aligned} \frac{1}{2}x - 3 &= 7 \\ \frac{1}{2}x &= 10 \\ x &= 20 \end{aligned}$$

Question 3:

$$\begin{aligned} 12k-1 &= 6k-25 \\ 6k &= -24 \\ k &= -4 \end{aligned}$$

Question 4:

$$\begin{aligned} 3(2m+6) &= 2(m-3) \\ 6m+18 &= 2m-6 \\ 4m &= -24 \\ m &= -6 \end{aligned}$$

Question 5:

$$\begin{aligned} \frac{x^2}{5} &= 31.25 \\ x^2 &= 156.25 \\ x &= \sqrt{156.25} \\ x &= \pm 12.5 \end{aligned}$$

6 Rearranging Formulas

Question 1:

$$\begin{aligned} F &= \frac{mv}{t} \\ Ft &= mv \\ m &= \frac{Ft}{v} \end{aligned}$$

Question 2:

$$A = \frac{1}{2}(a+b)$$
$$2A = (a+b)h$$
$$2A = ah + bh$$
$$2A - bh = ah$$
$$a = \frac{2A - bh}{h} = \frac{2A}{h} - b$$

Question 3:

$$F = \frac{kq}{r^2}$$
$$Fr^2 = kq$$
$$r^2 = \frac{kq}{F}$$
$$r = \pm\sqrt{\frac{kq}{F}}$$

7 Factorising Quadratics

Question 1: We are looking for two numbers which add to make 1 and multiply to make -30.The factors of 30 that satisfy theses two requirements are -5 and 6. Therefore, the full factorisation of $a^2 + a - 30$ is $(a-5)(a+6)$

Question 2: We are looking for two numbers which add to make -5 and multiply to make 6. The factors of 6 that satisfy theses two requirements are -2 and -3. Therefore, the full factorisation of $k^2 - 5k + 6$ is $(k-2)(k-3)$

Question 3: We are looking for two numbers which add to make 7 and multiply to make 12. The factors of 12 that satisfy theses two requirements are 3 and 4. Therefore, the full factorisation of $x^2 + 7x + 12$ is, $(x+3)(x+4)$

Question 4: $x^2 - 4 = (x+2)(x-2)$

8 Solving Quadratics By Factorisation

Question 1: The quadratic on the left hand side of the equation factorises so that, $p^2 - 3p - 10 = (p+2)(p-5) = 0$
For the left-hand side to be zero we require one of the brackets to be zero, hence, the two solutions are, $p = -2$ and $p = 5$.

Question 2: The quadratic on the left hand side of the equation factorises so that, $x^2 - 8x + 15 = (x-5)(x-3) = 0$.
Hence, $x = 3$ and $x = 5$.

Question 3: This quadratic factorises so that, $x^2 - 6x + 8 = 0 = (x-2)(x-4) = 0$.
Hence, $x = 2$ and $x = 4$.

9 Sequences and Nth Term (Linear)

Question 1: a) Substituting $n = 12$ into the formula. $4(12) + 1 = 49$. So, the 12^{th} term is 49

b) Every term in this sequence is generated when an integer value of n is substituted into $4n+1$. Hence if we set 77 to equal $4n+1$, we can determine its position in the sequence. Hence, $4n+1 = 77$, then making n the subject by subtracting 1 then dividing by 4,

$$n = \frac{77-1}{4} = 19$$

Hence 77 is the 19^{th} term in the sequence.

Question 2: a) To generate the first 5 terms of this sequence, we will substitute $n = 1, 2, 3, 4, 5$ into the formula given.
$1 = 5(1) - 4 = 1$
$2 = 5(2) - 4 = 6$
$3 = 5(3) - 4 = 11$
$4 = 5(4) - 4 = 16$
$5 = 5(5) - 4 = 21$
So, the first 5 terms are 1, 6, 11, 16, and 21

b) Every term in this sequence is generated when an integer value of n is substituted into $5n-4$. If we set 108 to equal $5n-4$, we can determine if it is a part of the sequence or not. If the value of n is a whole number then it is part of the sequence. Hence $5n - 4 = 108$, then making n the subject by adding 4 then dividing by 5,

$$n = \frac{112}{5} = 22.4$$

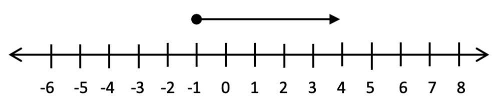
As there is no 22.4^{th} position in the sequence, it must be the case that 108 is not a term in this sequence.

Question 3: We are told it is an arithmetic progression and so must have n^{th} formula: $an + b$. To find a, we must inspect the difference between each term which is 5, hence $a = 5$. Then, to find b, lets consider the sequence generated by $5n$: 5, 10, 15, 20, 25

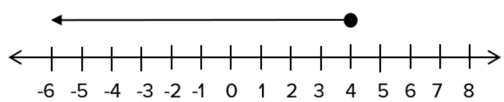
Every term is this sequence is bigger than the corresponding terms in the original sequence by 8. So, to get to the original sequence, we will have to subtract 8 from every term in this sequence. In other words, the n^{th} term formula for our sequence in question is $5n - 8$

10 Inequalities

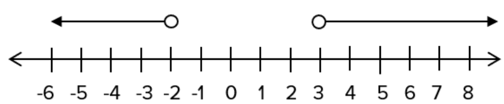
Question 1: The inequality, $x \geq -1$, will require a closed circle at -1 and an arrow pointing right.



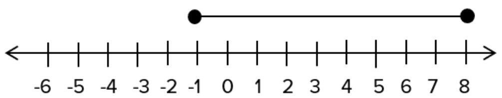
Question 2: The inequality, $x \leq 4$, will require an closed circle at 4 and an arrow pointing left.



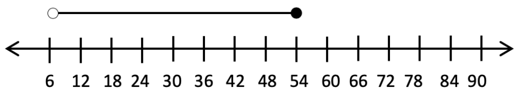
Question 3: The first inequality, $y > 3$, will require an open circle at 3 and an arrow pointing right. The other inequality, $y < -2$, will require an open circle at -2 and an arrow pointing left.



Question 4: The lower bound, $-1 \le x$, will require a closed circle at $x = -1$. The upper bound $x \le 8$, will require an closed circle at $x = 8$



Question 5: Forming the correct inequality $6 < b \le 54$ and displaying with an open circle for representing the strict inequality (6) and a closed circle representing the non-strict inequality (54).



11 Solving Inequalities

Question 1:

$$\begin{aligned} 7-3k &> -5k+12 \\ 7+2k &> 12 \\ 2k &> 5 \\ k &> \frac{5}{2} \end{aligned}$$

Question 2:

$$\begin{aligned} \frac{5x-1}{4} &> 3x \\ 5x-1 &> 12x \\ -1 &> 7x \\ x &< -\frac{1}{7} \end{aligned}$$

Question 3:

$$\begin{aligned} 2x+5 &> 3x-2 \\ 5 &> x-2 \\ 7 &> x \\ x &< 7 \end{aligned}$$

Question 4:

$$\begin{aligned} 4-3x &\le 19 \\ -3x &\le 15 \\ x &\ge -5 \end{aligned}$$

Question 5:

$$\begin{aligned} -5 < 2x-3 < 10 \\ -2 < 2x < 13 \\ -1 < x < \frac{13}{2} \end{aligned}$$

12 Simultaneous Equations

Question 1: Subtracting equation 2 from equation 1 so that,

$$\begin{aligned} y &= 2x-6 \\ y &= \frac{1}{2}x+6 \\ (y-y) &= (2x-\frac{1}{2}x)-6-6 \\ 0 &= \frac{3}{2}x-12 \end{aligned}$$

If we rearrange to make x the subject we find, $x = \frac{2\times 12}{3} = \frac{24}{3} = 8$. Substituting $x = 8$ back into the original first equation,

$$\begin{aligned} y &= 2(8)-6 \\ y &= 10 \end{aligned}$$

Hence, the solution is, $x = 8, y = 10$

Question 2: If we multiply the second equation by 2, we have two equations both with a $2x$ term, hence subtracting our new equation 2 from equation 1 we get

$$\begin{aligned} 2x-3y &= 16 \\ 2x+4y &= -12 \end{aligned}$$

$$\begin{aligned} (2x-2x)+(-3y-4y) &= 16-(-12) \\ 0x-7y &= 28 \end{aligned}$$

If we rearrange to make y the subject we find, $y = \frac{28}{-7} = -4$. Substituting $y = -4$ back into the original second equation,

$$\begin{aligned} x+2(-4) &= -6 \\ x-8 &= -6 \\ x &= 2 \end{aligned}$$

Hence, the solution is, $x = 2, y = -4$

Question 3:

$$\begin{aligned} 5x+2y+16 &= 0 \\ 2x+3y+13 &= 0 \end{aligned}$$

$$\begin{aligned} 15x+6y+48 &= 0 \\ -\quad 4x+6y+26 &= 0 \end{aligned}$$

$$11x+22 = 0$$

$$\begin{aligned} 11x &= -22 \\ x &= -2 \end{aligned}$$

Substituting x back into the original first equation,

$$\begin{aligned} 5(-2)+2y &= -16 \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

Hence, the solution is, $x = 2, y = -3$

Question 4: Let A be the cost of an adult ticket and let C be the cost of a child ticket, thus we have two simultaneous equations,

$$\begin{aligned} 2A+3C &= 20 \\ A+C &= 8.5 \end{aligned}$$

If we multiply the second equation by 2, we have two equations both with a $2A$ term, hence subtracting our new equation 2 from equation 1 we get,

$$\begin{aligned} 2A+3C &= 20 \\ 2A+2C &= 17 \end{aligned}$$

$$\begin{aligned} (2A-2A)+(3C-2C) &= (20-17) \\ C &= 3 \end{aligned}$$

Then, substituting this value back into the original equation 2, we get,

$$A + 3 = 8.5$$
$$A = 5.5$$

Therefore, the cost of a child ticket is £3, and the cost of an adult ticket is £5.50.

---

13 Proof

Question 1: We will try the first few even numbers (squaring them and adding 3) until we find an example that isnt prime. So, we get
$2^2 + 3 = 4 + 3 = 7,$ which is prime.
$4^2 + 3 = 16 + 3 = 19,$ which is prime.
$6^2 + 3 = 36 + 3 = 39,$ which is not prime.
Since 39 is divisible by 3, it must not be prime, so we have proved Luke's statement to be false.

Question 2: To show that the left and right hand sides of the equation are identical we expand the brackets on the left hand side, $5(3x-5) - 2(2x+9) = 15x - 25 - 4x - 18 = 11x - 43$. Hence we have shown the identity is true.

Question 3: The left hand side is equal to
$(n-2)^2 - (n-5)^2 = n^2 - 4n + 4 - (n^2 - 10n + 25) = 6n - 21 = 3(2n-7)$
Hence, we have shown that the identity is true.

Question 4: Even number = $2n$, so the product of two even numbers is $2n \times 2n = 4n^2$.
n^2 is just a whole number, odd or even.
So, 4 × any number = $Even$, since an even number multiplied by an odd or even number is always even.

---

14 Function Machines

Question 1: a) Inputting 35, we first multiply by 3: $35 \times 3 = 105$. Then, add 15 to get, $105 + 15 = 120$
b) We must work backwards and do the opposite operations. So, first subtracting 15 from the given output, we get, $48 - 15 = 33$. Then, dividing by 3 we get, $33 \div 3 = 11$. Meaning that 11 is the input required.

Question 2: Inputting -5, we first multiply by -2: $-5 \times -2 = 10$. Then, adding 7 we get, $10 + 7 = 17$. Meaning that 17 is the output required.

Question 3: Inputting $3x$, we first multiply by $\frac{1}{2}$, $3x \times \frac{1}{2} = \frac{3}{2}x$
Then, dividing by 3 we get, $\frac{3}{2}x \div 3 = \frac{1}{2}x$, meaning that $\frac{1}{2}x$ is the output.

Question 4: An example of two operations are, multiplying by 4, so $9 \times 4 = 36$, subtracting 9, so $36 - 9 = 27$

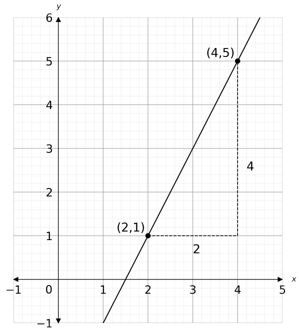
$$9 \longrightarrow \boxed{\times 4} \longrightarrow \boxed{-9} \longrightarrow 27$$

Question 5: We have to see what we get if we input x into the function machine. First, multiplying x by 12, $12 \times x = 12x$. Then, subtracting 25 to get, $12x - 25$. This is the output of inputting x but we know the output is equal to $2x$, so we are left with the equation, $12x - 25 = 2x$. Rearrange to get,

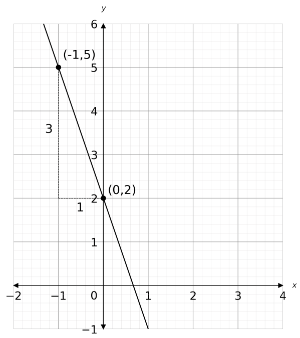
$$10x - 25 = 0$$
$$10x = 25$$
$$x = \frac{25}{10} = 2.5$$

1 Gradients of Straight Line Graphs

Question 1: Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{4}{2} = 2$

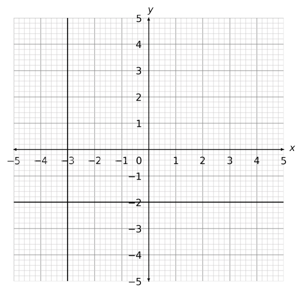


Question 2: Gradient $= -\frac{3}{1} = -3$



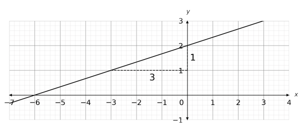
Question 3: Gradient $= \frac{-1-(-6)}{-8-2} = \frac{5}{-10} = -\frac{1}{2}$

Question 4: The lines $y = -2$ and $x = -3$ should be a straight line perpendicular to the axis at that point,



2 The Equation of a Straight Line

Question 1: The y-intercept is 2, so $c = 2$
$m = \text{gradient} = \frac{1}{3}$
Therefore, the equation of the line is $y = \frac{1}{3}x + 2$

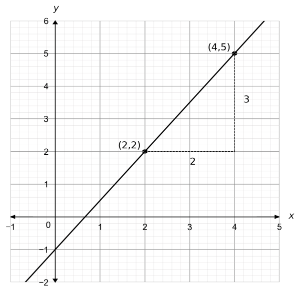


Question 2: $m = \text{gradient} = \frac{-6-34}{-3-2} = \frac{-40}{-5} = 8.$
Substitute one pair of coordinates into the equation $y = mx + c$ to find c, e.g. $(2, 34)$. So,

$$34 = 8 \times 2 + c$$
$$34 = 16 + c$$
$$c = 34 - 16$$
$$= 18$$

Therefore, the equation of the line is, $y = 8x + 18$.

Question 3: The y-intercept is -1, so $c = -1$.
$m = \text{gradient} = \frac{3}{2}$
Hence, $y = \frac{3}{2}x - 1$



3 Coordinates

Question 1: $A = (-2, 2)$, $B = (-1, -2)$, $C = (3, 0)$.

Question 2: $A = (-2, -2)$, $B = (0, 3)$. By taking the average of the x and y coordinates of A and B separately, the midpoint is

$$\left(\frac{-2+0}{2}, \frac{-2+3}{2}\right) = \left(-1, \frac{1}{2}\right)$$

Question 3: $A = (0, 5)$, $B = (-11, -10)$. By taking the average of the x and y coordinates of A and B separately, the midpoint is

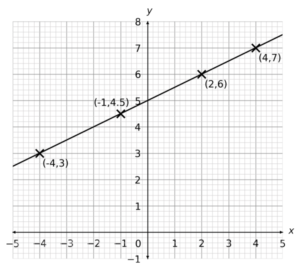
$$\left(\frac{0+(-11)}{2}, \frac{5+(-10)}{2}\right) = \left(-\frac{11}{2}, -\frac{5}{2}\right)$$

4 Drawing Straight Line Graphs

Question 1: Substitute the given values into the equation, e.g.when $x = -1$, $y = \frac{1}{2} \times (-1) + 5 = 4.5$, and so on. The completed table looks like:

x	-4	-1	2	4
y	3	4.5	6	7

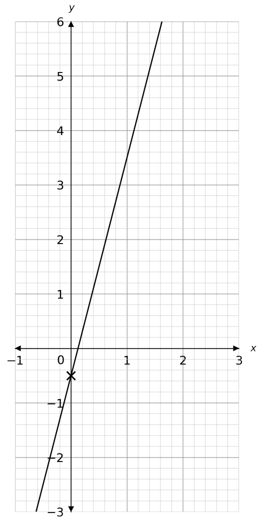
Plotting these points should give the following graph:



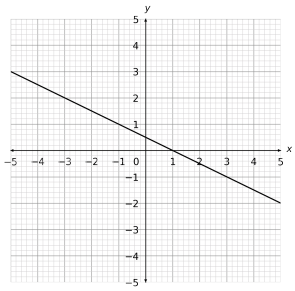
Question 2: Rearrange the equation to the form $y = mx + c$

$$2y = 8x - 1$$
$$y = 4x - \frac{1}{2}$$

So, the y-intercept is $-\frac{1}{2}$, and the gradient is 4 This is enough to draw the graph:



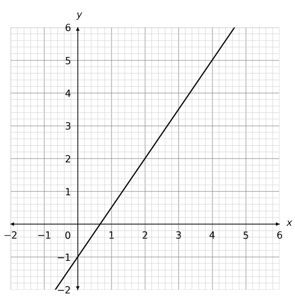
Question 3: Rearrange the equation to the form $y = mx + c$
$y = -0.5x + 0.5$
So, the y-intercept is $\frac{1}{2}$, and the gradient is $-\frac{1}{2}$. The result should look like the figure below



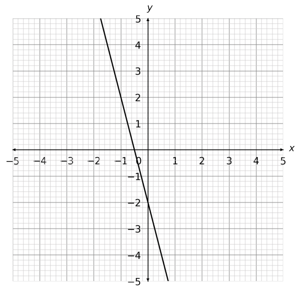
Question 4: Rearrange the equation to the form $y = mx + c$

$$2y = 3x - 2$$
$$y = \frac{3}{2}x - 1$$

So, the y-intercept is -1, and the gradient is $\frac{3}{2}$, giving the following graph:



Question 5: Rearrange the equation to the form $y = mx + c$
$y = -4x - 2$. So, the y-intercept is -2, and the gradient is -4, giving the following graph:

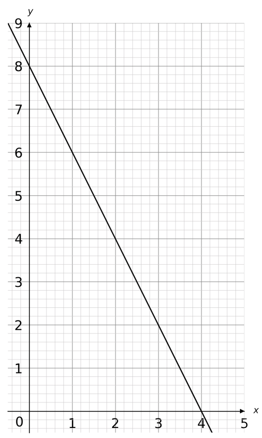


5 Parallel Lines

Question 1:
a) Rearrange to get, $y = \frac{1}{2}x + \frac{1}{4}$
b) Rearrange to get, $y = -2x + \frac{5}{2}$
c) Rearrange to get, $y = \frac{1}{2}x + 45$

a) and c) have the same gradient $\left(\frac{1}{2}\right)$ so they are parallel.

Question 2: Rearrrange the equation to the form $y = mx + c$
$y = -2x - \frac{3}{5}$.
The new line must have the same gradient (-2) and pass through $(1, 6)$.



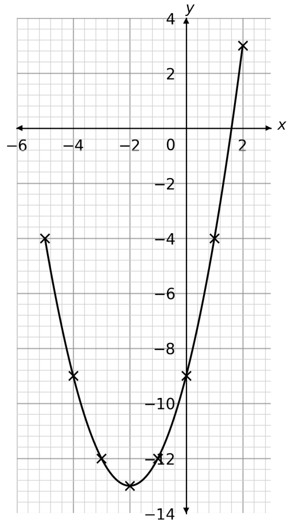
Question 3: $m = 3$
a) $y = -3x + 3$
b) $y = \dfrac{1}{3}x + 2$
c) $y = 6x + 4$
d) $y = 3x + 5$
Only (d) is parallel, since it has a gradient of 3.

6 Harder Graphs

Question 1: Substitute in the values of x to get the missing values of y. The completed table should look like:

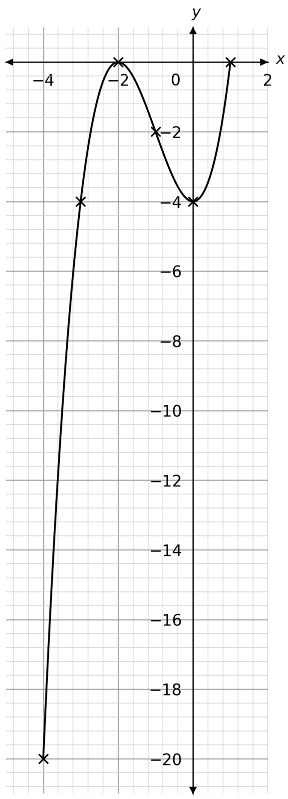
x	-5	-4	-3	-2	-1	0	1	2
y	-4	-9	-12	-13	-12	-9	-4	3

Plotting these coordinates on a pair of axes and joining them with a curve:



Question 2: The completed table and graph should look like:

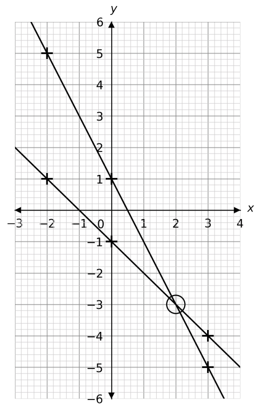
x	-4	-3	-2	-1	0	1
y	-20	-4	0	-2	-4	0



7 Solving Simultaneous Equations with Graphs

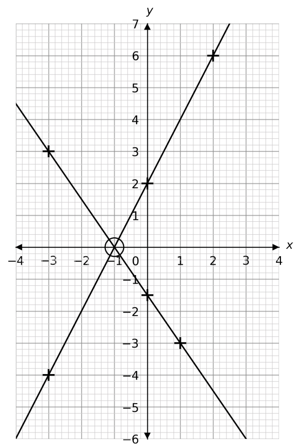
Question 1: Substitute in some values of x into the first equation, we get

$x = -2$ gives $y = -2 \times (-2) + 1 = 5$
$x = 0$ gives $y = -2 \times (0) + 1 = -1$
$x = 3$ gives $y = -2 \times (3) + 1 = -5$
So 3 coordinates on the line are $(-2, 5)$, $(0, -1)$, $(3, -5)$.
Then, doing the same for the second equation,
$x = -2$ gives $y = -(-2) - 1 = 1$
$x = 0$ gives $y = -(0) - 1 = -1$
$x = 3$ gives $y = -(3) - 1 = -4$
So, 3 coordinates on the line are $(-2, 1)$, $(0, -1)$, $(3, -4)$.
Plotting these points and drawing the lines gives the following graph.



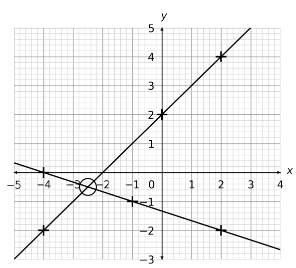
The two lines intersect at $(2, -3)$, therefore the solution is $x = 2$, $y = -3$

Question 2: Rearrange the second equation to be in a form we can use. We get: $y = -1.5x - 1.5$.
Plotting points for each equation gives the graph shown below.



The two lines intersect at $(-1, 0)$, therefore the solution is $x = -1$, $y = 0$

Question 3: Rearrange the second equation to the form $y = mx + c$, giving: $y = -\frac{1}{3}x - \frac{4}{3}$.
Plotting points for each equation gives the graph shown below.

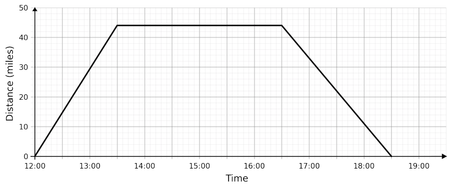


The two lines intersect at $(-2.5, -0.5)$, therefore the solution is, $x = -2.5$, $y = -0.5$

8 Distance-Time Graphs

Question 1: The journey can be described as follows:

$12:00 - 13:30$, he travels from 0 miles away to 44 miles away;
$13:30 - 16:30$, he stays in one place;
$16:30 - 18:30$, he travels from 44 miles away to 0 miles away.
On a graph, this looks like:



Question 2: The fastest speed is given by the steepest gradient. Eliminating the middle period (i.e. the least steep section) and comparing the other two:

Period 1:

$$\text{Gradient } = \frac{\text{distance travelled}}{\text{time taken}} = \frac{600-0}{72-0} = 8.33 \text{ m/s}$$

Period 3:

$$\text{Gradient } = \frac{\text{distance travelled}}{\text{time taken}} = \frac{1,500-880}{282-180} = 6.08 \text{ m/s}$$

Therefore, the fastest speed travelled by Chris during the race was 8.33 m/s, to 3 sf.

Question 3: (a) Total Distance $= 48 + 10 = 58$ km
(b) She stopped for 30 mins at the 32 km mark.

Question 4: The fastest average speed is given the steepest section of the graph. This is the final section which covered 48 km in one hour, thus, Maximum speed $= 48$ km/h

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9 Real Life Graphs

Question 1: Diesel price per litre $\approx \frac{40}{35} = £1.14$.
Petrol price per litre $\approx \frac{35}{35} = £1.00$.
Hence the difference in price per litre, $£1.14 - £1.00 = £0.14$

Question 2: Cost per bike $\approx £38.00$
Total cost (4 bikes) $= £38.00 \times 4 = £152$

Question 3: $£8 = £11.20$. Thus, $£800 = \$1120$

1 Ratios

Question 1: a) The sum of the ratio is $4+5=9$. Since the ratio share for blond students is 4, this means that the fraction of blond students is $\frac{4}{9}$

b) $\frac{4}{9}$ of the students have blond hair, so the fraction of students with brown hair is $\frac{5}{9}$. If there is a total of 450 students in the school, so the number of students with brown hair is:

$$\frac{5}{9}\times 450 = 250 \text{ students.}$$

Question 2: Total parts $= 2+5=7$
7 parts $= 35$, therefore 1 part $= 35\div 7 = 5$
2 parts $= 2\times 5 = 10$
5 parts $= 5\times 5 = 25$
Hence the ratio is $10:25$

Question 3: The ratio is 2 parts blue to 13 parts white $(2:13)$. Lucy buys 16 blue tiles, which is 2 parts. so, 1 part $= 16\div 2 = 8$
No. of white tiles $= 13\times 8 = 104$ tiles
Cost of blue tiles $= £2.80\times 16 = £44.80$
Cost of white tiles $= 104\times £2.35 = £244.40$
Hence, the total cost is

$$£44.80 + £244.40 = £289.20$$

Question 4: Deducting the 20% spent on the magazine subscription gives 80% of the original amount:
$0.8\times £200 = £160$.
Steve therefore has £160 pounds remaining which he spends on football stickers, sweets and fizzy drinks in the ratio of $5:2:1$.
Total parts $= 5+2+1=8$
Thus, the amount spent on football stickers is:

$$\frac{5}{8}\times £160 = £100$$

Question 5: a) The ratio of books read by Jon to books read by Kate is $2:1$. The ratio of books read by Alieke to books read by Jon is $4:1$. Scaling up the second ratio so Jon has 2 parts gives the following 3 way ratio:
Alieke : Jon : Kate $= 8:2:1$

b) The difference between the ratio share is 7 parts $(8-1=7)$.
The difference in the ratio share is 7 parts, and the difference in the number of books read is 63. Thus, 1 part $= 63\div 7 = 9$ books
Total parts $= 8+2+1 = 11$
Hence the total number of books is

$$11\times 9 \text{ books} = 99 \text{ books}$$

2 Direct and Inverse Proportion

Question 1: There is enough food for 3 days for a total of 5 people, so there is a total of $3\times 5 = 15$ days worth of food for one person. Given that there are now 3 people going camping, the food will last $15\div 3 = 5$ days.

Question 2: It took 8 staff 20 minutes to complete the check-in, which is a total of $8\times 20 = 160$ minutes of work. Therefore, if 10 staff are working, it will take $160\div 10 = 16$ minutes.

Question 3: It took 5 people half a day to cover 1 square metre, which is a total of $5\times 0.5 = 2.5$ work days for one person. Therefore, to complete the whole dig (24 square metres) it would take one person $24\times 2.5 = 60$ work days. Thus, to complete the dig in 3 days, $60\div 3 = 20$ people are required.

3 Percentages

Question 1: 10% of $180 = 180\div 10 = 18$
Therefore, 30% of $180 = 3\times 18 = 54$
1% of $180 = 180\div 100 = 1.8$
Therefore, 3% of $180 = 3\times 1.8 = 5.4$
33% of $180 = 30\% + 3\% = 54 + 5.4 = 59.4$

Question 2: $(99\div 150)\times 100 = 66\%$

Question 3:

$$\begin{aligned}\text{Percentage change} &= \left(\frac{\text{change}}{\text{original}}\right)\times 100\\ &= \left(\frac{£25,338 - £24,600}{£24,600}\right)\times 100\\ &= \left(\frac{£738}{£24,600}\right)\times 100\\ &= 3\%\end{aligned}$$

Question 4: A 10% price reduction means the new value is 90% of the original value. A further 10% price reduction is therefore 90% of the new value, i.e. 90% of 90:
$90\times 0.9 = 81$
Thus, the new value is 81% of the original price, which is a reduction of $100-81 = 19\%$

4 Reverse Percentages

Question 1: The original price of the t-shirt is:
$£13.50\div 0.75 = £18$

Question 2: The original price of the car is:
$£11,550\div 1.05 = £11,000$

Question 3: The total mass of the bar is:
$9.90\text{g}\div 0.18 = 55\text{g}$

Question 4: The total capacity of the stadium is:
$19805\div 0.85 = 23,300$

Question 5: The total population of the U.K. is:
$9,300,000\div 0.14 = 66,428,571$

Question 6: The original price of the car is:
$£9,680\div 0.44 = £22,000$

5 Compound Growth and Decay

Question 1: Using the compound growth formula:

$$\text{Amount after 4 years} = \$1,400,000 \times \left(1 + \frac{2.4}{100}\right)^4$$
$$= \$1,400,000 \times 1.024^4$$
$$= \$1,539,316.28$$

Question 2: Using the compound decay formula:

$$\text{No. of tigers in 5 years} = 234 \times \left(1 - \frac{18}{100}\right)^5$$
$$= 234 \times 0.82^5$$
$$= 87 \text{ tigers (nearest whole number)}$$

This is less than 100, therefore Riley is correct.

Question 3: Using the compound growth formula and solving for x:

$$£292,662.70 = £268000 \times \left(1 + \frac{x}{100}\right)^2$$
$$\frac{£292,662.70}{£268,000} = \left(1 + \frac{x}{100}\right)^2$$
$$1.0920 = \left(1 + \frac{x}{100}\right)^2$$
$$\sqrt{1.0920} = 1 + \frac{x}{100}$$
$$1.0450 = 1 + \frac{x}{100}$$
$$0.0450 = \frac{x}{100}$$
$$x = 4.5\%$$

Question 4: Using the compound decay formula:

$$\text{Value after 3 years} = £850,000 \times \left(1 - \frac{6}{100}\right)^3$$
$$= £850,000 \times 0.94^3$$
$$= £705,996.40$$

Using this as the new value for N_0 in the compound decay formula:

$$\text{Value after a further 2 years} = £705,996.40 \times \left(1 - \frac{4}{100}\right)^2$$
$$= £705,996.40 \times 0.96^2$$
$$= £650,646.28$$

Which as a percentage of the original value is:

$$\left(\frac{£850,000 - £650,646.28}{£850,000}\right) \times 100 = 23\%$$

Question 5: Using the compound decay formula:

$$£15,187.50 = £36,000 \times \left(1 - \frac{x}{100}\right)^3$$

Solving for x gives,

$$\frac{15,187.50}{36,000} = \left(1 - \frac{x}{100}\right)^3$$
$$\sqrt[3]{\frac{15,187.50}{36,000}} = 1 - \frac{x}{100}$$
$$\frac{x}{100} = 1 - \sqrt[3]{\frac{15,187.50}{36,000}}$$
$$\frac{x}{100} = 0.25$$
$$x = 25\%$$

Hence using the compound decay formula with $n = 5$,

$$\text{Value after 5 years} = £36,000 \times \left(1 - \frac{25}{100}\right)^5$$
$$= £8543 \text{ (nearest pound)}$$

6 Conversions

Question 1: Deducting the 3% fee gives 97% remaining. So, money left $= \frac{97}{100} \times 500 = £485$
Hence the amount in dollars is, $485 \times 1.56 = \$756.60$

Question 2: 2.3 km = 2,300 m. So, the distance in feet is, $2,300 \div 0.3048 = 7,546$ feet (to the nearest foot).

Question 3: The volume of his fish tank in cm^3 is: 120 cm × 180 cm × 100 cm = 2160000 cm^3, where 1 m^3 = 1000000 cm^3. So the volume of his fish tank in m^3 is: $2160000 \div 1000000 = 2.16$ m^3

Question 4: 13.1 miles × 1.61 kilometres = 21.091 kilometres
The total time to complete the half marathon at a pace of 5.5 minutes per kilometre is therefore:

$$21.091 \text{ kilometres} \times 5.5 \text{ minutes per kilometre} = 116 \text{ mins}$$
$$\equiv 1 \text{ hour } 56 \text{ minutes}$$

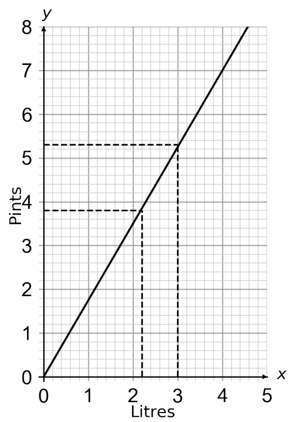
(Remember that 30 seconds is $\frac{1}{2}$ a minute = 0.5 minutes)

Question 5: Value in Jan = £5,000 × €1.16 = €5,800
Converting back into pounds in Feb gives, €5,800 ÷ €1.15 = £5,043
Hence the profit is £5,043 − £5,000 = £43 to the nearest pound.

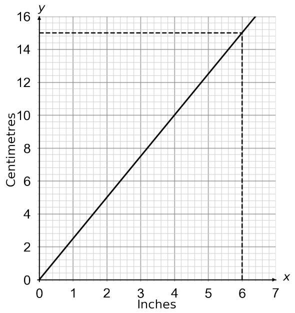
7 Conversion Graphs

Question 1: a) Locate the value 3 on the horizontal (x) axis and draw a line vertically upwards until it touches the black solid line of the graph. Then draw a line from this point horizontally to the left to find the corresponding value on the vertical (y) axis. The value falls between 5.2 and 5.4 pints, so the approximate answer is 3 litres = 5.3 pints (accept ±0.1 pints, see graph below)

b) Locate 3.8 on the vertical (y) axis and draw a line horizontally to the right until it touches the black solid line of the graph. Then draw a line from this point vertically down to find the corresponding value on the horizontal (x) axis. This value falls between 2 and 2.2 litres. Since it is closer to 2.2 than 2, the approximate answer is 3.8 pints = 2.15 litres (accept ±0.1 litres, see graph below)



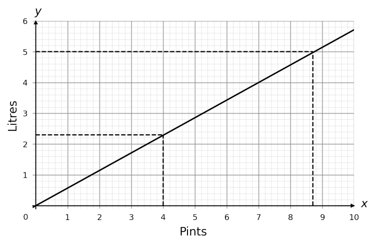
Question 2: a) 1 inch $\approx$ 2.5 cm. Plot the points $(1,2.5),(2,5),(3,7.5)$ on the graph and draw a line through each point.



b) To convert 15cm to inches, locate 15cm on the vertical (y) axis and draw a horizontal line to the right until it touches the line of the graph. Then draw a line vertically down to find the corresponding value on the horizontal (x) axis. The value on the x-axis is 6, so 15 cm $\approx$ 6 inches

Question 3: a) Locate 4 on the horizontal (x) axis and draw a line up until it touches the line of the graph. Then go across to the corresponding value on the vertical (y) axis. This line touches between 2.2 litres and 2.4 litres, so 4 pints $\approx$ 2.3 litres

b) Locate 5 on the vertical (y) axis and draw a line across to the right until it touches the line of the graph. Then go down to the corresponding value on the horizontal x axis. This line touches between 8.6 pints and 8.8 pints, so 5 litres $\approx$ 8.7 pints



8 Best Buys

Question 1: Brand B contains 3 times as much as brand A $(200 \text{ ml} \times 3 = 600 \text{ ml})$
600 ml of brand A costs $= £0.80 \times 3 = £2.40$. This is more than £2.20, so brand B is better value for money.

Question 2: 30% of 120 $= 0.3 \times 120 = 36$ extra pencils. This means that for £4.20 you receive $120 + 36 = 156$ pencils. So, the price per pencil for Brand A is $£4.20 \div 156 = £0.0269$.
Brand B sells 200 pencils for £6.20, so the price per pencil is: $£6.20 \div 200 = £0.031$.
Therefore brand A is better value for money since the price per pencil is less.

Question 3:

$$\text{Supermarket A} : 2.40 \div 215 = £0.0111... \text{ per gram}$$

$$\text{Supermarket B} : 4.10 \div 403 = £0.0101... \text{ per gram}$$

$$\text{Supermarket C} : 3.40 \div 297 = £0.0114... \text{ per gram}$$

Therefore supermarket B offers the best value for money.

Question 4: a) 250g $= 0.25$ kg of Gorgonzola. So the cost of Gorgonzola is $0.25 \times £11.60 = £2.90$.

b) The price of Edam is: $£4.48 \div 400 = £0.0112$ per gram.
The price of Gorgonzola is: $£11.60 \div 1000 = £0.0116$ per gram.
Therefore the Edam cheese is better value as it costs less.
Converting the prices from pounds per gram to prices in pence per gram:
Edam: $£0.0112 \times 100 = 1.12$ pence per gram
Gorgonzola: $£0.0116 \times 100 = 1.16$ pence per gram
The difference between one gram of Edam and one gram of Gorgonzola is therefore: $1.16 - 1.12 = 0.04$ pence per gram

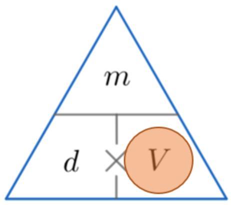
Question 5:

$$\text{Swift Cabs total cost} = £3.00 + £3.80 + (£1.60 \times 10) = £22.80$$
$$\text{Zoom Taxis total cost} = £3.80 + (11 \times £1.60) = £21.40$$
$$\text{Relaxi Cabs total cost} = 11 \times £2.10 = £23.10$$

The best value company is Zoom Taxis, the next best value is Swift Cabs, and the worst value is Relaxi Cabs.

9 Density Mass Volume

Question 1: 2 kg = 2000 g

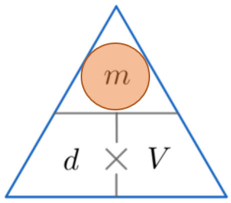


Therefore, the volume of the olive oil can be calculated as follows:

$$\text{Volume} = 2000 \text{ g} \div 0.925 \text{ g/cm}^3 = 2162 \text{ cm}^3$$

Question 2: The cube has a side length of 7 m, so the volume of the cube is: $7 \times 7 \times 7 = 343 \text{ m}^3$. Thus,

$$\text{Mass} = 343 \times 10,800 = 3,704,400 \text{ kg}$$



Question 3: Mass = 2460 kg and Volume = 1.2 m^3. Substituting these values into the formula: Density $= 2460 \text{ kg} \div 1.2 \text{ m}^3 = 2050 \text{ kg/m}^3$

Question 4: a) Total Volume = Volume of A + Volume of B
Rearranging the density formula to make volume the subject gives, volume = mass $\div$ density. So,

$$\text{Volume of A} = 1200 \text{ g} \div 5 \text{ g/cm}^3 = 240 \text{ cm}^3$$
$$\text{Volume of B} = 600 \text{ g} \div 3 \text{ g/cm}^3 = 200 \text{ cm}^3$$

So, total volume $= 240 \text{ cm}^3 + 200 \text{ cm}^3 = 440 \text{ cm}^3$

b) Density = mass ÷ volume
where total mass = 1200 g + 600 g = 1800 g
Therefore, density = 1800 g ÷ 440 cm^3 = 4.09 g/cm^3

Question 5: If the ratio of metal A to metal B is $3:7$, that means that $\frac{3}{10}$ of the mass of metal C comes from metal A and the remaining $\frac{7}{10}$ is metal B. So,

The mass of metal A: 2500 g $\times \frac{3}{10} = 750g$
The mass of metal B: 2500 g $\times \frac{7}{10} = 1750g$

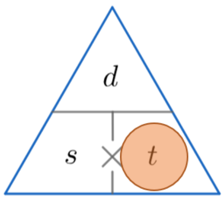
Since density = mass ÷ volume, then volume = mass ÷ density

The volume of metal A: 750 g ÷ 3.2 g/cm^3 = 234.375 cm^3
The volume of metal A: 1750 g ÷ 5.5 g/cm^3 = 318.18 cm^3

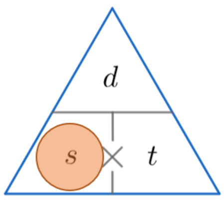
If metal A has a volume of 234.375 cm^3 and metal B has a volume of 318.18 cm^3, then their combined volume is the volume of metal C.
Volume of metal C = 234.375 + 318.18 = 552.5568 cm^3.
Hence, the density of metal C = 2500 g ÷ 552.5568 cm^3 = 4.5 g/cm^3

10 Speed Distance Time

Question 1: Time $= \frac{d}{s} = \frac{100}{8.5} = 11.76$ s (2 dp)



Question 2: 30 minutes = 0.5 hours, so Gustavo's speed can be calculated as follows: speed $= \frac{d}{t} = \frac{36}{0.5} = 72$ mph. Gustavo is exceeding the speed limit.



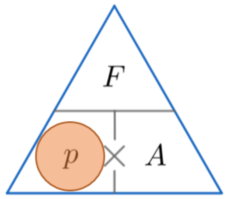
Question 3: Dividing the journey into two parts, A and B
Distance in Part A = 3 hours × 55 mph = 165 miles.
90 minutes = 1.5 hours. Thus,
Distance in Part B = 1.5 hours × 48 mph = 72 miles.
Therefore the total distance travelled is 165 + 72 = 237 miles

Question 4: 210 million = 210,000,000. Converting the time from minutes and seconds to seconds, 11 minutes = 11 × 60 seconds = 660 seconds. 11 minutes and 40 seconds = 660 seconds + 40 seconds = 700 seconds.
Hence the speed of light can be calculated as follows:
Speed of light = 210,000,000 km ÷ 700 seconds = 300,000 km/s

Question 5: Converting 35 years to seconds: 35 years = $35 \times 365 \times 24 \times 60 \times 60 = 1.104 \times 10^9$ seconds. Hence, distance $= 17 \times (1.104 \times 10^9) = 1.88 \times 10^{10}$ km

11 Pressure Force Area

Question 1: Since the square has a side length of 3 m, the area is: 3 m × 3 m = 9 m^2

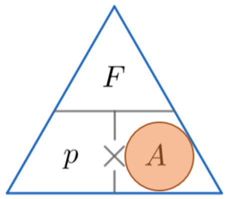


Substituting the values for the area and the force into the pressure equation as follows:

$$\text{pressure} = \frac{F}{A} = \frac{185.6}{9} = 20.6222... = 20.6 \text{ N/m}^2 \text{ (3 sf)}$$

Question 2:

$$A = \frac{F}{p} = \frac{740}{2312.5} = 0.32 \text{ m}^2$$



Question 3:

$$\text{Force} = 16 \text{ m}^2 \times 2480 \text{ N/m}^2 = 39,680 \text{ N}$$

Question 4: The area of the circular face of the cylinder in contact with the ground is: 4872 N ÷ 812 N/m^2 = 6 m^2

The formula for the area of a circle is $A = \pi r^2$. Rearranging to make the radius, r, the subject gives

$$\sqrt{\frac{a}{\pi}} = r$$

Substituting in the values gives,

$$\sqrt{\frac{6 \text{ m}^2}{\pi}} = 1.38 \text{ m}$$

Hence, diameter = 2 × 1.38 = 2.76 m

Question 5: The base of the pyramid has an area of 8 × 8 = 64 m^2
So,

$$\text{Pressure exerted by the pyramid} = 440 \text{ N} \div 64 \text{ m}^2 = 6.875 \text{ N/m}^2$$

The cube exerts the same pressure as the square-based pyramid, so the pressure exerted by the cube is also 6.875 N/m^2.
Hence, the area of the cube in contact with the ground can be calculated as follows:

$$\text{Area} = 110 \text{ N} \div 6.875 \text{ N/m}^2 = 16 \text{ m}^2$$

Thus, the side length of the cube can be calculated by taking the square root of the area:

$$\text{Side length of cube} = \sqrt{16} = 4 \text{ m}$$

1 Geometry Basics

Question 1:
$\angle CDB = 180° - 103° = 77°$ (angles on a straight line sum to 180°).

Question 2:
$x = 360° - 100° - 105° - 50° = 105°$ (angles around a point sum to 360°).

Question 3: Base angles in an isosceles triangle are equal and angles in a triangle add up to 180°. $y = 180° - 61° - 61° = 58°$.

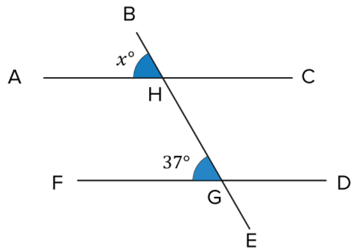
Question 4: Base angles in an isosceles triangle are equal and angles in a triangle add up to 180°,

$$x + x + 55° = 180°$$
$$2x = 180° - 55° = 125°$$
$$x = 62.5°$$

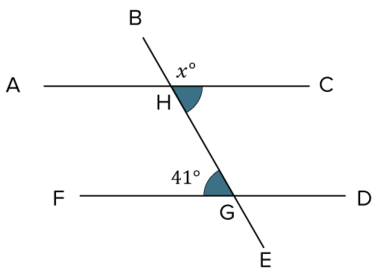
.
Question 5: $x = 180° - 115° = 65°$ (angles on a straight line sum to 180°). $y = 180° - 25° - 65°$ (angles in a triangle sum to 180°), so $y = 90°$

2 Corresponding and Alternate Angles

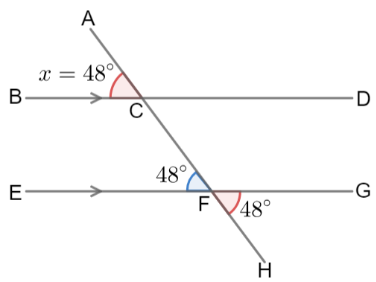
Question 1: $\angle AHB = \angle FGH$ (corresponding angles), so $\angle x = 37°$.



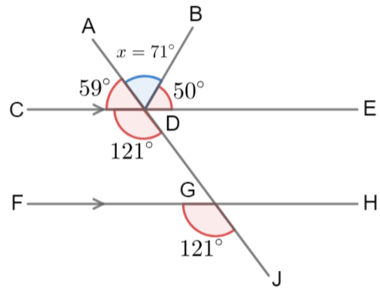
Question 2: $\angle FGH = \angle GHC$ (alternate angles), so $\angle GHC = 41°$. $x = 180° - 41° = 139°$ (angles on a straight line sum to 180°).



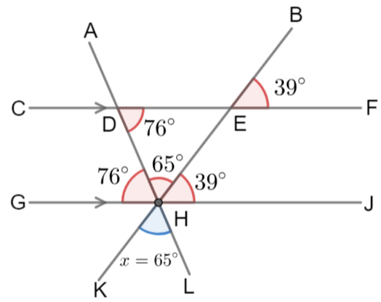
Question 3: $\angle HFG = \angle EFC$ (vertically opposite angles), so $\angle EFC = 48°$. $\angle EFC = \angle BCA$ ($\angle x$) (corresponding angles), so: $\angle BCA = x = 48°$.



Question 4: $\angle FGJ = CDG$ (corresponding angles), so $\angle CDG = 121°$. $\angle CDA = 180 - 121 = 59°$ (angles on a straight line sum to 180°). $x = 180 - 59 - 50 = 71°$ (angles on a straight line sum to 180°).



Question 5: $\angle BEF = \angle EHJ$ (corresponding angles), so $\angle EHJ = 39°$. $\angle EDH = \angle DHG$ (alternate angles), so angle DHG $= 76°$. $\angle DHE = 180 - 76 - 39 = 65°$ (angles on a straight line sum to 180°). $\angle DHE = \angle x$ (vertically opposite angles), so $x = 65°$.



3 2D Shapes

Question 1: Irregular pentagon.

Question 2: Trapezium.

Question 3: $x° = 180° - 56° = 124°$ (adjacent angles in a parallelogram sum to 180°).

4 Interior and Exterior Angles

Question 1: Sum of interior angles $= 180 \times (5 - 2) = 540°$. Hence each interior angle is, $x° = 540° \div 5 = 108°$.

Question 2: Sum of interior angles $= 180 \times (8 - 2) = 1080°$. Hence each interior angle is, $x° = 1080° \div 8 = 135°$.

Question 3: Sum of interior angles $= 180 \times (5 - 2) = 540°$. Hence,

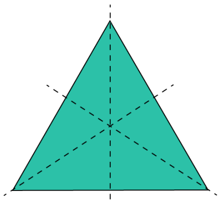
$$33 + 140 + 2x + x + (x + 75) = 540$$
$$4x + 248 = 540$$
$$4x = 292$$
$$x = 292 \div 4 = 73°$$

Question 4: Sum of interior angles $= 180 \times (4 - 2) = 360°$. $\angle CDB = 180 - (y + 48) = 132 - y$ (angles on a straight line sum to 180°). $\angle CAB = 180 - 68 = 112$ (angles on a straight line sum to 360°). So, $112 + 90 + 2y + (132 - y) = 360°$ (sum of interior angles).

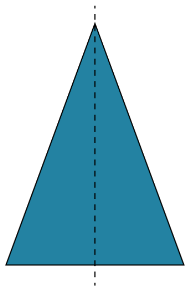
$$y + 334 = 360$$
$$y = 360 - 334 = 26°$$

5 Symmetry

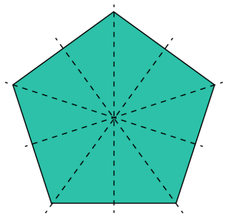
Question 1: An equilateral triangle has three lines of symmetry.



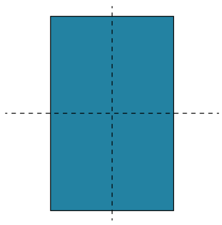
Question 2: An isosceles triangle only has one line of symmetry.



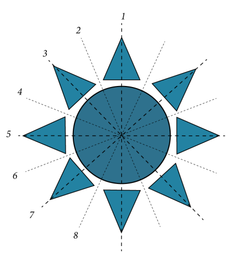
Question 3: A regular pentagon has 5 lines of symmetry.



Question 4: There are a number of shapes you could make with two lines of symmetry, the most straightforward being a rectangle.



Question 5: The shape has 8 lines of symmetry .



6 Areas of Shapes

Question 1: Area $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 11.5 \times 12 = 69$ cm^2

Question 2: Form a right-angled triangle with hypotenuse = 5 cm and base = 3 cm (8 cm − 5 cm = 3 cm).
Thus, perpendicular height $= \sqrt{5^2 - 3^2} = \sqrt{16} = 4$ cm.
Hence, Area $= \frac{1}{2}(a+b)h = \frac{1}{2}(5+8) \times 4 = 26$ cm^2.

Question 3: Area = base × height = 8 × 15 = 120 cm^2

7 Circles

Question 1: a) Circumference $= \pi d = \pi \times 8.4 = \frac{42}{5}\pi$ mm
b) Area $= \pi r^2$, $r = 8.4 \div 2 = 4.2$ mm.
So, area $= \pi \times 4.2^2 = 55.417... = 55.4$ mm^2 (3sf).

Question 2: Area $= \pi r^2 = \pi \times 5^2 = 25\pi$ cm^2

Question 3: Area $= \pi r^2 = 200$ cm^2, and $r = x$.

$$200 = \pi x^2$$
$$x = \sqrt{\frac{200}{\pi}} = 7.97... = 8.0 \text{ cm (1 dp)}$$

Question 4: Circumference $= \pi d = 120$mm, $d = x$

$$120 = \pi \times x$$
$$x = 38.2 \text{ mm (3 sf)}$$

8 Perimeter

Question 1: Area $= x^2 = 64$, so $x = 8$.
Perimeter $= 8 + 8 + 8 + 8 = 32$ m.

Question 2: Length of one side $= 21 \div 6 = 3.5$ cm.

Question 3: Length of diameter $= 2r = 2 \times 5 = 10$ cm.
Length of curved edge $= \frac{1}{2}\pi d = \frac{1}{2} \times \pi \times 10 = 5\pi$ cm.
Total perimeter $= 10 + 5\pi = 25.7$ cm (1 dp).

Question 4: Missing lengths: $120 - 55 = 65$ cm , $195 - 70 = 125$ cm.
Total Perimeter $= 120 + 70 + 65 + 125 + 55 + 195 = 630$ cm.

Question 5: $AB = BC = x + 5$

$$(x+5) + (x+5) + 3x = 45\text{cm}$$
$$5x + 10 = 45$$
$$5x = 35$$
$$x = 35 \div 5 = 7\text{cm}$$

9 Circle Sector, Segments and Arcs

Question 1: Area $= \pi r^2$, $r = 5.24 \div 2 = 2.62$ cm.
Thus, Area $= 2.62^2 \times \pi = 21.6$ cm (3 sf).

Question 2: Area of sector $= \frac{\text{angle}}{360} \times \pi r^2$.
Thus, Area $= \frac{72°}{360°} \times \pi(5)^2 = \frac{72°}{360°} \times 25\pi = 5\pi$ m^2.

Question 3:

$$\text{Area of sector} = 26.15 = \frac{x^\circ}{360^\circ} \times \pi \times 15^2$$
$$26.15 = \frac{x^\circ}{360^\circ} \times 225\pi$$
$$\frac{26.15}{225\pi} = \frac{x^\circ}{360^\circ}$$
$$x = \frac{26.15}{225\pi} \times 360^\circ$$
$$x = 13.3^\circ \text{ (1 dp)}$$

Question 4: Arc length $= \frac{\text{angle}}{360} \times 2\pi r = \frac{165}{360} \times 2\pi \times 14 = 40.3$mm. Thus, total perimeter $= 14 + 14 + 40.3 = 68.3$mm.

Question 5:

$$\text{Area of sector} = 160 = \frac{x^\circ}{360^\circ} \times \pi \times 9^2$$
$$160 = \frac{x^\circ}{360^\circ} \times 81\pi$$
$$\frac{160}{81\pi} = \frac{x^\circ}{360^\circ}$$
$$x = \frac{160}{81\pi} \times 360^\circ$$
$$x = 226^\circ \text{ (to 3 sf)}$$

10 Congruent Shapes

Question 1: B and F are congruent, E and G are congruent.

Question 2: P and Q are congruent, M and K are congruent.

Question 3: A and H are congruent, D and G are congruent.

11 Similar Shapes

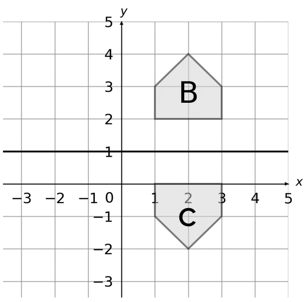
Question 1:
Scale factor, $SF = 5 \div 2 = 2.5$.
So $x = 2.5 \times 3 = 7.5$ cm.

Question 2:
a) $SF = 42 \div 14 = 3$.
b) AC $= 51 \div 3 = 17$ cm.

Question 3:
a) $SF = 6 \div 3 = 2$.
b) BE $= 4.4 \times 2 = 8.8$ cm.

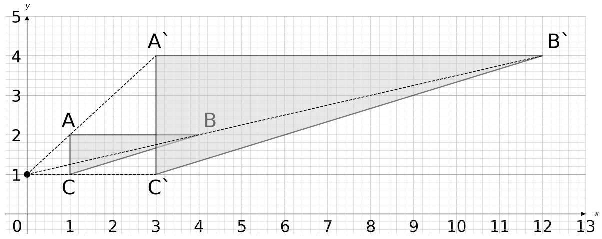
12 Transformations

Question 1:

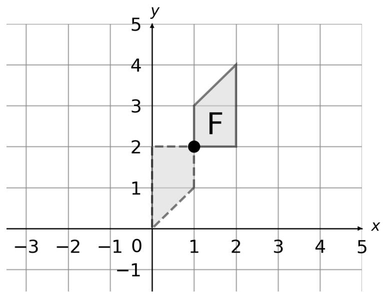


Question 2: Translation, by the vector $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

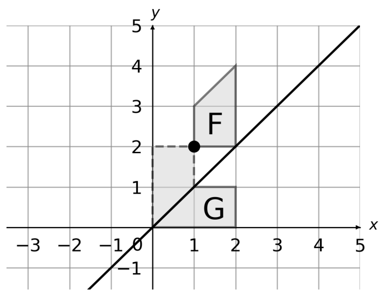
Question 3:



Question 4: First, perform the rotation,

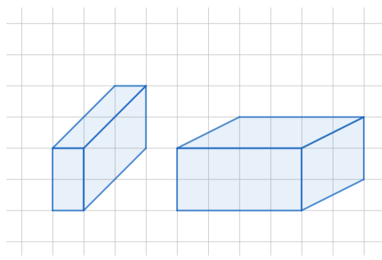


Then reflect in the line $y = x$



13 3D Shapes

Question 1: Two examples of cuboids are shown below.



Question 2:
a) Square-based pyramid
b) Cone
c) Cylinder

Question 3: Total faces $= 2$ (ends) $+ 3$ (sides) $= 5$.

Question 4: 6 faces, 12 edges, 8 vertices.

Question 5: 1 face, 0 edges, 0 vertices.

14 Volume of 3D shapes

Question 1: Volume $= 3 \times 12 \times 16 = 576$ cm^3

Question 2: Volume $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 5^2 \times 12 = 100$ m^3

Question 3:

$$\begin{aligned} \text{Area of cross section } &= \frac{1}{2} \times (45 + 60) \times 20 = 1{,}050 \text{ cm}^2 \\ \text{Volume of prism } &= 1{,}050 \times 80 = 84{,}000 \text{ cm}^3 \end{aligned}$$

Question 4:

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \text{base area} \times \text{height} \\ 54 &= \frac{1}{3} \times 18 \times (x + 5) \\ 54 &= 6(x + 5) \\ x + 5 &= 9 \\ x &= 4 \text{ cm} \end{aligned}$$

Question 5:

$$\begin{aligned} \text{Volume of cylinder } &= \pi \times r^2 h = \pi \times (2.3)^2 \times 5.6 \approx 93.07\text{m}^3 \\ \text{Volume of hemisphere } &= \frac{1}{2} \times (\frac{4}{3}\pi \times (2.3)^3) \approx 25.48\text{m}^3 \\ \text{Total volume } &= 93.0665... + 25.4825... = 119 \text{ m}^3 \text{ (3 sf)} \end{aligned}$$

15 Surface Area

Question 1: 3 pairs of faces:
front/back area $= 2 \times (4 \times 2.5) = 20$ mm^2.
top/bottom area $= 2 \times (2.5 \times 6) = 30$ mm^2.
left/right area $= 2 \times (4 \times 6) = 48$ mm^2.
total area $= 20 + 30 + 48 = 98$ mm^2.

Question 2: Let l = slant height

$$\begin{aligned} \text{Surface area} &= \pi r l + \pi r^2 \\ 3l\pi + 3^2\pi &= 120 \\ 3l\pi + 9\pi &= 120 \\ l &= \frac{120 - 9\pi}{3\pi} = 9.7 \text{ cm (1 dp)} \end{aligned}$$

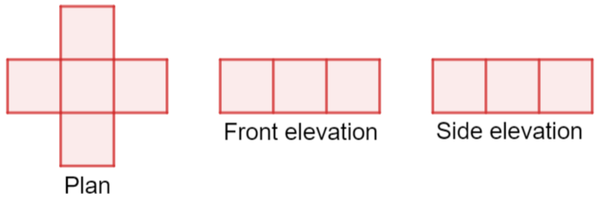
Question 3:
Surface area of sphere $= 4\pi r^2 = 4\pi(8.5)^2 = 907.9$ m^2.
No. of tins required $= 907.9 \div 10 = 90.8$, i.e. 91 tins are required.
Total cost $= 91 \times £9.60 = £873.60$.

Question 4:
Area of the two triangular faces $= 2 \times (\frac{1}{2} \times 6 \times 8) = 48$ cm^2.
Area of the rectangular base $= 6 \times 11 = 66$ cm^2.
Slanted height of the prism $=$ AB $= \sqrt{8^2 + 3^2} = \sqrt{73}$ cm.
Area of the two sides $= 2 \times 11 \times \sqrt{73} = 22\sqrt{73}$.
Total surface area $= 48 + 66 + 22\sqrt{73} = 301.97$ cm^2.

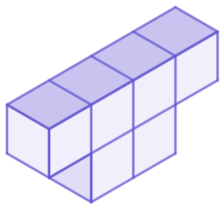
Question 5:
Area of square base $= 12 \times 12 = 144$ cm^2.
Length of midpoint of DC to E $= \sqrt{10^2 + 6^2} = \sqrt{136}$ mm.
Hence, area of 4 triangle faces $= 4 \times (\frac{1}{2} \times 12 \times \sqrt{136}) = 279.89$ cm^2.
Total surface area $= 279.89 + 144 = 423.89$ cm^2.

16 Projections, Plans and Elevations

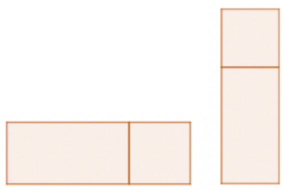
Question 1: All 3 projections are shown below.



Question 2: See 3D diagram below.

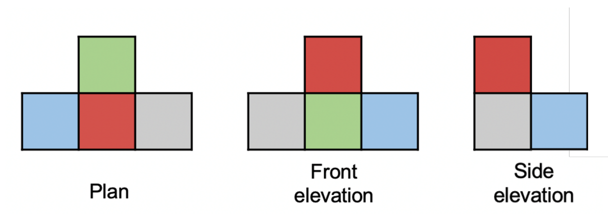


Question 3: The plan (left) and side (right) elevations are shown below.



Question 4: The plan (left) and side (right) elevations are shown below.

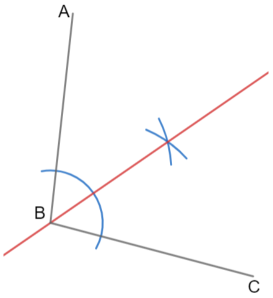
Question 5: All 3 projections are as seen below.



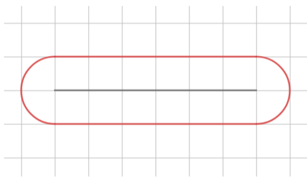
17 Loci and Construction

Question 1: The bisector of an angle is a line segment which divides the angle into two equal parts.

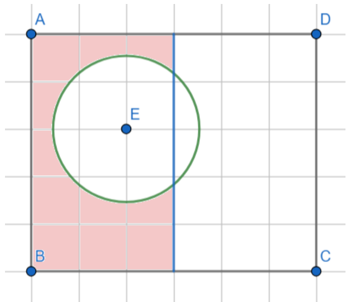
Question 2: The correct construction is a bisection of an angle, as shown below.



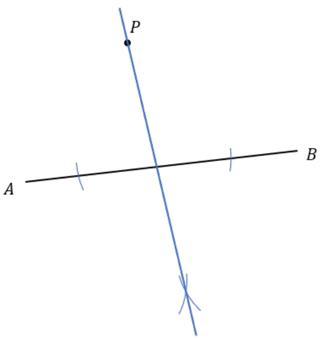
Question 3:



Question 4: Considering only the area 3m away from the house, draw a line parallel to CD and 3 cm away from it. The locus of points which are 1.5m away from the tree at E will be a circle of radius 1.5cm. Shade the area outside the two excluded regions, as shown below.

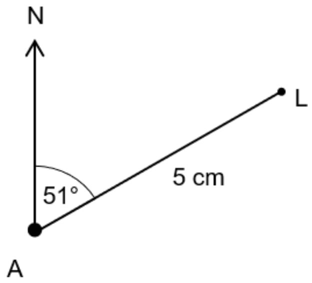


Question 5: Construction of a line perpendicular to AB passing through point P as shown:

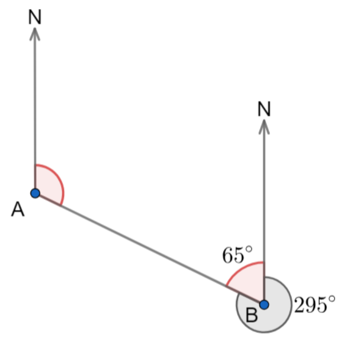


18 Bearings

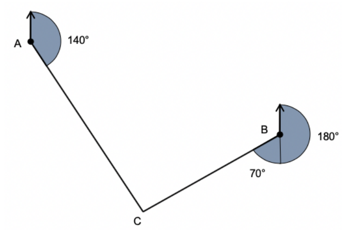
Question 1: Let the lighthouse be L and the boat be B. L from B is given by an angle of 051° and a distance of 5 cm. The final diagram should look like,



Question 2: $360° - 295° = 65°$ (angles around a point sum to 360°). Two North lines are parallel, so $180 - 65° = 115$ (co-interior/allied angles sum to 180°). Hence, Bearing of B from A $= 180° - 65° = 115°$.



Question 3: C is the point of intersection lines drawn along both bearings.



Question 4: By use of a protractor or otherwise, the angle is measured to be 60 degrees, so the bearing is, 060°

Question 5: Two North lines are parallel, so $180° - 60° = 120°$ (co-interior/allied angles sum to 180°).
So, $360° - 120° = 240°$ (angles around a point sum to 360°).
Hence, Bearing of A from B = 240°.

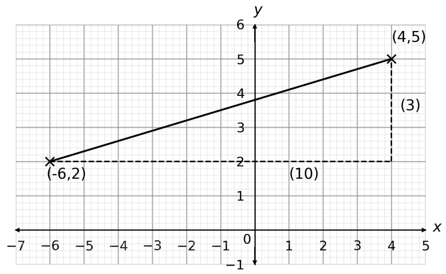
1 Pythagoras

Question 1: The missing side is the hypotenuse. Substituting the other 2 sides into the equation $a^2+b^2=c^2$ gives:

$$c^2 = 8^2 + 14^2$$
$$c^2 = 64 + 196 = 260$$
$$c = \sqrt{260} = 16.1245...$$

So $BC = c = 16.1$ cm (1 dp)

Question 2: Sketch a right-angled triangle by plotting the given points:



The distance between the points is given by the hypotenuse of the right-angled triangle. Substituting the known sides into the equation $a^2+b^2=c^2$ gives:

$$c^2 = 10^2 + 3^2$$
$$c^2 = 100 + 9 = 109$$
$$c = \sqrt{109} = 10.4403... = 10.4 \text{ cm (3 sf)}$$

Question 3: The missing side is the hypotenuse. Substituting the known sides into the equation $a^2+b^2=c^2$ gives:

$$c^2 = 5.9^2 + 6.7^2$$
$$c^2 = 34.81 + 44.89 = 79.7$$
$$c = \sqrt{79.7} = 8.927... = 8.9 \text{ cm (1 dp)}$$

Question 4: Substituting the known sides into $c^2 = a^2 + b^2$ gives: $5.1^2 = LN^2 + 3.1^2$
Solving for LN,

$$5.1^2 = LN^2 + 3.1^2$$
$$26.01 = LN^2 + 9.61$$
$$LN^2 = 16.4$$
$$LN = \sqrt{16.4} = 4.0496... = 4.0 \text{ cm (1 dp)}$$

Question 5: Substituting the known sides into $c^2 = a^2+b^2$ (and letting the height of the wall be a) gives:

$$2.9^2 = a^2 + 1.3^2$$
$$a^2 = 8.41 - 1.69 = 6.72$$
$$a = \sqrt{6.72} = 2.592 = 2.6 \text{ cm (1 dp)}$$

2 Trigonometry

Question 1: 'CAH': $\cos(43°) = \frac{35}{p}$.
$p = \frac{35}{\cos(43°)} = 47.85646... = 47.9$ m (3 sf).

Question 2: 'SOH': $\sin(q) = \frac{13}{15}$
$q = \sin^{-1}\left(\frac{13}{15}\right) = 60.0735.. = 60.1°$(1 dp)

Question 3: According to 'SOH CAH TOA', the sin of w must be equal to the opposite side divided by the hypotenuse. We can use Pythagoras to find the hypotenuse. If the hypotenuse is c, then a and b are both 2, so the equation $a^2+b^2=c^2$ becomes: $c^2 = 2^2+2^2 = 4+4 = 8$, so $c = \sqrt{8} = 2\sqrt{2}$.

'SOH': $\sin(w) = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

Question 4: 'SOH': $\sin(30°) = \frac{CB}{12}$
$CB = 12\sin(30°) = 6.0$ cm (1 dp).

Question 5: 'TOA': $\tan(x) = \frac{4}{7}$.
$x = \tan^{-1}\left(\frac{4}{7}\right) = 29.7448813...$, $x = 29.7°$(1 dp).

3 Column Vectors

Question 1:

$$2\mathbf{a} = 2 \times \begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 \\ 16 \end{pmatrix}$$
$$2\mathbf{a} + \mathbf{b} = \begin{pmatrix} 6 \\ 16 \end{pmatrix} + \begin{pmatrix} -7 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 18 \end{pmatrix}$$

Question 2: $3\mathbf{a} - 2\mathbf{b} = 3 \times \begin{pmatrix} 2 \\ 7 \end{pmatrix} - 2 \times \begin{pmatrix} -5 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 21 \end{pmatrix} - \begin{pmatrix} -10 \\ 6 \end{pmatrix} = \begin{pmatrix} 16 \\ 15 \end{pmatrix}$

Question 3:
$\mathbf{a} + 2\mathbf{b} - \mathbf{c} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + 2 \times \begin{pmatrix} 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 10 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 14 \\ -5 \end{pmatrix}$

1 Probability Basics & Listing Outcomes

Question 1: $P(A) + P(B) = 55\% + 40\% = 95\%$
$P(C) = 100\% - 95\% = 5\%$

Question 2: a) Probability of Jimmy not watching a romantic comedy:
$1 - 0.56 = 0.44$

Since the probability of Jimmy watching a sci-fi movie or a horror film is equal, the probability of Jimmy watching a sci-fi movie must be half of this amount: $0.44 \div 2 = 0.22$

b) $0.22 + 0.56 = 0.78$

Question 3: BY was already given in the question, so the full list of other possible outcomes is:
BO, BW, NO, NY, NW, PO, PY, PW

Question 4: a)

$$1 = 0.25 + 5x + 4x$$
$$0.75 = 9x$$
$$x = \frac{1}{12}$$

Blue: $5x = \frac{5}{12}$

b) Green: $4x = \frac{4}{12} = \frac{1}{3}$

Question 5: a)
There are 25 odd numbers in total between 1 and 50:
1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

There are 10 multiples of 5 between 1 and 50:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50

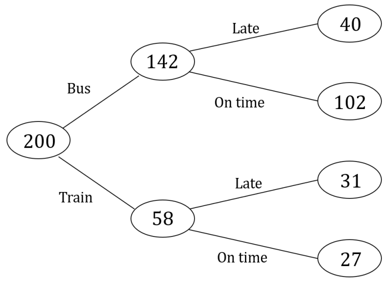
However, some of these multiples of 5 also feature on the odd number list, so cannot be counted twice. So, ignoring the odd multiples of 5, there are only 5 multiples of 5 remaining.
25 odd numbers + 5 (even) multiples of 5 = 30 numbers in total.
Hence, P(Multiple of 5 or odd) $= \frac{30}{50} = \frac{3}{5}$

b) The factors of 48 are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8. This means that 10 numbers out of the 50 in the hat are factors of 48.
Hence, P(factor of 48) $= \frac{10}{50} = \frac{1}{5}$

2 Frequency Trees

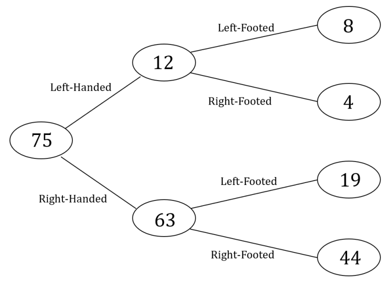
Question 1: 200 people in total.
58 travelled by train.
$200 - 58 = 142$ travelled by bus .
$142 - 40 = 102$ by bus and on time.

$71 - 40 = 31$ by train and late.
$58 - 31 = 27$ by train and on time.



Question 2: a) 75 people in total.
63 right-handed.
$75 - 63 = 12$ left-handed people.
Of the left-handed people, one third are right-footed:
$12 \div 3 = 4$ left-handed and right-footed people.

$12 - 4 = 8$ left-handed and left-footed people.
27 people in total are left-footed.
$27 - 8 = 19$ right-handed and left-footed people.
$63 - 19 = 44$ right-handed and right-footed people.



b) There are 63 people who are right-handed, and 44 of them are right-footed: $\frac{44}{63}$

c) There are 75 people in total and the number of right-footed people is $44 + 4 = 48$: $\frac{48}{75} = \frac{16}{25}$

Question 3: a) 13 students failed French and this was $\frac{1}{12}$ of the entire French group.
French students: $12 \times 13 = 156$.
$156 - 13 = 143$ took French and passed.

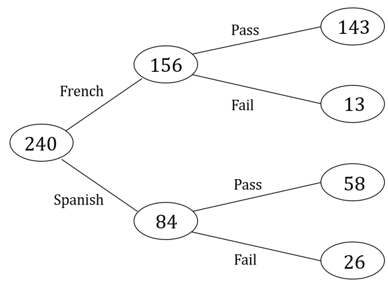
65% of the year group took French. This represents 156 students.
Total students:

$$156 \div 65 \times 100 = 240 \text{ students}$$

35% of the year group took Spanish:

$$0.35 \times 240 \text{ students} = 84 \text{ Spanish students}$$

The number of students who failed Spanish was twice the number that failed French. 13 failed French, so,
$2 \times 13 = 26$ failed Spanish and
$84 - 26 = 58$ passed Spanish



b) The total in the year group was 240 students. Of these 240, 143 passed French. $\frac{143}{240} \times 100 = 60\%$

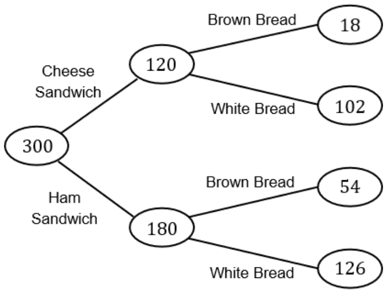
Question 4: a) 15% of the pupils chose a cheese sandwich on brown bread and that this figure represents a total of 18 pupils:

$$(18 \div 15) \times 100 = 120 \text{ pupils who select cheese}$$

120 pupils chose a cheese sandwich, 18 of them had brown bread, then $120 - 18 = 102$ cheese sandwich on white bread.
The cheese sandwich to ham sandwich ratio is $2:3$. So $\frac{2}{5}$ of these pupils had a cheese sandwich and $\frac{3}{5}$ of these pupils had a ham sandwich. $\frac{3}{5} = 180$ pupils chose ham.

$120 \text{ cheese} + 180 \text{ ham} = 300$ pupils total.
Of the people that chose a ham sandwich, $\frac{3}{10}$ opted for brown bread.
$\frac{3}{10} \times 180 = 54$ ham sandwich on brown bread.

$180 - 54 = 126$ ham sandwich on white bread.



b) There were a total of 300 pupils and 54 pupils chose a ham sandwich on brown bread: $\frac{54}{300}$ or $\frac{9}{50}$

Question 5: Votes were shared between the 3 parties in the ratio of $7:6:3$. So the Conservative Party received $\frac{7}{16}$ of the votes, the Labour party $\frac{6}{16}$ and the Green Party $\frac{3}{16}$. If 24,750 voted for the Green party and this represented $\frac{3}{16}$ of the total number of votes received, then the total number of votes is

$$(24,750 \div 3) \times 16 = 132,000 \text{ votes}$$

The total number of votes received by the Conservative party was:

$$\frac{7}{16} \times 132,000 = 57,750 \text{ votes}$$

The number of votes received by the Labour party was:

$$\frac{6}{16} \times 132,000 = 49,500 \text{ votes}$$

The votes cast by men and by women for the Labour party were in a ratio of $3:2$. This means that $\frac{3}{5}$ of the votes were cast by men and $\frac{2}{5}$ by women. The total number of votes cast by men was:

$$\frac{3}{5} \times 49,500 = 29,700 \text{ votes}$$

The total number of votes cast by women was:

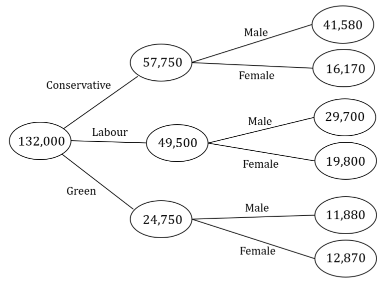
$$\frac{2}{5} \times 49,500 = 19,800 \text{ votes}$$

The number of female votes received by the Green party was 65% of the number of female votes received by the Labour party.

$0.65 \times 19,800 = 12,870$ votes

Green party male votes:
$24,750 - 12,870 = 11,880$ male votes

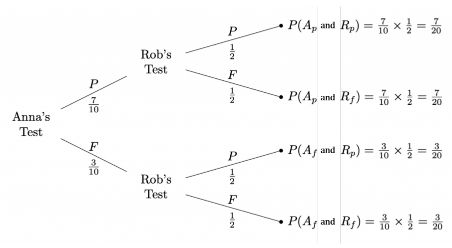
The number of male votes for the Conservative party was 40% more than the male votes received by the Labour party. The number of male votes received by the Labour party was 29,700, so there are $29,700 \times 1.4 = 41,580$ male Conservative votes.
The Conservative party received 57,750 votes in total:
$57,750 - 41,580 = 16,170$ female Conservative votes



3 Probability and Tree Diagrams

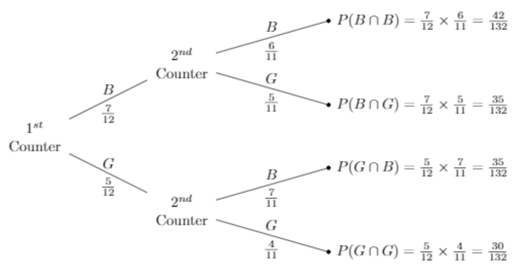
Question 1: (a) Let "Anna passing" be event A_p and "Rob" passing be event R_p. The probability of both passing is: $P(A_p \text{ and } R_p) = 0.35$
$0.7 \times P(R_p) = 0.35$
$P(R_p) = 0.35 \div 0.7 = 0.5$

(b) The probability of both Anna and Rob failing their driving test can be found using a tree diagram:

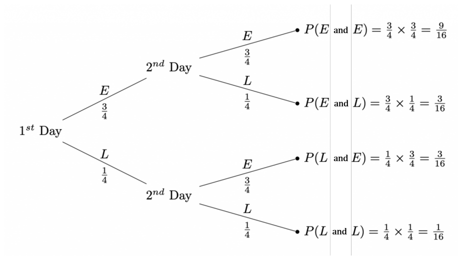


Hence the probability of them both failing is $\frac{3}{20} = 0.15$.

Question 2: Draw a tree diagram without replacement. Adding together the probabilities of the result being blue then blue or green then green: $\frac{7}{22} + \frac{5}{33} = \frac{31}{66}$



Question 3: Bottom line: Probability of being late of both days is $\frac{1}{16}$



4 Relative Frequency

Question 1: a) 12 students out of the total of 52 used public transport: $\frac{12}{52} = 0.231$ to 3 decimal places.

b) 16 students out of the total of 52 walked to school: $\frac{16}{52} = 0.308$ to 3 decimal places

c) 9 of the 52 students cycled, so the number who didn't cycle is: $52 - 9 = 43$.
$\frac{43}{52} = 0.827$ to 3 decimal places

Question 2: In Bev's experiment, the relative frequency of rolling a 6 is: $\frac{63}{400} = 0.1575$. The probability of rolling a 6 is: $\frac{1}{6} = 0.166666....$ The two results are fairly similar, i.e. the die does not appear to be biased, so Bev's statement is false.

Question 3: a) $\frac{32}{32+48} = 0.4 = 40\%$.
Hannah's statement is correct.

b) The total number of cars spotted was $32 + 41 + 48 + 111 = 232$ cars. The total number of silver cars spotted was $32 + 41 = 73$ silver cars.
$\frac{73}{232} = 0.315$ to 3 decimal places.

c) $\frac{73}{232} \times 5,000 = 1,573$ silver cars

Question 4: a) Bob throws the die 100 times and throws a 7 on 15 occasions: $15 \div 100 = 0.15$

b) Susan throws the die 500 times and throws a 7 on 60 occasions: $60 \div 500 = 0.12$

c) Susans results are more accurate because the more times you conduct an experiment, the more accurate the estimate will be. The expected probability of rolling a 7 is $\dfrac{1}{8} = 0.125$, which is closer to Susan's result.

Question 5: a) 16 students $= \dfrac{1}{10} = 0.1$.
Total students $= 16 \times 10 = 160$.

b) $\frac{4}{10} \times 160$ students $= 64$ students chose Pizza Cottage.

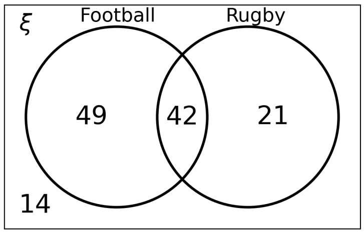
c) $\frac{1}{2} \times 160$ students $= 80$ students chose Dazza's Fish and Chips.

5 Venn Diagrams

Question 1: a) 14 outside, 42 in the intersection.

7 : 3 means that $\frac{7}{10}$ of the 70 students play football only and $\frac{3}{10}$ of the 70 students play rugby only.

The number of students who play football only is: $\frac{7}{10} \times 70 = 49$ students
The number of students who play rugby only is: $\frac{3}{10} \times 70 = 21$ students



b) 21 students play rugby only out of the total 126 students: $\frac{21}{126} = \frac{1}{6}$

Question 2: a) The number of students in year 10 is

$$40 + 52 + 14 + 36 = 142 \text{ students}$$

b) There are 142 students in total and, of these, 14 study both history and geography: $\frac{14}{142} = \frac{7}{71}$

Question 3: a) All **80** students like at least **1** of the animals. So, the numbers in the Venn diagram should add up to 80.

15 students like all **3** animals. So, insert '15' in the intersection of all 3 circles.

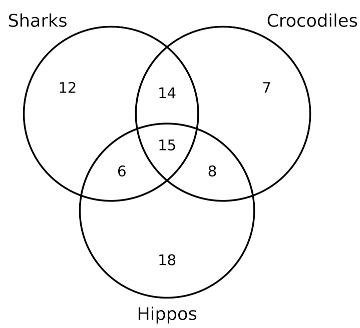
14 students like sharks and crocodiles but do not like hippos. So, insert 14 in the section where sharks and crocodiles intersect.

23 students like crocodiles and hippos. 15 students like sharks, crocodiles and hippos, so there are $23 - 15 = 8$ students who like crocodiles and hippos only. So, insert 8 in the intersection of crocodiles and hippos.

21 students like sharks and hippos. 15 students like sharks, crocodiles and hippos. So, $21 - 15 = 6$ students like sharks and hippos, but not crocodiles. Therefore insert 6 in the intersection of sharks and hippos.

44 students like crocodiles. 14 students like crocodiles and sharks, 15 students like sharks, crocodiles and hippos, and 8 like crocodiles and hippos, so $14+15+8 = 37$ students like crocodiles. Therefore, there are $44 - 37 = 7$ students who like crocodiles only. Therefore insert 7 in the part of the crocodile circle that does not overlap with any other section.

12 students only like sharks. So insert 12 in the sharks only section. Finally, $80 - 12 - 14 - 7 - 6 - 15 - 8 = 18$ students that only like hippos.



b) There are 80 students in total, of which 18 only like hippos: $\frac{18}{80}$ or $\frac{9}{40}$

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6 Set Notation

Question 1: a) A is the subset consisting of even numbers: $A = \{104, 110, 112, 114\}$

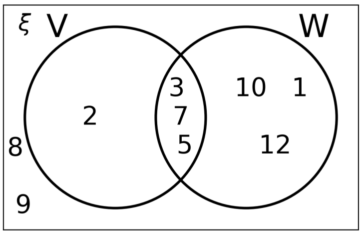
$A \cup B = \{103, 104, 110, 112, 114\}$

b) A' is the group of odd (not even) numbers from the universal set ξ: $A = \{103, 105, 109\}$

Question 2: $V = \{2, 3, 5, 7\}$

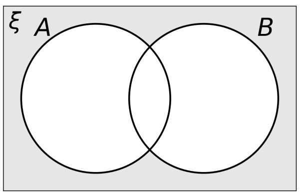
$V \cap W = \{3, 5, 7\}$. So, 3, 5 and 7 go in the intersection. The only remaining number from V, the number 2, needs to be placed inside the V circle, but outside the W circle.

1, 10 and 12 from $V \cup W$ need to be placed inside W, but outside V. Finally, 8 and 9 need to placed outside the circles, but still inside the rectangle.



Question 3: Write the variable followed by a colon before the inequality, and then put everything inside curly brackets. The results are as follows:
a) $\{x : x \geq 12\}$
b) $\{z : z < -2\}$
c) $\{a : a > 0\}$
d) $\{x : 13 < x\}$

Question 4: $(A' \cap B')$ means anything not in A and not in B, which is everything outside of the circles, so the Venn diagram should be shaded similar to:



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7 Types of Data

Question 1:
a) Continuous.
b) Continuous.
c) Categorical.
d) Discrete.

Question 2:
a) Primary.
b) Continuous.

Question 3: Tahani is wrong because although a shoe size is based on foot length, the length of a person's foot can be of any value, whereas shoe sizes have limited values (5, 5 and a half, 6, 6 and a half etc.).

Question 4:
a) Primary.
b) By collecting the data himself, he can ensure that the numbers are all accurately recorded. A second advantage is that he can make efforts to make sure his sample is representative (he can ask people of different genders, races, ages, etc.).

Question 5: a) Since the data that Steve collects from the first half of the class is worded data, this is categorical data.

b) Since the data that Steve collects from the second of the class is numerical, this is quantitative data. Since the data can only take certain values (numbers between 1 and 10), the data is discrete quantative data.

c) The first disadvantage of collecting data in this way is that it is harder to analyse. The second disadvantage is that there are only 6 options for the worded responses, whereas there are eleven options for numbered responses between 0 and 10.

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8 Mean, Median, Mode and Range

Question 1:
Since the number 350 occurs 3 times, it is the most common value, so:
Mode $= 350$.
Range $= 590 - 280 = 310$.

Question 2: Order the set of values:
$154, 163, 164, 168, 170, 179, 185, 188$.

There are 8 values, so: $\frac{8+1}{2} = 4.5$ So the median is half-way between the 4th value and the 5th value. The 4th value is 168 and the 5th value is 170, so the median is 169.

Question 3: a) The sum of the reaction times is $0.25 + 0.34 + 0.39 + 0.38 + 0.39 + 1.67 + 0.28 + 0.3 + 0.42 + 0.46 = 4.88$.
Then, Mean $= \frac{4.88}{10} = 0.488$

b) 1.67 is the outlier as it is vastly higher than all the other values. If this outlier were removed, then the mean would be lower.

Question 4: Total length: $7 \times 1.35\text{m} = 9.45\text{m}$

When the extra plank of wood is added, the mean length of a plank of wood increases to 1.4m. This means there are now 8 planks of wood, with a combined length of: $8 \times 1.40\text{m} = 11.2\text{m}$. Therefore, the length of this extra plank of wood is: $11.2\text{m} - 9.45\text{m} = 1.75\text{m}$.

Question 5: The combined weight of all 8 members is:
$63 + 60 + 57 + 66 + 62 + 65 + 69 + 58 = 500\text{kg}$.
The combined weight of the team is: $1.02 \times 500 = 510\text{kg}$.
The mean weight following this weight gain is: $510\text{kg} \div 8 = 63.75\text{kg}$.

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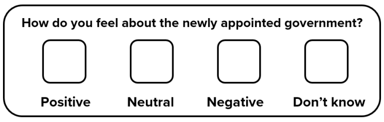
9 Data Sampling and Questionnaires

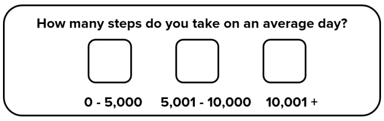
Question 1: First criticism: leading question - she leads people into agreeing with her opinion that the new government will be a disaster.
Second criticism: not enough options - somebody might have no opinion on the matter, or they could be neutral about it.
Here is an example of an improved question:



Question 2: The first criticism: crossovers between the options (values such as £30 appear twice in different boxes).
Second criticism: not enough options - there is no suitable box to tick for someone who spends more than £120 on food every week.

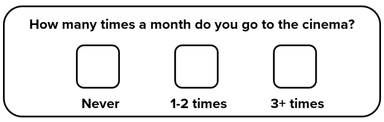
Because he is only asking people at the end of his street, Saru will probably get answers from people with a similar level of income so the range of expenditure will not be representative of a larger group.

Question 3: a) Although it does specify the time frame, the results will depend on the time of day the person is completing the questionnaire. Overlapping response boxes. Someone who has taken 5000 steps could tick two of the responses.
b) Example of improved question:



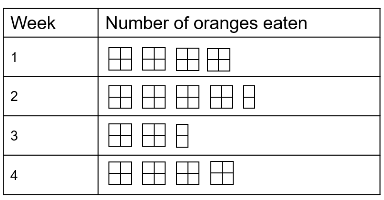
Question 4: There needs to be some time frame referenced in the question, otherwise people will answer over varying time frames. Including an option for zero and having no overlapping response boxes is also important.

Question 5: The original questionnaire has very subjective answers that are qualitative rather than quantitative. A better questionnaire has options with a specified number of average visits over a certain time frame, giving more useful data.



10 Pictographs

Question 1: Using a key of 1 large square = 2 oranges eaten etc. gives the following pictograph:



Question 2: a) $4\frac{1}{4}$ pictures $= (\frac{1}{4} \times 2) + (4 \times 2) = 8.5$km

b) $6 = 3$ pictures.
There are 3 days - Tuesday, Thursday, and Friday - where she achieved her goal.

Question 3: a) 1 square = £20, and $\frac{1}{2}$ circle = £10 etc.
Sally: 3 squares $= 3 \times £20 = £60$
Ahmed: 3 squares $= 3 \times £20 = £60$
Delaine: $1\frac{1}{4}$ squares $= 1.25 \times £20 = £25$
Priti: 3 squares $= 3 \times £20 = £60$
Annabelle: $4\frac{1}{2}$ squares $= 4.5 \times £20 = £90$
Derek: $3\frac{3}{4}$ squares $= 3.75 \times £20 = £75$

Sally, Ahmed, Priti, Annabelle and Derek raised more than £50, so 5 people raised more than £50.

b) There was just 1 student who raised less than £60 and that was Delaine (do not count Sally or Ahmed since £60 is not less than £60).
As a fraction, this is $\frac{1}{6}$.

c) Total of money raised: $£60 + £60 + £25 + £60 + £90 + £75 = £370$
Mean: $£370 \div 6 = £62$ to the nearest pound

Question 4: 60 Purple Emperors $= 1\frac{1}{2}$ butterflies. Therefore, each large square in the pictograph must represent: 1 image $= 60 \div 1.5 = 40$ butterflies.

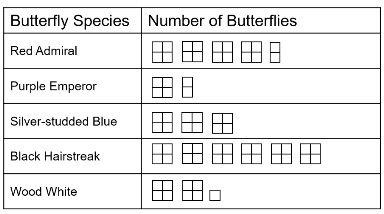
There are 3 times as many Red Admirals as there are Purple Emperors.
$3 \times 60 = 180$ Red Admirals.

The number of Silver-studded Blue butterflies is $\frac{2}{3}$ the number of Red Admirals.
$180 \times \frac{2}{3} = 120$ Silver-studded Blues.

There are 100% more Black Hairstreaks than there are Silver-studded Blue butterflies.
$2 \times 120 = 240$ Black Hairstreaks.

Finally, the number of Wood Whites is 37.5% of the number of Black Hairstreaks.
$240 \times 0.375 = 90$ Wood Whites.

Red Admiral: $180 \div 40 = 4.5$ full squares
Silver-studded Blue: $120 \div 40 = 3$ full squares
Black Hairstreak: $240 \div 40 = 6$ full squares
Wood White: $90 \div 40 = 2.25$ full squares



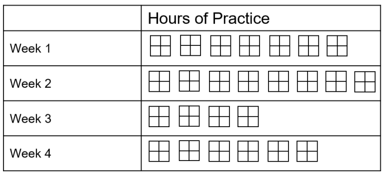
Question 5: $2:3$ is equivalent to $x:24$. So $x = 16$, where x represents the week 3 value.

$2:1$ is equivalent to $y:16$, so $y = 32$, where y is the week 2 value.

$7:8$ is equivalent to $z:32$, so $z = 32$, where z is the week 1 value.

Since all of the above numbers are divisible by 4, each pictograph image will represent 4 hours of practice.

For week 1, $28 \div 4 = 7$ complete images
For week 2, $32 \div 4 = 8$ complete images
For week 3, $16 \div 4 = 4$ complete images
For week 4, $24 \div 4 = 6$ complete images



11 Stem and Leaf Diagrams

Question 1: The first digits will be in the stem section, and the second digits will be in the leaf section:

Stem	Leaf				
1	4	9			
2	2	4	9		
3	2	5	5	6	8
4	1	7			

Key: 1 | 4 means 14 cm

Question 2: a) The mode is 12.

b) The largest value is 51 and the smallest is 9, so the range is $51 - 9 = 42$.

c) All the journeys under 20 minutes are the ones that appear in the 0 stem or the 1 stem. There are 8 journeys in these two sections from a total of 15: $\frac{8}{15} = 53.3\%$ (1 dp).

Question 3: a) The range of the calves' weights is $143 - 93 = 50$ pounds.

b) Mode = 121 pounds.

c) There are 24 calves in total. The sum of their weights is:

$93+97+97+98+101+103+106+106+109+112+112+114+114+115+118+120+121+121+121+123+126+132+138+143 = 2740$ pounds
The mean weight of the calves is, $\frac{2740 \text{ pounds}}{24 \text{ calves}} = 114$ pounds

d) The mean weight of the 24 calves is 114 pounds. Introducing another calf to the data set that is greater than 114 pounds will increase the mean weight of the calves.

Question 4: a) $n = 17$, so: $\frac{17+1}{2} = 9$.

The median is the 9th value which is 19.

b) Jamal has played in 17 matches in total. In these matches, he has scored 10 points or less on 4 occasions: $\frac{4}{17}$

c) In the 17 matches that Jamal plays, he scores between 20 and 35 points on 5 occasions: $\frac{5}{17} = 29\%$.

d) Jamal has scored more than 25 points on 7 out of 17 occasions. Since the team always wins when Jamal scores over 25 points (and loses when he doesn't), then the team wins 7 times out of 17. Therefore the probability of winning the next game is: $\frac{7}{17} = 0.4$ (1 dp).

Question 5: a) The modal score for spanish is 57.

b) In the French side of the stem and leaf diagram, there are 5 values listed in the 7 stem group. The modal group is therefore the 70s or the 70 - 79 group.

c) The range for French is $87 - 42 = 45$.
The range for Spanish is $89 - 32 = 57$.
The difference is: $57 - 45 = 12$.

d) $n = 19$, so $\frac{19+1}{2} = 10$. hence the median is the 10th value or both.

Spanish median = 60.
French Median = 65.

The difference between the two median scores is 5, so 5 as a percentage of the median Spanish score is: $\frac{5}{60} \times 100 = 8\%$. Hence, the median French score is 8% greater than the median Spanish score (nearest %).

12 Frequency Tables

Question 1: a) The highest frequency is the 1 bathroom category, so the mode is 1.
There are $30 + 21 + 5 + 7 + 3 = 66$ values in total, so $n = 66$, and $\frac{66+1}{2} = 33.5$. This means that the median is halfway between the 33rd and the 34th value.

The first 30 values are in the 1 bathroom category, and the following 23 values are in the 2 bathroom category. Therefore values 33 and 34 are in the 2 bathroom category so the median is 2 bathrooms.

b) It is not possible to calculate the mean due to the fact that there is a category of 5 bathrooms or more. It is not clear exactly how many bathrooms people in this category have.

Question 2: a) The mode (most common value) is 1 goal per game.

b) There are $7 + 14 + 13 + 8 + 3 + 4 + 1 = 50$ values in total, so $n = 50$, and $\frac{50+1}{2} = 25.5$. This means that the median is halfway between the 25th and the 26th value.

The first 7 values are in the 0 goals category, and the following 14 values are in the 1 goal category. This means that the first 21 values fall in the 0 goal or the 1 goal category. The following 13 values fall into the 2 goal category, so values 25 and 26 must be in this category, so the median is 2 goals.

c) Multiply the number of goals by the frequency:

7×0 goals = 0 goals

14×1 goal $= 14$ goals
13×2 goals $= 26$ goals
8×3 goals $= 24$ goals
3×4 goals $= 12$ goals
4×5 goals $= 20$ goals
1×6 goals $= 6$ goals

Total number of goals scored $= 0 + 14 + 26 + 24 + 12 + 20 + 6 = 102$
Hence, the mean number of goals scored is, 102 goals $\div 50$ games $= 2$ goals (to the nearest goal)

Question 3: a) The total of frequency column is 240. $240 - 15 - 76 - 32 - 9 = 108$ divers. Therefore, $x + y = 108$ divers.
The ratio of x to y is $7:5$. This means that x is $\frac{7}{12}$ of the total and y is $\frac{5}{12}$ of the total.

$x = \frac{7}{12} \times 108$ divers $= 63$ divers
$y = \frac{5}{12} \times 108$ divers $= 45$ divers
b) The modal number of shark encounters is 2 shark encounters.

c) There are 240 divers, so $n = 240$, and $\frac{240+1}{2} = 120.5$. This means that the median is halfway between the 120th and the 121st value.

The first 9 values are in the 0 shark encounters category, and the following 32 values are in the 1 shark encounter category, so the first 41 values fall in the 0 or the 1 shark encounter categories. The following 76 values fall into the 2 shark encounters category, so the first 117 values fall in the 0 or 1 or 2 shark encounter categories. The following 63 values fall in the 3 shark encounters category, so values 120 and 121 must be in this category. Hence, the median is simply 3 shark encounters.

d) Multiply the number of shark encounters by the frequency:

9×0 shark encounters $= 0$ shark encounters
32×1 shark encounters $= 32$ shark encounters
76×2 shark encounters $= 152$ shark encounters
63×3 shark encounters $= 189$ shark encounters
45×4 shark encounters $= 180$ shark encounters
15×5 shark encounters $= 75$ shark encounters

Total number of shark encounters $= 0 + 32 + 152 + 189 + 180 + 75 = 628$
The mean number of shark encounters is: 628 shark encounters $\div 240$ divers $= 3$ shark encounters (to the nearest whole number)
Question 4:
a)

Name	Frequency
Abigail	4
Dawn	6
Elizabeth	4
Gemma	8
Leanne	3
Sophie	5
Tanya	2

b) Total: $4 + 6 + 4 + 8 + 3 + 5 + 2 = 32$ students

If 8 out of the 32 students voted for Gemma, then $32 - 8 = 24$ did not.

So, $\frac{24}{32} = \frac{3}{4}$ of students did not vote for her.

13 Grouped Frequency Tables

Question 1: Rewriting the list of heights for each group,

The $0 < h \le 20$ group: $7, 9, 15, 19, 19$
The $20 < h \le 30$ group: $21, 22, 25, 25, 27, 28, 30$
The $30 < h \le 40$ group: $31, 32, 32, 33, 35, 37, 38, 39$
The $40 < h \le 70$ group: $46, 51, 55, 61$

Height, h (cm)	Frequency
$0 < h \le 20$	5
$20 < h \le 30$	7
$30 < h \le 40$	8
$40 < h \le 70$	4

Question 2: a) The bottom two groups in the table amount to the total number of people who took over 2 minutes, which is: $19 + 19 = 38$ people

b) 90 seconds $= 90 \div 60 = 1.5$ minutes.
The first two groups in the table represent the people who completed the puzzle in under 90 seconds, so: $8 + 22 = 30$ people. 30 people out of 100 completed the puzzle in under 90 seconds: $\frac{30}{100} = \frac{3}{10}$

Question 3: a)

Score	Frequency
0 – 10	1
11 – 20	8
21 – 30	4
31 – 40	4
41 – 50	5
51 – 60	3

b) 5 students scored between 41 and 50 marks and 3 students scored between 51 and 60 marks. Therefore, 8 students in total scored above 40. 25 students in total, of which 8 scored higher than 40: $\frac{8}{25} = 32\%$

Question 4: a)

Time in shop (minutes)	Frequency
0 – 5	3
6 – 10	5
11 – 15	4
16 – 20	6
21 +	2

b) The number of people who spent more than 10 minutes in the bike shop is $4 + 6 + 2 = 12$ customers. If their average spend was £12.50

each, then the total spent is: £12.50 × 12 = £150.

c) There is a total of $4+6=10$ customers who spent more than 10 minutes but less than 21 in the shop.

In total there were $3+5+4+6+2=20$ customers in total. 10 out of the 20 customers spent more than 10, but less than 21 minutes: $\frac{10}{20}=\frac{1}{2}$.

d) The total time spent in the shop by all the customers: $16+23+4+9+4+18+45+20+8+6++3+14+12+17+12+19+9+16+10+15=280$ minutes.
There was a total of 20 customers in the shop, so the mean amount of time spent in the shop was:

$$\frac{280 \text{ minutes}}{20 \text{ customers}} = 14 \text{ minutes}$$

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14 Estimating the Mean

Question 1: Find the midpoints of the first column:

Journey time, t (mins)	Frequency	Midpoint
$0 < t \leq 10$	2	5
$10 < t \leq 20$	45	15
$20 < t \leq 30$	25	25
$30 < t \leq 40$	3	35

Then, multiply the frequency of each group by its midpoint to estimate the total journey time per group.

2×5 minutes $= 10$ minutes
45×15 minutes $= 675$ minutes
25×25 minutes $= 625$ minutes
3×35 minutes $= 105$ minutes

Estimated total journey time (all groups):
10 minutes + 675 minutes + 625 minutes + 105 minutes = 1415 minutes.

Total number of journeys: $2+45+25+3=75$ journeys.
Then the estimated mean can be calculated as follows:

$$\frac{1415 \text{ minutes}}{75 \text{ journeys}} = 18.9 \text{ minutes (1 dp)}$$

Question 2: Find the midpoints of the first column:

Distance, d (cm)	Frequency	Midpoint
$0 < d \leq 50$	4	25
$50 < d \leq 100$	18	75
$100 < d \leq 150$	56	125
$150 < d \leq 200$	32	175
$200 < d \leq 250$	8	225

Then, multiply the frequency of each group by its midpoint.

$4 \times 25\text{cm} = 100\text{cm}$
$18 \times 75\text{cm} = 1,350\text{cm}$
$56 \times 125\text{cm} = 7,000\text{cm}$
$32 \times 175\text{cm} = 5,600\text{cm}$
$8 \times 225\text{cm} = 1,800\text{cm}$

Estimated total jump length: $50\text{cm}+1,350\text{cm}+7,000\text{cm}+5,600\text{cm}+1,800\text{cm}=15,850\text{cm}$

Total number of jumps: $4+18+56+32+8=118$ jumps.
Then the estimated mean is:

$$\frac{15,850 \text{ cm}}{118 \text{ jumps}} = 134.3 \text{ cm (1 dp)}$$

Question 3: Find the midpoints of the first column.
Once you have calculated all the midpoints, create a new column and then you should have a table that looks like the table below:

Time, t (mins)	Frequency	Midpoint
$0 < t \leq 10$	12	5
$10 < t \leq 14$	18	12
$14 < t \leq 20$	34	17
$20 < t \leq 32$	33	26
$32 < t \leq 45$	19	38.5

Then, multiply the frequency of each group by its midpoint.

12×5 minutes $= 60$ minutes
18×12 minutes $= 216$ minutes
34×17 minutes $= 578$ minutes
33×26 minutes $= 858$ minutes
19×38.5 minutes $= 731.5$ minutes

Estimated total journey time:
60 minutes+216 minutes+578 minutes+858 minutes+731.5 minutes = 2443.5 minutes

Total number of journeys: $12+18+34+33+19=116$ journeys.
Then the estimated mean can be calculated as follows:

$$\frac{2443.5 \text{ minutes}}{116 \text{ journeys}} = 21.1 \text{ minutes (1 dp)}$$

Question 4: a) Find the midpoints of the first column:

Lengths of fish (cm)	Frequency	Midpoint
$0 < l \leq 20$	16	10
$20 < l \leq 30$	27	25
$30 < l \leq 50$	9	40
$50 < l \leq 70$	13	60
$70 < l \leq 90$	8	80
$90 < l \leq 300$	6	195

Then, multiply the frequency of each group by its midpoint.

16×10 cm $= 160$ cm
27×25 cm $= 675$ cm
9×40 cm $= 360$ cm
13×60 cm $= 780$ cm
8×80 cm $= 640$ cm
6×195 cm $= 1,170$ cm

Estimated total length: $160\text{cm}+675\text{cm}+360\text{cm}+780\text{cm}+640\text{cm}+1,170\text{cm}=3,785\text{cm}$.

Total number of fish caught: $16 + 27 + 9 + 13 + 8 + 6 = 79$ fish caught. Then the estimated mean can be calculated as follows:

$$\frac{3,785\text{cm}}{79 \text{ fish}} = 47.9 \text{ cm (1 dp)}$$

b) Ignoring the 6 fish that were in the $90 - 300$ cm category, then the total number of fish caught is reduced from 79 to 73. The estimated total length of all the 79 fish caught was 3,785cm. Subtracting the 6 fish in the $90 - 300$cm category: $3,785\text{cm} - (6 \times 195\text{cm}) = 2,615\text{cm}$

Therefore, excluding the 6 biggest fish, the estimated total length of the other 73 fish was 2,615cm. The estimated mean can therefore be calculated as follows:

$$\frac{2,615\text{cm}}{73 \text{ fish}} = 35.8 \text{ cm (1 dp)}$$

Question 5: a) Find the midpoints of the first column:

Suzanna		
Number of hits, h	Frequency	Midpoint
$0 < h \leq 10$	3	5
$10 < h \leq 20$	25	15
$20 < h \leq 30$	28	25
$30 < h \leq 40$	19	35
$40 < h \leq 50$	8	45
$50 < h \leq 60$	2	55

Then, multiply the frequency of each group by its midpoint.

3×5 hits $= 15$ hits
25×15 hits $= 375$ hits
28×25 hits $= 700$ hits
19×35 hits $= 665$ hits
8×45 hits $= 360$ hits
2×55 hits $= 110$ hits

Total number of times the bullseye was hit:
15 hits + 375 hits + 700 hits + 665 hits + 360 hits + 110 hits = 2,225 hits

Total number of participants: $3 + 25 + 28 + 19 + 8 + 2 = 85$ participants. Then, the estimated mean can be calculated as follows:

$$\frac{2,225 \text{ bullseyes hit}}{85 \text{ participants}} = 26.2 \text{ hits (1 dp)}$$

b) By organising the data in batches of 20 hits, rather than batches of 10 hits, each row in Flolellas table will combine 2 of Suzannas rows, so the table will be half the size. It will look as follows:

Number of hits, h	Frequency
$0 < h \leq 20$	28
$20 < h \leq 40$	47
$40 < h \leq 60$	10

To calculate the estimated mean, work out the new midpoints, as follows:

Number of hits, h	Frequency	Midpoint
$0 < h \leq 20$	28	10
$20 < h \leq 40$	47	30
$40 < h \leq 60$	10	50

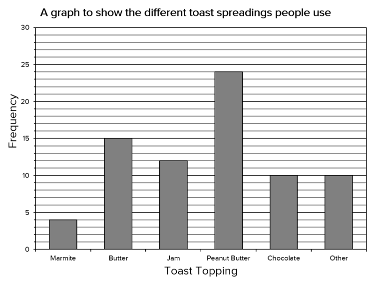
Using Floellas table, the estimated mean can be calculated as follows:

$$\frac{(28 \times 10) + (47 \times 30) + (10 \times 50)}{85} = 25.8 \text{ hits (1 dp)}$$

When the answer is rounded to the nearest whole number, the estimated mean is the same.

15 Bar Graphs

Question 1: The completed bar graph should look like the one shown below. There must be gaps between the bars and everything (including the axes and the individual bars) should be clearly labelled.

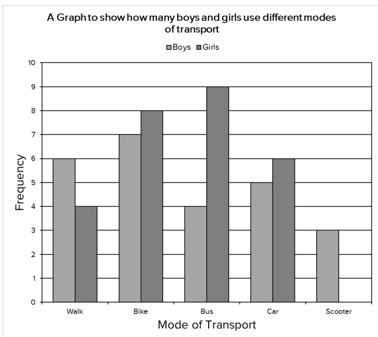


Question 2: a) According to the scale on the y-axis, 1 small line accounts for 1,000 book sales. So, the number of audiobooks sold is 3,000, the number of hardbacks sold is 5,000, and the number of paperbacks sold is 12,000.
Therefore the percentage of sales that were audiobooks is:

$$\frac{3,000}{3,000 + 5,000 + 12,000} \times 100 = 15\%$$

b) The ratio is: $5,000 : 12,000 = 5 : 12$

Question 3:



Question 4: a) The number of children who have between 0 and 5 servings per week is a total of 16. Therefore, the number of adults and children combined who have between 0 and 5 servings is $4 + 16 = 20$.

b) The number of children who eat over 20 servings of fruit and vegetables is 4. The number of children who eat between 6 and 10 servings of fruit and vegetables is 25. Therefore, the difference between these two categories is $25 - 4 = 21$.

c) The mode is the most frequently occurring value. The $16 - 20$ bar is the highest with 22. Therefore the mode is $16 - 20$ servings.

d) Adults eat more portions of fruit and vegetables per week than children.
Question 5: a) 12 girls.

b) The number of boys who played video games for $11 - 15$ hours was 8 and the number of boys who played $6 - 10$ hours was 2. Therefore the number of boys who played for less than 16 hours was $8+2=10$ boys.

c) The number of girls who played for $6-10$ hours was 12 and the number who played for $0-5$ hours was 8. Therefore the number of girls who played video games for 10 hours or less was $12+8=20$ girls.
In total there are $8+12+20+14+8+10+6+2=80$ girls. Thus 20 out of the 80 girls play video games for 10 hours or less, which is 25%.

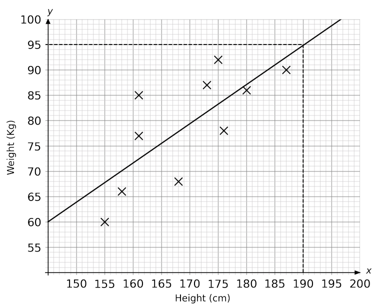
16 Scatter Graphs

Question 1: A - As the x variable increases, the y variable also increases. This indicates that there is a positive correlation. Since all the points are close together in a straight line, this graph has strong positive correlation.

B - There is no clear pattern here, so this graph has no correlation.

C - As the x variable increases, the y variable decreases, so there is a negative correlation. Since all the points are reasonably close to the line of best fit, this graph has moderate negative correlation.

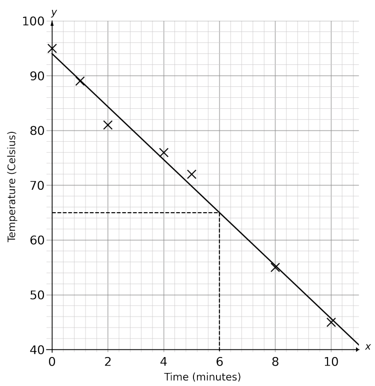
Question 2: a) The results of plotting the ten points on a graph should look like:



b) The line of best fit will cut through what you believe to be the middle of all the points,

To predict the weight of someone with a height of 190cm, locate 190 on the horizontal x-axis and draw a vertical line up to your line of best fit. Then draw across from this point to the corresponding value on the y-axis. The prediction, according to this line of best fit, is 95kg.
(Your line of best fit may be slightly different, in which case any answers between 93kg and 97kg are acceptable.)

Question 3: a)



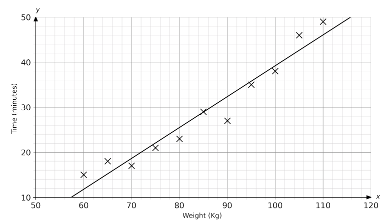
b) Draw a line of best fit that cuts through the middle of as many of the dots as possible. As the x variable increases, the y variable decreases, so there is a negative correlation. Since all the points are very close to the line of best fit, this graph has strong negative correlation.

c) Since the y variable decreases as the x variable increases, the temperature of the cup of tea is reducing over time.

d) The estimated temperature is 66°.

e) It would be inappropriate to find an estimate for the temperature after 45 minutes as 45 minutes is beyond the range of the data and tea would not get colder than room temperature.

Question 4: a)

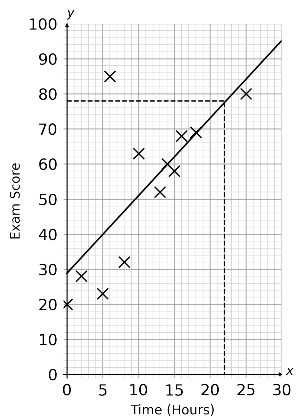


b) As the x variable increases, the y variable increases, so there is a positive correlation. Since all the points are very close to the line of best fit, this graph has strong positive correlation.

c) Since the y variable increases as the x variable increases, this tells us that the time taken to run 5 kilometres is greater for a heavier runner.

d) It would be inappropriate to find an estimate for the time taken for a runner of 40 kilograms since 40 kilograms is beyond the range of the data.

Question 5: a)



b) Draw in a line of best fit. Since the line of best fit goes up and, generally, the points are close to this line, there is a positive correlation.

c) Since the y values (the exam scores) increase as the x values (time spent revising) increase, more time spent revising is likely to give a better exam score.

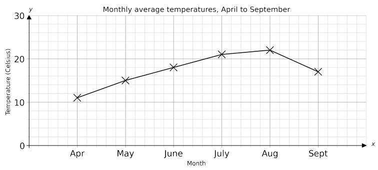
d) An outlier is any point which is a long way from the line of best fit. This is the point that corresponds to the student who scored 85 with only 6 hours of revision.

e) 22 hours of revision corresponds to an exam score of approximately 78 marks (accept 77-79 marks).

f) It would be inappropriate to find an estimate for an exam score for a student doing 85 hours of revision as this is beyond the range of the data.

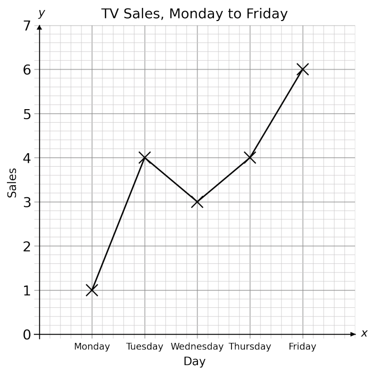
17 Line graphs

Question 1: The line graph should have the months on the x-axis and the temperature on the y-axis. It should also have the axes clearly labelled and an appropriate title at the top.

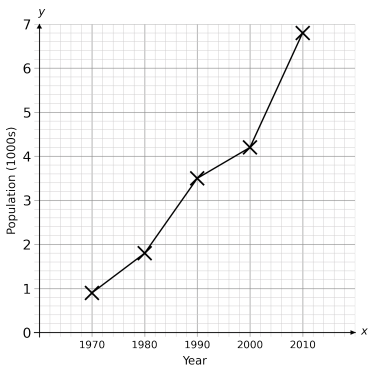


Question 2: The first mistake Roger made is that he did not label one of his axes.
The second mistake he made is that he plotted the 2014 point at 600 when it should be at 700.

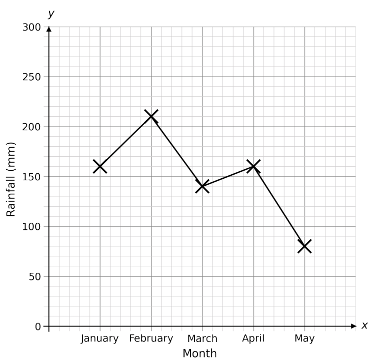
Question 3:



Question 4:



Question 5:



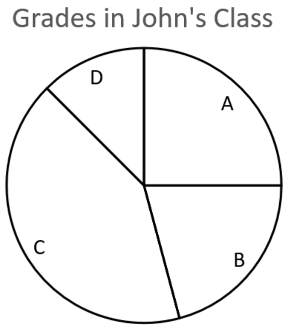
18 Pie Charts

Question 1: $\frac{90°}{360°} \times 32 = 8$ students

Question 2: $\frac{60°}{360°} \times 510$ cars $= 85$ yellow cars.

Question 3: John recorded the grades of 24 pieces of homework $(6+5+10+3=24)$.
The angle for the grade A slice must be: $\frac{6}{24} \times 360° = 90°$.
The angle for the grade B slice must be: $\frac{5}{24} \times 360° = 75°$.
The angle for the grade C slice must be: $\frac{10}{24} \times 360° = 150°$.
The angle for the grade D slice must be: $\frac{3}{24} \times 360° = 45°$.

Drawing the circle with a compass, and measuring the angles with a protractor,



Question 4: a) $40° = 600$ cars.
$1° = 600 \div 40 = 15$ cars per $1°$.
$85° = 85 \times 15 = 1275$ Renault cars.

b) $1°$equates to 15 cars, so, $15 \times 360 = 5,400$ cars.

Question 5: Oliver has 12 hours of leisure time in total and this is represented by a $60°$ slice of the pie chart. Hence, Oliver plays golf for: $\frac{60°}{360°} \times 12$ hours $= 2$ hours.

If Lewis spends 2 hours of the available 9 playing golf, this will be represented by $\frac{2}{9} \times 360° = 80°$.