1 Types of Numbers

Question 1: $1\times45=45,\ 3\times15=45,\ 5\times9=45$
There are no more factor pairs, so the complete list of factors is

$$1,\ 3,\ 5,\ 9,\ 15,\ 45$$

Question 2:
a) $1^3=1$, so 1 is a cube number.
b) $3^3=27$, so 27 is a cube number.
c) $4^3=64$, so 64 is a cube number.
d) The next cube number is $5^3=125$. So, 100 is not a cube number.

Question 3: 0.89 is a rational number, as can be written as a fraction:

$$0.89=\frac{89}{100}$$

Question 4: b) $2\sqrt{4}$ is an integer $(2\sqrt{4}=2\times\sqrt{4}=2\times2=4)$

Question 5: c) $0.\dot{3}$ is the only rational number $(0.\dot{3}=\frac{1}{3})$

2 BIDMAS/ BODMAS

Question 1: There are two brackets (B), (2×3^3) and $(15-9)$. Inside the first bracket, there is a power or index number (I or O), $2\times3^3=2\times27$. Carry out any divisions or multiplications (DM) then additions or subtractions (AS) inside the brackets, $(2\times27=54)$ and $(15-9=6)$. Completing the calculation,$(2\times3^3)\div(15-9)=54\div6=9$

Question 2: (B), $12\div4=3$, (I),$(3)^2=9$, (M), $16\times9=144$

Question 3: Numerator: (B) substitute in the given value of x and apply the power (I), before the addition (A).$((-3)^2+3)=(9+3)=12$. Denominator: (B) there is a subtraction inside the bracket (S), $(10-6)=4$. Finally simplify the fraction (D), so,$\frac{12}{4}=3$

Question 4: (B), $y^2+5y^2=6y^2$.(M), $3y\times7y=21y^2$.(S), and we get, $6y^2-21y^2=-15y^2$

Question 5: Numerator: multiplication (M), so

$$42q^2\times pq=42q^2\times q\times p=42q^3p$$

Denominator: (B), subtraction inside the bracket (S) $9p-5p=4p$. Then, the division (D) operator can be applied, $28p^3\div4p=7p^2$. So, we are left with a fraction with p on the top and bottom, as well as a factor of 7. Both cancel so,

$$\frac{42pq^3}{7p^2}=\frac{6q^3}{p}$$

3 Place Value

Question 1: 800 or eight hundred

Question 2: $\frac{1}{10}$ or one tenth

Question 3: $\frac{6}{1000}$ or six thousandths

Question 4: 500,000 or five hundred thousand

Question 5: 3 thousands = 3,000 5 tens = 50 1 hundredth = 0.01
Adding these all together, we get Beckys number to be

$$3,000+50+0.01=3,050.01$$

4 Long Division

Question 1: When the result is not an integer the remaining bit left over can be written either as a decimal or as a remainder.

$$\begin{array}{r} 0\,2\,2\,.\,5 \\ 14\overline{)3^3 1^3 5.^7 0} \end{array}$$

$$\begin{array}{r} 0\,2\,2\,r\,7 \\ 14\overline{)3^3 1^3 5.^7 0} \end{array}$$

Question 2: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 1\ 5 \\ 15\overline{)2^2 2^7 5} \end{array}$$

Question 3: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 2\ 3 \\ 13\overline{)2^2 9^3 9} \end{array}$$

Question 4: Using long division or the bus stop method.

$$\begin{array}{r} 0\ 2\ 6 \\ 9\overline{)2^2 3^5 4} \end{array}$$

5 Long Multiplication

Question 1: Using the long multiplication method, multiplying 619 first by 5 then by 40 and summing the results,

$$\begin{array}{r} 619 \\ \times45 \\ \hline 3095 \\ +24760 \\ \hline 27855 \end{array}$$

Question 2: Using the long multiplication or the grid method

$$\begin{array}{r} 52 \\ \times31 \\ \hline 52 \\ +1560 \\ \hline 1612 \end{array}$$

Question 3: Using the long multiplication or the grid method

$$\begin{array}{r} 760 \\ \times24 \\ \hline 3040 \\ +15200 \\ \hline 18240 \end{array}$$

Question 4: Using the long multiplication or the grid method

$$\begin{array}{r} 364 \\ \times52 \\ \hline 728 \\ +18200 \\ \hline 18928 \end{array}$$

Question 5: Using the long multiplication or the grid method

$$\begin{array}{r} 473 \\ \times 326 \\ \hline 2838 \\ +9460 \\ +141900 \\ \hline 154198 \end{array}$$

6 Decimals

Question 1: By means of column addition or otherwise,

$$\begin{array}{r} 82.070 \\ +31.865 \\ \hline 113.935 \end{array}$$

Question 2: We can make the calculation easier by converting the divisor to a whole number by multiplying both 2.3 and 18.63 by 10, so,

$$\begin{array}{r} 8\,.\,1 \\ 23\overline{)\,\not{1}\,{}^{1}\not{8}\,{}^{18}6\,.\,{}^{2}3} \end{array}$$

Question 3: To make the first number whole: $3.566 \times 1,000 = 3,566$. Thus the column multiplication is,

$$\begin{array}{r} 3566 \\ \times 14 \\ \hline 14264 \\ 35660 \end{array}$$

Adding the the two parts,

$$\begin{array}{r} 14264 \\ +35660 \\ \hline 49924 \end{array}$$

We multiplied one of our numbers by 1000, which means our result is 1000 times too big. Therefore, the final answer is, $3.566 \times 14 = 49924 \div 1000 = 49.924$

Question 4:

$$\begin{array}{r} 0.113 \\ +0.890 \\ \hline 1.003 \end{array}$$

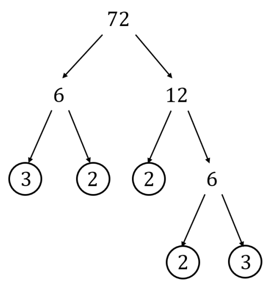
Question 5: Converting the decimals to a whole numbers. If we multiply 0.002 and 0.043 by 1000, we have a simple integer multiplication,

$$\begin{array}{r} 2 \\ \times 43 \\ \hline 86 \end{array}$$

However this value is $1000 \times 1000 = 1000000$ times too big, so we have to divide the result by this, $86 \div 1000000 = 0.000086$

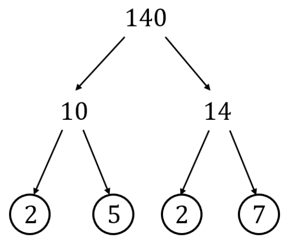
7 Prime Factors, LCM and HCF

Question 1: Using a prime factor tree:



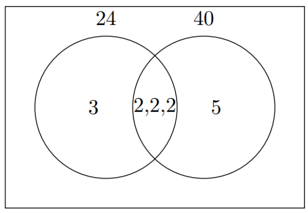
The prime factorisation of 72 is, $72 = 2 \times 2 \times 2 \times 3 \times 3$
Index notation: $72 = 2^3 \times 3^2$

Question 2: Using a prime factor tree:



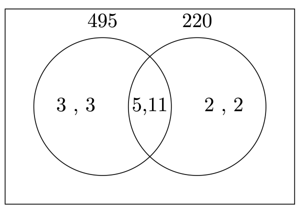
The prime factorisation of 140 is, $140 = 2 \times 2 \times 5 \times 7$
Index notation: $140 = 2^2 \times 5 \times 7$

Question 3:
Prime factors of 24: $2 \times 2 \times 2 \times 3$.
Prime factors of 40: $2 \times 2 \times 2 \times 5$



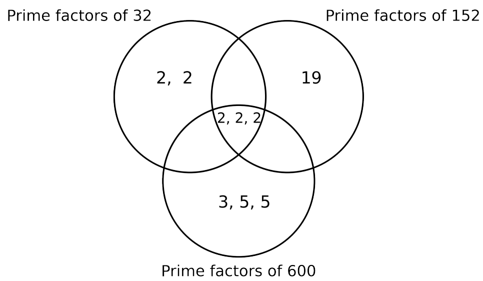
HCF $= 2 \times 2 \times 2 = 8$.
LCM $= 8 \times 3 \times 5 = 120$

Question 4:
Prime factors of $495 = 3 \times 3 \times 5 \times 11$
Prime factors of $220 = 2 \times 2 \times 5 \times 11$



HCF $= 5 \times 11 = 55$
LCM $= 2 \times 2 \times 3 \times 3 \times 5 \times 11 = 1980$

Question 5: Prime factors of $32 = 2\times2\times2\times2\times2$
Prime factors of $152 = 2\times2\times2\times19$
Prime factors of $600 = 2\times2\times2\times3\times5\times5$



$\text{HCF} = 2\times2\times2 = 8$
$\text{LCM} = 2\times2\times2\times2\times2\times3\times5\times5\times19 = 45600$

8 Fractions

Question 1:

$$\frac{6}{13}\times\frac{4}{3}=\frac{6\times4}{3\times13}=\frac{24}{39}=\frac{8}{13}$$

Question 2:

$$\frac{7}{10}-\frac{8}{3}=\left(\frac{7}{10}\times\frac{3}{3}\right)-\left(\frac{8}{3}\times\frac{10}{10}\right)=\frac{21}{30}-\frac{80}{30}=-\frac{59}{30}$$

Question 3:

$$\frac{9}{11}\div\frac{6}{7}=\frac{9}{11}\times\frac{7}{6}=\frac{9\times7}{11\times6}=\frac{63}{66}=\frac{21}{22}$$

Question 4:

$$\frac{5}{4}\times\frac{2}{3}=\frac{5\times2}{4\times3}=\frac{10}{12}=\frac{5}{6}$$

Question 5: First, convert the mixed fraction to an improper fraction,

$$12\frac{1}{2}=\frac{25}{2},\quad \frac{25}{2}\div\frac{5}{8}=\frac{25}{2}\times\frac{8}{5}=\frac{25\times8}{2\times5}=\frac{200}{10}=20$$

9 Fractions, Decimals, and Percentages

Question 1: $54.4\%\div100=0.544$

Question 2: $16.4\%=\frac{164}{1000}=\frac{41}{250}$

Question 3: $\frac{17}{40}=\frac{42.5}{100}=0.425$

Question 4: $0.256=\frac{256}{1000}=\frac{32}{125}$

Question 5: $\frac{13}{20}=\frac{65}{10}=0.65$

10 Rounding Numbers

Question 1: $560,180$ rounded to the nearest thousand is $560,000$

Question 2: 97.96 rounded to 1 decimal place is 98.0

Question 3: 0.02345 rounded to 3 significant figures is 0.0235

Question 4: 1.0093 rounded to 3 significant figures is 1.01

Question 5: 55.099 rounded to 2 decimal places is 55.10

11 Ordering Numbers

Question 1: Descending order means from largest to smallest. Hence,

$$23,\ 4,\ 1,\ -23.5,\ -42$$

Question 2: Ascending order means from smallest to largest, hence,

$$2.04,\ 2.5,\ 2.58,\ 2.8,\ 3.5$$

Question 3:

$$4.092,\ 4.87,\ 5.01,\ 5.12,\ 5.23$$

Question 4: Convert all numbers to the same form. $64\%=0.64$ and, $64.4\%=0.644$. $\frac{5}{8}=0.625$. Hence,

$$0.625,\ 0.633,\ 0.64,\ 0.644$$

Finally, putting them in order in the original forms,

$$\frac{5}{8},\ 0.633,\ 64\%,\ 64.4\%$$

Question 5: Substitute in $x=3$ for each of the terms, $\frac{1}{x}=\frac{1}{3}$, $x^2=3^2=9$, $x=3$, $(x+1)=4$, $2x=6$. Hence,

$$\frac{1}{3},\ 3,\ 4,\ 6,\ 9$$

Hence we can place the original terms in order,

$$\frac{1}{x},\ x,\ (x+1),\ 2x,\ x^2$$

12 Fractions and Recurring Decimals

Question 1: Treat the fraction as a division using the bus stop method.

$$\begin{array}{r}0.1\,1\,1\,1\,1\\ 9\overline{)1.0^{1}0^{1}0^{1}0^{1}0}\end{array},\quad \frac{1}{9}=0.\dot{1}$$

Question 2: Let $x=0.\dot{3}$ and $10x=3.\dot{3}$.

$$10x-x=9x=3.\dot{3}-0.\dot{3}=3$$

Making x the subject then, $x=\frac{3}{9}=\frac{1}{3}$

Question 3: Let $x=0.\dot{3}9\dot{0}$ and $1,000x=390.\dot{3}9\dot{0}$.

$$1,000x-x=999x=390.\dot{3}9\dot{0}-0.\dot{3}9\dot{0}=390$$

Making x the subject then, $x=\frac{390}{999}=\frac{130}{333}$

This simplifies to, $x=0.\dot{3}9\dot{0}=\frac{130}{333}$

Question 4: Using the bus stop method.

$$\begin{array}{r}0\ .9\ 0\ 9\ 0\ 9\ 0\ 9\ 0\\ 11\overline{)10.0^{1}0^{10}0^{1}0^{10}0^{1}0^{10}0^{1}0}\end{array},\quad \frac{10}{11}=0.\dot{9}\dot{0}$$

Question 5: Let $x=1.5\dot{4}$ then $10x=15.\dot{4}$ and $100x=154.\dot{4}$

$$100x-10x=90x=154.\dot{4}-15.\dot{4}=154-15=139$$

Making x the subject then, $x=\frac{139}{90}$

13 Estimating

Question 1: Round each number to 1 significant figure:

$$\frac{9.02+6.65}{0.042\times 11}\approx\frac{9+7}{0.04\times 10}=\frac{16}{0.4}=\frac{160}{4}=40$$

Question 2: Round each number to 1 significant figure:

$$\frac{57.33-29.88}{8.66-5.55}\approx\frac{60-30}{9-6}=\frac{30}{3}=10$$

Question 3: Round each number to 1 significant figure:
45p = £0.45
1.89 rounds to 2 and 0.45 rounds to 0.5
(Pens) £2 × 5 = £10
(Pencils) £0.50 × 3 = £1.50
(Total) £10 + £1.50 = £11.50

Question 4: Round each number to 1 significant figure:
32.60 rounds to 30, 17.50 rounds to 20.
(Children) £20 × 3 = £60
(Adults) £30 × 2 = £60
(Total) £60 + £60 = £120

Question 5: We know that, $9^2 = 81$ and $10^2 = 100$. So the answer must be between 9 and 10. Since 98 is only 2 away from 100, but 17 away from 81, we can conclude that the solution is going to be much closer to 10. Therefore, it is a slight overestimate at $\sqrt{98}\approx 9.9$.

14 Upper and Lower Bounds

Question 1:
Lower bound: $5.43-0.005=5.425$
Upper bound: $5.43+0.005=5.435$
The interval is therefore $5.425\le C<5.435$

Question 2:
Lower bound: $175-0.5=174.5$
Upper bound: $175+0.5=175.5$
The interval is therefore $174.5\le h<175.5$

Question 3:
Lower bound: $5.45+0.005=5.455$
Upper bound: $5.45-0.005=5.445$
The interval is therefore £5.445 B $\le C<$ £5.455 B

Question 4: To make the fraction as small as possible, use the lower bound for P, which is 144.5 and the upper bound for v, which is 23.45 Therefore, the lower bound for the driving force of the car is,

$$F=\frac{144.5}{23.45}=6.16\text{ Newtons (3 s.f.)}$$

This is a suitable degree of accuracy as it is the same degree to which the values are given in the question.

Question 5: To find the upper bound for the average speed, we will need the upper bound for the distance, 80.35, and the lower bound for the time, 1.865.
To find the lower bound for the average speed we will need the lower bound for the distance, 80.25, and the upper bound for the time, 1.875. Therefore, we get:

$$\text{Max average speed}=\frac{\text{distance upper bound}}{\text{time lower bound}}=\frac{80.35}{1.865}=43.1\text{(3 sf)}$$

$$\text{Min average speed}=\frac{\text{distance lower bound}}{\text{time upper bound}}=\frac{80.25}{1.875}=42.8\text{(3 sf)}$$

15 Standard Form

Question 1: $1.15\times 10^{-6}=0.00000115$.

Question 2: $5,980,000=5.98\times 10^{6}$

Question 3: $0.0068=6.8\times 10^{-3}$

Question 4: 5.6×10^{6} and 8×10^{2}

$$(5.6\times 10^6)\div(8\times 10^2)=(5.6\div 8)\times(10^6\div 10^2)$$

Using the formula $10^a\div 10^b=10^{a-b}$ we can rewrite the equation as,

$$(5.6\div 8)\times 10^{6-2}=0.7\times 10^4$$

Standard form requires the number be between 1 and 10, thus

$$0.7\times 10^4=7\times 10^{-1}\times 10^4=7\times 10^3$$

Question 5:

$$(2.5\times 10^4)\times(6\times 10^{-9})=2.5\times 6\times 10^4\times 10^{-9}=15\times 10^{-5}$$

Standard form requires the number be between 1 and 10, thus

$$15\times 10^{-5}=1.5\times 10\times 10^{-5}=1.5\times 10^{-4}$$

1 Collecting Like Terms

Question 1:

$$5x+5-2x+3-4-x=(5x-2x-x)+(5+3-4)=2x+4$$

Question 2:

$$ab+bc+2ab-bc+a=a+(ab+2ab)+(bc-bc)=a+3ab$$

Question 3:

$$11x+7y-2x-13y=(11x-2x)+(7y-13y)=9x-6y$$

Question 4:

$$2m+6n-3+8n+5m=(2m+5m)+(6n+8n)-3=7m+14n-3$$

Question 5:

$$2a^2+5b-2a-3b+5a^2=(2a^2+5a^2)+(5b-3b)-2a=7a^2+2b-2a$$

2 Powers and Roots

Question 1: $a^b \times a^c = a^{b+c}$, so, $a^2 \times a^3 = a^{2+3}$, $a^2 \times a^3 = a^5$

Question 2: We can recognise, $12^2 = 144$ and $14^2 = 196$
So, $\sqrt{144}+\sqrt{196} = 12+14 = 26$

Question 3: Using the laws of indices, $(3^2)^3 = 3^{2\times3} = 3^6$. Hence, the expression now looks like, $3^6 \div 3^4$.
Then, $3^6 \div 3^4 = 3^{6-4} = 3^2 = 9$

Question 4: First considering the numerator, the laws of indices tell us, $7^5 \times 7^3 = 7^{5+3} = 7^8$. Thus the expression now is, $\frac{7^8}{7^6}$. This can be simplified to, $\frac{7^8}{7^6} = 7^{8-6} = 7^2 = 49$.

Question 5: We know that, $20^1 = 20$ and $100^0 = 1$. So $20+1 = 21$

3 Rules of Indices

Question 1: $9 = 3^2$, so we can write the first term as, $\left(3^2\right)^5$. Then, $\left(3^2\right)^5 = 3^{2\times5} = 3^{10}$. Therefore, the whole expression becomes, $3^{10}\times3^{-5}$. This can be written as, $3^{10+(-5)} = 3^5$. So, we have written the expression as a power of 3.

Question 2: Firstly, as $3^2 = 9$, the inverse operation gives, $\sqrt{9} = 3$. So, that leaves $6^{-2} = \frac{1}{6^2}$. We know that $6^2 = 6\times6 = 36$, so $6^{-2} = \frac{1}{36}$. Multiplying our two answers together, we get

$$\sqrt{9}\times6^{-2} = 3\times\frac{1}{36} = \frac{3}{36} = \frac{1}{12}$$

Question 3: This expression can be rewritten as, $\sqrt{4}\times(\sqrt{4})^3$. Given we know that $\sqrt{4} = 2$, this becomes, 2×2^3. Hence, $2\times2^3 = 2\times8 = 16$.

Question 4: As it is a negative power we can rewrite this as, $8^{-\frac{5}{3}} = \frac{1}{8^{\frac{5}{3}}}$
Now, we can work out the denominator, which we will write as, $8^{\frac{5}{3}} = \sqrt[3]{8^5} = (\sqrt[3]{8})^5$ We know that $\sqrt[3]{8} = 2$. So this simplifies to, $(\sqrt[3]{8})^5 = 2^5 = 32$. Therefore, the answer is, $8^{-\frac{5}{3}} = \frac{1}{32}$.

4 Expanding Brackets - Single and Double Brackets

Question 1:

$$\begin{aligned}3xy(x^2+2x-8) &= 3xy\times x^2+3xy\times2x+3xy\times(-8)\\ &= 3x^3y+6x^2y-24xy\end{aligned}$$

Question 2:

$$\begin{aligned}9pq(2-pq^2-7p^4) &= 9pq\times2-9pq\times pq^2-9pq\times7p^4\\ &= 18pq-9p^2q^3-63p^5q\end{aligned}$$

Question 3:

$$\begin{aligned}(y-3)(y-10) &= y\times y+y\times(-10)+(-3)\times y+(-3)\times(-10)\\ &= y^2-10y-3y+30\\ &= y^2-13y+30\end{aligned}$$

Question 4:

$$\begin{aligned}(m+2n)(m-n) &= m\times m+m\times(-n)+2n\times m+2n\times(-n)\\ &= m^2-mn+2mn-2n^2\\ &= m^2+mn-2n^2\end{aligned}$$

Question 5: We can write this as two sets of brackets,

$$\begin{aligned}(2y^2+3x)(2y^2+3x) &= 2y^2\times2y^2+2y^2\times3x+3x\times2y^2+3x\times3x\\ &= 4y^4+6xy^2+6xy^2+9x^2\\ &= 4y^4+12xy^2+9x^2\end{aligned}$$

5 Expanding Brackets - Triple Brackets

Question 1: Firstly, we expand the second two brackets into a normal quadratic

$$\begin{aligned}(m-9)(m+1) &= m^2+m-9m-9\\ &= m^2-8m-9\end{aligned}$$

Then,

$$(m+8)(m-9)(m+1) = (m+8)(m^2-8m-9)$$

Now,

$$(m+8)(m^2-8m-9) = m^3-8m^2-9m+8m^2-64m-72$$

Collecting and simplifying like terms, we get the simplified expansion to be,

$$m^3-73m-72$$

Question 2: As $(k+4)^2$ is the same as $(k+4)(k+4)$, the expression can also be written like, $(2k-3)(k+4)(k+4)$. Firstly,

$$\begin{aligned}(k+4)(k+4) &= k^2+4k+4k+16\\ &= k^2+8k+16\end{aligned}$$

Then,

$$(2k-3)(k+4)(k+4) = (2k-3)(k^2+8k+16)$$

Now,

$$(2k-3)(k^2+8k+16) = 2k^3+16k^2+32k-3k^2-24k-48$$

Collecting and simplifying like terms, we get the simplified expansion to be,

$$2k^3+13k^2+8k-48$$

Question 3: Firstly,

$$(x+2)(x-4) = x^2+2x-4x-8$$
$$= x^2-2x-8$$

Then,

$$(x+1)(x+2)(x-4) = (x+1)(x^2-2x-8)$$

Now,

$$(x+1)(x^2-2x-8) = x^3-2x^2-8x+x^2-2x-8$$

Collecting and simplifying like terms, we get the simplified expansion to be,

$$x^3-x^2-10x-8$$

Question 4: We can write the expression given in the question as, $(a+b)^3 = (a+b)(a+b)(a+b)$. Firstly,

$$(a+b)(a+b) = a^2+ab+ba+b^2$$
$$= a^2+2ab+b^2$$

Then,

$$(a+b)^3 = (a+b)(a^2+2ab+b^2)$$

Now,

$$(a+b)(a^2+2ab+b^2) = a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$$

Collecting and simplifying like terms, we get the simplified expansion to be,

$$a^3+3a^2b+3ab^2+b^3$$

Question 5: Firstly,

$$(2x+5)(x+2) = 2x^2+4x+5x+10$$
$$= 2x^2+9x+10$$

Then,

$$(x-3)(2x+5)(x+2) = (x-3)(2x^2+9x+10)$$

Now,

$$(x-3)(2x^2+9x+10) = 2x^3+9x^2+10x-6x^2-27x-30$$

Collecting and simplifying like terms, we get the simplified expansion to be,

$$2x^3+3x^2-17x-30$$

6 Factorising (Foundation)

Question 1: $5pq(2+3r)$

Question 2: $u(u^2+3v^3+2)$.

Question 3: $y^5(4x+1+12y^2)$

Question 4: $5xy(y-x-xy)$

Question 5: $7abc(1+2a+3b+7c^2)$

7 Surds

Question 1: $\sqrt{75} = \sqrt{3\times 25} = \sqrt{3}\times\sqrt{25} = 5\sqrt{3}$

Question 2: $\sqrt{63} = \sqrt{9\times 7} = \sqrt{9}\times\sqrt{7} = 3\sqrt{7}$

Question 3: Using, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, the expression can be simplified to,

$$\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$$

Question 4: Multiplying top and bottom of the fraction by: $\sqrt{3}$.

$$\frac{12}{\sqrt{3}} = \frac{12\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{12\sqrt{3}}{3} = \frac{4\sqrt{3}}{1} = 4\sqrt{3}$$

Question 5: We will multiply top and bottom of this fraction by $(\sqrt{10}+1)$. So, the numerator becomes, $7\times(\sqrt{10}+1) = 7\sqrt{10}+7$ and the denominator becomes

$$(\sqrt{10}-1)(\sqrt{10}+1) = \sqrt{10}\times\sqrt{10}+1\times\sqrt{10}-1\times\sqrt{10}-1\times 1$$
$$= (\sqrt{10})^2+\sqrt{10}-\sqrt{10}-1$$
$$= 10-1 = 9$$

Thus, the fraction is $\frac{7\sqrt{10}+7}{9} = \frac{7(1+\sqrt{10})}{9}$

8 Solving Linear Equations

Question 1:

$$2x+1 = 2$$
$$2x = 1$$
$$x = \frac{1}{2}$$

Question 2:

$$\frac{1}{2}x-3 = 7$$
$$\frac{1}{2}x = 10$$
$$x = 20$$

Question 3:

$$12k-1 = 6k-25$$
$$6k = -24$$
$$k = -4$$

Question 4:

$$3(2m+6) = 2(m-3)$$
$$6m+18 = 2m-6$$
$$4m = -24$$
$$m = -6$$

Question 5:

$$\frac{x^2}{5} = 31.25$$
$$x^2 = 156.25$$
$$x = \sqrt{156.25}$$
$$x = \pm 12.5$$

9 Rearranging Formulas

Question 1:

$$F = \frac{mv}{t}$$
$$Ft = mv$$
$$m = \frac{Ft}{v}$$

Question 2:

$$A = \frac{1}{2}(a+b)$$
$$2A = (a+b)h$$
$$2A = ah + bh$$
$$2A - bh = ah$$
$$a = \frac{2A - bh}{h} = \frac{2A}{h} - b$$

Question 3:

$$F = \frac{kq}{r^2}$$
$$Fr^2 = kq$$
$$r^2 = \frac{kq}{F}$$
$$r = \pm\sqrt{\frac{kq}{F}}$$

Question 4:

$$\frac{x}{x+c} = \frac{a}{b}$$
$$\frac{bx}{x+c} = a$$
$$bx = a(x+c)$$
$$bx = ax + ac$$
$$bx - ax = ac$$
$$x(b-a) = ac$$
$$x = \frac{ac}{b-a}$$

Question 5:

$$a = \frac{3-2b}{b-4}$$
$$a(b-4) = 3-2b$$
$$ab - 4a = 3 - 2b$$
$$ab + 2b - 4a = 3$$
$$ab + 2b = 3 + 4a$$
$$b(a+2) = 3 + 4a$$
$$b = \frac{3+4a}{a+2}$$

10 Factorising Quadratics

Question 1: We are looking for two numbers which add to make 1 and multiply to make −30.The factors of 30 that satisfy theses two requirements are −5 and 6. Therefore, the full factorisation of a^2+a-30 is $(a-5)(a+6)$

Question 2: We are looking for two numbers which add to make −5 and multiply to make 6. The factors of 6 that satisfy theses two requirements are −2 and −3. Therefore, the full factorisation of k^2-5k+6 is $(k-2)(k-3)$

Question 3: We are looking for two numbers which add to make 7 and multiply to make 12. The factors of 12 that satisfy theses two requirements are 3 and 4. Therefore, the full factorisation of $x^2+7x+12$ is, $(x+3)(x+4)$

Question 4: In the quadratic, $a = 3$, −11 is negative and c is positive. We can set up the brackets as follows: $(3x\ -\ )(x\ -\ )$.

We are looking for two positive numbers which multiply to make 6. The possible factors of 6 are

$$(6)\times(1) = 6$$
$$(3)\times(2) = 6$$

We now test all the combinations

$$(3x-6)(x-1) = 3x^2 - 6x - 3x + 6$$
$$(3x-1)(x-6) = 3x^2 - x - 18x + 6$$
$$(3x-3)(x-2) = 3x^2 - 6x - 3x + 6$$
$$(3x-2)(x-3) = 3x^2 - 9x - 2x + 6$$

Hence the correct factorisation is $(3x-2)(x-3)$

Question 5: We can see this is a sub-type (c) meaning it will contain both + and −

Factors of −6:
$(-1)\times 6 = -6$, $(-6)\times 1 = -6$, $(-2)\times 3 = -6$, $(-3)\times 2 = -6$,
Lets find the options which give $-5m$

$$(2m+2)(2m-3) = 4m^2 - 2m - 6$$
$$(4m+1)(m-6) = 4m^2 - 23m - 6$$
$$(4m-1)(m+6) = 4m^2 + 23m - 6$$
$$(4m+3)(m-2) = 4m^2 - 5m - 6$$

We can see that last option with +3 and −2 is the correct combination.This gives the final answer to be: $(4m+3)(m-2)$

11 Difference of Two Squares

Question 1: Identifying that both of the coefficients of each term are square numbers, then the square roots are $a = \sqrt{9x^2} = 3x$ and $b = \sqrt{49y^2} = 7y$. So the factorisation is, $(3x+7y)(3x-7y)$

Question 2: Here we can remove a factor of 2 first so, $2x^2-8 = 2(x^2-4)$. Thus the factorisation is, $2(x+2)(x-2)$

Question 3: the factorisation is, $4x^2-9 = (2x-3)(2x+3)$.

Question 4: This is a difference of two squares so we can apply the formula the same way, $99^2-98^2 = (99+98)(99-98) = (197)(1) = 197$

12 Solving Quadratics By Factorisation

Question 1: The quadratic on the left hand side of the equation factorises so that, $p^2-3p-10 = (p+2)(p-5) = 0$
For the left-hand side to be zero we require one of the brackets to be zero, hence, the two solutions are, $p = -2$ and $p = 5$.

Question 2: The quadratic on the left hand side of the equation factorises so that, $x^2-8x+15 = (x-5)(x-3) = 0$.
Hence, $x = 3$ and $x = 5$.

Question 3: This quadratic factorises so that, $x^2-6x+8 = 0 = (x-2)(x-4) = 0$.
Hence, $x = 2$ and $x = 4$.

Question 4: The quadratic on the left hand side of the equation factorises so that, $2x^2+13x+15 = (2x+3)(x+5) = 0$.

Hence, $x = -\frac{3}{2}$ and $x = -5$.

Question 5: The quadratic on the left hand side of the equation factorises so that, $(3k-1)(k+6) = 0$.
Hence, $k = \frac{1}{3}$ and $k = -6$.

13 The Quadratic Formula

Question 1: Here, $a = 1, b = 11$, and $c = 16$.

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 1 \times 16}}{2} = \frac{-11 \pm \sqrt{57}}{2}$$

$x = -1.73$ (3 s.f.), and $x = -9.27$ (3 s.f.)

Question 2: Here, $a = 1, b = -2$, and $c = -44$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-44)}}{2} = \frac{2 \pm \sqrt{180}}{2}$$

$x = 7.71$ (3 s.f.), and $x = -5.71$ (3 s.f.)

Question 3: Here, $a = 4, b = 7$, and $c = -1$.

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 4 \times (-1)}}{2 \times 4} = \frac{-7 \pm \sqrt{65}}{8}$$

$x = 0.13$ (2 d.p.), and $x = -1.88$ (2 d.p.)

Question 4: Here, $a = 1, b = 8$, and $c = 13$.

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 13}}{2 \times 1} = \frac{-8 \pm \sqrt{12}}{2}$$

$x = -2.27$ (2 d.p.), and $x = -5.73$ (2 d.p.)

Question 5: Here, $a = 25, b = -30$, and $c = 7$.

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \times 25 \times 7}}{2 \times 25} = \frac{30 \pm \sqrt{200}}{50} = \frac{3 \pm \sqrt{2}}{5}$$

14 Completing the Square

Question 1: The coefficient of m term is 5, and half of 5 is $\frac{5}{2}$, so we get

$$\begin{aligned} m^2 + 5m + 6 &= \left(m + \frac{5}{2}\right)^2 + 6 - \left(\frac{5}{2}\right)^2 \\ &= \left(m + \frac{5}{2}\right)^2 + 6 - \frac{25}{4} \\ &= \left(m + \frac{5}{2}\right)^2 - \frac{1}{4} \end{aligned}$$

Question 2: In order to be able to apply our normal process of completing the square, we need to take a factor of 2 out of this whole expression:

$$2x^2 - 8x + 10 = 2(x^2 - 4x + 5)$$

Now it looks more familiar, the coefficient of the x term is -4, half of which is -2, so we get:

$$\begin{aligned} 2(x^2 - 4x + 5) &= 2\left[(x-2)^2 + 5 - (-2)^2\right] \\ &= 2[(x-2)^2 + 1] \\ &= 2(x-2)^2 + 2 \end{aligned}$$

Question 3: The coefficient of x term is $-2m$, and half of $-2m$ is $-m$, so we get

$$x^2 - 2mx + n = (x-m)^2 + n - (-m)^2 = (x-m)^2 + n - m^2$$

Question 4: The coefficient of the z term is 14, half of which is 7, so we get:

$$z^2 + 14z - 1 = (z+7)^2 - 1 - 7^2 = (z+7)^2 - 50$$

Meaning our equation is now, $(z+7)^2 - 50 = 0$
Now we must rearrange this equation to make z the subject,

$$\begin{aligned} (z+7)^2 &= 50 \\ z + 7 &= \pm\sqrt{50} \\ z &= -7 \pm \sqrt{50} = -7 \pm 5\sqrt{2} \end{aligned}$$

Question 5: We have to start by writing the equation in a more familiar form, $x^2 + 4x - 3 = 0$. In this case, we have $a = 1$ so now completing the square we get,

$$x^2 + 4x - 3 = (x+2)^2 - 3 - 4$$

Now to solve this quadratic we must rearrange it to make x the subject

$$\begin{aligned} (x+2)^2 - 7 &= 0 \\ (x+2)^2 &= 7 \\ x + 2 &= \pm\sqrt{7} \\ x &= -2 \pm \sqrt{7} \end{aligned}$$

15 Algebraic Fractions

Question 1: We need to find a common denominator between all three fractions before we can do the addition and subtraction. As 30 is the lowest common multiple of 2, 3 5, we will choose $30x$ as the common denominator.

$$\begin{aligned} \frac{1}{2x} + \frac{1}{3x} - \frac{1}{5x} &= \left(\frac{1}{2x} \times \frac{15}{15}\right) + \left(\frac{1}{3x} \times \frac{10}{10}\right) - \left(\frac{1}{5x} \times \frac{6}{6}\right) \\ &= \frac{15}{30x} + \frac{10}{30x} - \frac{6}{30x} = \frac{15 + 10 - 6}{30x} = \frac{19}{30x} \end{aligned}$$

Question 2: We need to find a common denominator between the fractions before we can do the addition, hence,

$$\frac{8}{x} - \frac{1}{x-3} = \frac{8(x-3)}{x(x-3)} - \frac{1(x)}{(x-3)(x)} = \frac{8x - 24 - x}{x(x-3)} = \frac{7x - 24}{x(x-3)}$$

There are no more common terms so this is fully simplified.

Question 3: First, well look at the numerator, before we can factorise it, we must expand the brackets,

$$2(8-k) + 4(k-1) = 16 - 2k + 4k - 4 = 2k + 12$$

Then, the most we can do is take the 2 out as a factor, leaving us with $2k + 12 = 2(k+6)$. Now, the denominator is $k^2 - 36 = (k+6)(k-6)$. As there is a factor of $(k+6)$ in both the numerator and denominator, these will cancel.

$$\frac{2\cancel{(k+6)}}{\cancel{(k+6)}(k-6)} = \frac{2}{k-6}$$

Question 4: Our first step when dividing any fractions should be to flip the second fraction over and turn the division into a multiplication.

$$\frac{7ab}{12} \div \frac{4a}{9b^2} = \frac{7ab}{12} \times \frac{9b^2}{4a}$$

Completing the multiplication,

$$\frac{7ab}{12} \times \frac{9b^2}{4a} = \frac{63ab^3}{48a}$$

There is a factor of a that we can cancel and can also take out a factor of 3 from 63 and 48,

$$\frac{3a \times 21b^3}{3a \times 16} = \frac{21b^3}{16}$$

Question 5: To do this subtraction, we need to find a common denominator so the left-hand fraction must be multiplied by $(z+5)$ on top and bottom.

$$\frac{z+2}{z-1} = \frac{(z+2)(z+5)}{(z-1)(z+5)}$$

For the right-hand fraction we will multiply $(z-1)$ on top and bottom.

$$\frac{z}{z+5} = \frac{z(z-1)}{(z-1)(z+5)}$$

Then, the subtraction is:

$$\frac{z+2}{z-1} - \frac{z}{z+5} = \frac{(z+2)(z+5)}{(z-1)(z+5)} - \frac{z(z-1)}{(z-1)(z+5)} = \frac{(z+2)(z+5)-z(z-1)}{(z-1)(z+5)}$$

Expanding the numerator, we get:

$$(z+2)(z+5) - z(z-1) = z^2 + 7z + 10 - z^2 + z = 8z + 10 = 2(4z+5)$$

Our final answer is:

$$\frac{2(4z+5)}{(z-1)(z+5)}$$

---

16 Sequences and Nth Term (Linear)

Question 1: a) Substituting $n = 12$ into the formula. $4(12)+1 = 49$. So, the 12^{th} term is 49

b) Every term in this sequence is generated when an integer value of n is substituted into $4n+1$. Hence if we set 77 to equal $4n+1$, we can determine its position in the sequence. Hence, $4n+1 = 77$, then making n the subject by subtracting 1 then dividing by 4,

$$n = \frac{77-1}{4} = 19$$

Hence 77 is the 19^{th} term in the sequence.

Question 2: a) To generate the first 5 terms of this sequence, we will substitute $n = 1,2,3,4,5$ into the formula given.

$1 = 5(1) - 4 = 1$

$2 = 5(2) - 4 = 6$

$3 = 5(3) - 4 = 11$

$4 = 5(4) - 4 = 16$

$5 = 5(5) - 4 = 21$

So, the first 5 terms are 1, 6, 11, 16, and 21

b) Every term in this sequence is generated when an integer value of n is substituted into $5n-4$. If we set 108 to equal $5n-4$, we can determine if it is a part of the sequence or not. If the value of n is a whole number then it is part of the sequence. Hence $5n-4 = 108$, then making n the subject by adding 4 then dividing by 5,

$$n = \frac{112}{5} = 22.4$$

As there is no 22.4^{th} position in the sequence, it must be the case that 108 is not a term in this sequence.

Question 3: We are told it is an arithmetic progression and so must have n^{th} formula: $an+b$. To find a, we must inspect the difference between each term which is 5, hence $a = 5$. Then, to find b, lets consider the sequence generated by $5n$: 5, 10, 15, 20, 25

Every term is this sequence is bigger than the corresponding terms in the original sequence by 8. So, to get to the original sequence, we will have to subtract 8 from every term in this sequence. In other words, the n^{th} term formula for our sequence in question is $5n-8$

---

17 Quadratic Sequences

Question 1: To generate the first 5 terms of this sequence, we will substitute $n = 1,2,3,4,5$ into the formula given.

$n = 1$ gives $(1)^2 + 6(1) - 10 = -3$

$n = 2$ gives $(2)^2 + 6(2) - 10 = 6$

$n = 3$ gives $(3)^2 + 6(3) - 10 = 17$

$n = 4$ gives $(4)^2 + 6(4) - 10 = 30$

$n = 5$ gives $(5)^2 + 6(5) - 10 = 45$

Hence the first five terms of the sequence are, $-3, 6, 17, 30, 45$

Question 2: a) To generate the first 4 terms of this sequence, we will substitute $n = 1,2,3,4$ into the formula given.

$n = 1$ gives $(1)^2 - 2 = -1$

$n = 2$ gives $(2)^2 - 2 = 2$

$n = 3$ gives $(3)^2 - 2 = 7$

$n = 4$ gives $(4)^2 - 2 = 14$

b) Every term in this sequence is generated when an integer value of n is substituted into $n^2 - 2$. Hence if we set 287 to equal $n^2 - 2$, we can determine its position in the sequence, $n^2 - 2 = 287$, making n the subject, $n = \sqrt{287+2} = 17$. Hence 287 is the 17^{th} term in the sequence.

Question 3: Every term in this sequence is generated when an integer value of n is substituted into $(n-1)^2$ Thus if we set 49 to equal $(n-1)^2$, we can determine its position in the sequence,

$$(n-1)^2 = 49$$
$$n - 1 = \pm\sqrt{49}$$
$$n = 1 + 7 = 8$$

Hence 49 is the 8^{th} term in the sequence, as n can only be a positive integer.

Question 4: The n^{th} term formula will take the form $an^2 + bn + c$ where $a, b,$ and c are numbers to be determined. Firstly, we have to find the differences between the terms in the sequences, and then find the difference between the differences. We then divide this by 2 to find a. Doing so, we find, $a = 1$. Now we need to find b and c by comparing the values generated by a sequence of n^2, to the original sequence.

$$u_n = 9, 20, 33, 48, 65$$
$$n^2 = 1, 4, 9, 16, 25$$
$$u_n - n^2 = 8, 16, 24, 32, 40$$

The difference, is a linear sequence whose n^{th} term formula is precisely $bn + c$, so. The difference is 8, so the n^{th} term must be $8n + c$ where $c = 0$. Therefore, we get the n^{th} term formula of the quadratic to be $n^2 + 8n$

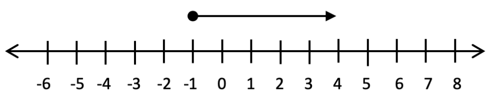
Question 5: We find, $a = 2$. Now we need to find b and c by comparing the values generated by a sequence of $2n^2$, to the original sequence.

$$u_n = 0, 1, 6, 15, 28$$
$$2n^2 = 2, 8, 18, 32, 50$$
$$u_n - 2n^2 = -2, -7, -12, -17, -22$$

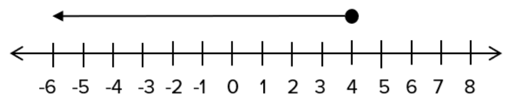
The difference, is a linear sequence whose n^{th} term formula is precisely $bn + c$, so the difference is -5, so the n^{th} term must be $-5n + c$. Then, for $n = 1$, we get $-5n = -5$, whereas the first term is not -5, but -2. To get from -5 to -2 we have to add 3, so we must have that $c = 3$, and thus the n^{th} term is $-5n + 3$. Therefore, we get the n^{th} term formula of the quadratic to be $2n^2 - 5n + 3$

18 Inequalities on a Number Line

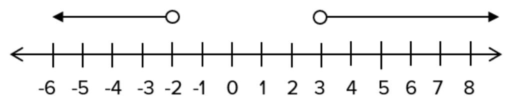
Question 1: The inequality, $x \geq -1$, will require a closed circle at -1 and an arrow pointing right.



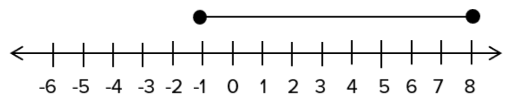
Question 2: The inequality, $x \leq 4$, will require an closed circle at 4 and an arrow pointing left.



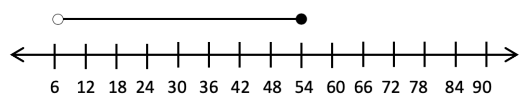
Question 3: The first inequality, $y > 3$, will require an open circle at 3 and an arrow pointing right. The other inequality, $y < -2$, will require an open circle at -2 and an arrow pointing left.



Question 4: The lower bound, $-1 \leq x$, will require a closed circle at $x = -1$. The upper bound $x \leq 8$, will require an closed circle at $x = 8$



Question 5: Forming the correct inequality $6 < b \leq 54$ and displaying with an open circle for representing the strict inequality (6) and a closed circle representing the non-strict inequality (54).



19 Solving Inequalities

Question 1:

$$7 - 3k > -5k + 12$$
$$7 + 2k > 12$$
$$2k > 5$$
$$k > \frac{5}{2}$$

Question 2:

$$\frac{5x - 1}{4} > 3x$$
$$5x - 1 > 12x$$
$$-1 > 7x$$
$$x < -\frac{1}{7}$$

Question 3:

$$2x + 5 > 3x - 2$$
$$5 > x - 2$$
$$7 > x$$
$$x < 7$$

Question 4:

$$4 - 3x \leq 19$$
$$-3x \leq 15$$
$$x \geq -5$$

Question 5:

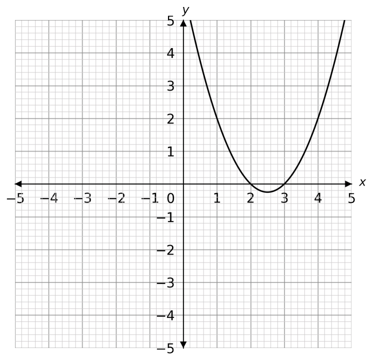
$$-5 < 2x - 3 < 10$$
$$-2 < 2x < 13$$
$$-1 < x < \frac{13}{2}$$

20 Quadratic Inequalities

Question 1: So, we will factorise this quadratic, and then use what would be the solutions to help us plot the graph. Observing that $(-2) \times (-3) = 6$ and $(-2) + (-3) = -5$, we get,

$$x^2 - 5x + 6 \leq 0$$
$$(x - 2)(x - 3) \leq 0$$

So, the roots of this quadratic would be $x = 2$ and $x = 3$.

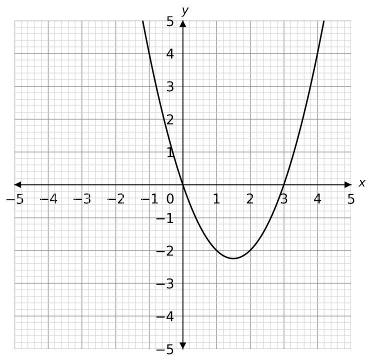


The inequality in the question is $x^2 - 5x + 6 \leq 0$, so the question is: when does the graph go below zero? We can see that it goes below zero when x is between 2 and 3. Therefore, the solution is, $2 \le x \le 3$

Question 2: Taking x out as a factor, we get

$$x^2 - 3x > 0$$
$$x(x-3) > 0$$

So, the roots of this quadratic would be $x = 0$ and $x = 3$.

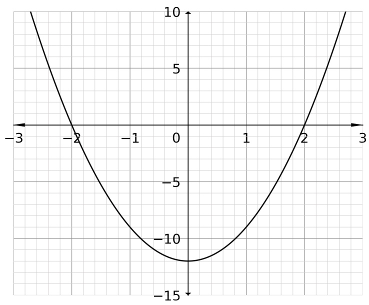


The inequality in the question is $x^2 - 3x > 0$, so the question is: when does the graph go above zero? We can see that it goes above zero when x is less than 0 and also when x is above 3. Therefore, $x < 0$ or $x > 3$

Question 3: We solve this inequality by simply rearranging it to make p the subject,

$$3p^2 + 8 > 20$$
$$3p^2 > 12$$
$$p^2 > 4$$

So, the roots of this quadratic would be $p = -2$ and $p = 2$.

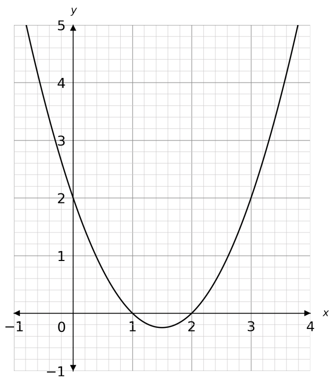


The inequality in the question is $3p^2 - 12 > 0$, so the question is: when does the graph go above zero? We can see that it goes above zero when p is less than -2 and also when p is above 2. Therefore, the solution is, $p < -2$ or $p > 2$

Question 4: We solve this inequality by simply rearranging,

$$x^2 - 3x + 2 < 0$$
$$(x-2)(x-1) < 0$$

Hence, the roots of this quadratic would be $x = 1$ and $x = 2$.

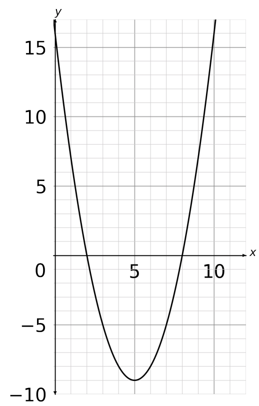


The inequality in the question is $x^2 - 3x + 2 < 0$, so the question is: when does the graph go below zero? We can see that it goes below zero when x is greater than 1 and x is less than 2. Therefore, the solution is, $1 < x < 2$

Question 5: We solve this inequality by simply rearranging,

$$x^2 - 5x + 24 < 5x + 8$$
$$x^2 - 10x + 16 < 0$$
$$(x-8)(x-2) < 0$$

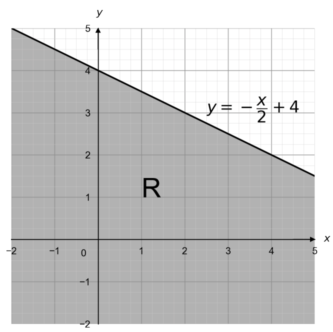
Hence, the roots of this quadratic would be $x = 2$ and $x = 8$.



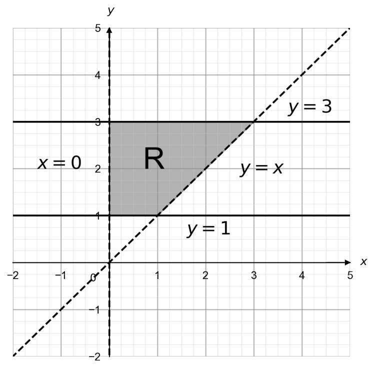
The inequality in the question is $x^2 - 10x + 16 < 0$, so the question is: when does the graph go below zero? We can see that it goes below zero when x is greater than 2 and x is less than 8. Therefore, $2 < x < 8$.

21 Graphical Inequalities

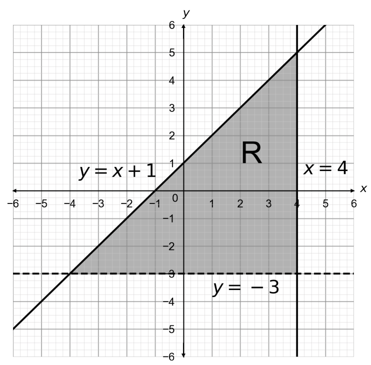
Question 1: Firstly, rearrange this equation to get, $y \le -\frac{x}{2} + 4$. Drawing this is an equation, the graph would be a solid line with gradient $-\frac{1}{2}$ and y-intercept 4. Once drawn, we should shade and mark the region below the line.



Question 2: We are going to treat the inequalities as equations and plot them as straight lines. The first one will be a solid plot of the line $y = 1$, the second will be a solid plot of the line $y = 3$, the third will be a dashed plot of the line $x = 0$, and the fourth will be a dashed plot of the line $y = x$. Now, we want to shade the area that is below the line $y = 1$, above the line $y = 3$, to the right of the line $x = 0$, and above the line $y = x$. The resulting graph looks like:



Question 3: We are going to plot the inequalities as straight lines. The first one will be a solid plot of the line $y = x+1$, the second will be a solid plot of the line $x = 4$, and the third will be a dashed plot of the line $y = -3$. Now, we want to shade the area that is below the line $y = x+1$, above the line $y = -3$, and to the left of the line $x = 4$



Question 4: The horizontal line is $y = -2$. The dashed line has its y-intercept at 2 and a gradient of 2, so it is $y = 2x+2$. The final line has its y-intercept at 5 and a gradient of -3, so it is $y = -3x+5$. Now, the shaded area is above $y = -2$ and the line is solid, so the inequality is, $y \geq -2$. The shaded area is below $y = 2x+2$ and the line is dashed, so the inequality is, $y < 2x+2$. The shaded area is below $y = -3x+5$ and the line is solid, so the inequality is, $y \leq -3x+5$ Therefore, the shaded area is described by the 3 inequalities, $y \geq -2$, $y < 2x+2$, and $y \leq -3x+5$

Question 5: The vertical line is $x = 2$. The dashed line has its y-intercept at 2 and a gradient of $\frac{1}{4}$, so it is $y = \frac{1}{4}x+2$. The final line has its y-intercept at -6 and a gradient of 2, so it is $y = 2x-6$. Therefore, the shaded area is described by the 3 inequalities: $x > 2$, $y < \frac{1}{4}x+2$, and $y \geq -2x-6$

22 Iterative methods

Question 1: We will form a table with one column of x values, one column on the results of calculating $2x^3-6x$, and one column stating if the answer is bigger or smaller than the desired 1.

x	$2x^3-6x$	Result
2	4	Too big
1	-4	Too small
1.5	-2.25	Too small
1.7	-0.374	Too small
1.8	0.864	Too small
1.9	2.318	Too big
1.85	1.56325	Too big

So, if 1.8 gives a result that is too small and 1.85 gives a result that is too big, then the actual solution must be somewhere between these two values. Given that any number between 1.8 and 1.85 must round to 1.8, the solution must be 1.8 to 1 decimal place.

Question 2: To find a solution we will use the recursive formula, until we get two consecutive terms which round to the same number to 2 decimal places.

$$x_1 = 2$$
$$x_2 = \sqrt[3]{3(2)+9} = 2.4662$$
$$x_3 = \sqrt[3]{3(2.4662)+9} = 2.54060$$
$$x_4 = \sqrt[3]{3(2.54060)+9} = 2.55207$$
$$x_5 = \sqrt[3]{3(2.55207)+9} = 2.55383$$

These last two results both round to 2.55 to 2d.p., so the solution must be 2.55 to 2 decimal places.

Question 3:

$$x_1 = 1$$
$$x_2 = \frac{-3}{(1)^2+5} = -0.5$$
$$x_3 = \frac{-3}{(-0.5)^2+5} = -0.5714$$
$$x_4 = \frac{-3}{(-0.5714)^2+5} = -0.5632$$
$$x_5 = \frac{-3}{(-0.5632)^2+5} = -0.5642$$

These last two results both round to -0.56 to 2 d.p., so the solution must be -0.56 to 2 decimal places.

Question 4:

$$x_1 = 4$$
$$x_2 = \frac{3}{(4)^2} + 3 = 3.1875$$
$$x_3 = \frac{3}{(3.1875)^2} + 3 = 3.29527$$
$$x_4 = \frac{3}{(3.29527)^2} + 3 = 3.27627$$
$$x_5 = \frac{3}{(3.27627)^2} + 3 = 3.27949$$
$$x_6 = \frac{3}{(3.27949)^2} + 3 = 3.27894$$

These last two results both round to 3.279 to 3 d.p., so the solution must be 3.279 to 3 decimal places.

Question 5:

$$x_1 = 2$$
$$x_2 = \sqrt[3]{6(2)+5} = 2.57128$$
$$x_3 = \sqrt[3]{6(2.57128)+5} = 2.73363$$
$$x_4 = \sqrt[3]{6(2.73363)+5} = 2.77641$$
$$x_5 = \sqrt[3]{6(2.77641)+5} = 2.78746$$
$$x_6 = \sqrt[3]{6(2.78746)+5} = 2.79031$$

These last two results both round to 2.79 to 2 d.p., so the solution must be 2.79 to 2 decimal places.

23 Simultaneous Equations

Question 1: Subtracting equation 2 from equation 1 so that,

$$y = 2x - 6$$
$$y = \frac{1}{2}x + 6$$
$$(y - y) = (2x - \frac{1}{2}x) - 6 - 6$$
$$0 = \frac{3}{2}x - 12$$

If we rearrange to make x the subject we find, $x = \frac{2\times 12}{3} = \frac{24}{3} = 8$. Substituting $x = 8$ back into the original first equation,

$$y = 2(8) - 6$$
$$y = 10$$

Hence, the solution is, $x = 8, y = 10$

Question 2: If we multiply the second equation by 2, we have two equations both with a $2x$ term, hence subtracting our new equation 2 from equation 1 we get

$$2x - 3y = 16$$
$$2x + 4y = -12$$

$$(2x - 2x) + (-3y - 4y) = 16 - (-12)$$
$$0x - 7y = 28$$

If we rearrange to make y the subject we find, $y = \frac{28}{-7} = -4$. Substituting $y = -4$ back into the original second equation,

$$x + 2(-4) = -6$$
$$x - 8 = -6$$
$$x = 2$$

Hence, the solution is, $x = 2, y = -4$

Question 3:

$$5x + 2y + 16 = 0$$
$$2x + 3y + 13 = 0$$

$$15x + 6y + 48 = 0$$
$$-\quad 4x + 6y + 26 = 0$$

$$11x + 22 = 0$$
$$11x = -22$$
$$x = -2$$

Substituting x back into the original first equation,

$$5(-2) + 2y = -16$$
$$2y = -6$$
$$y = -3$$

Hence, the solution is, $x = 2, y = -3$

Question 4: Let A be the cost of an adult ticket and let C be the cost of a child ticket, thus we have two simultaneous equations,

$$2A + 3C = 20$$
$$A + C = 8.5$$

If we multiply the second equation by 2, we have two equations both with a $2A$ term, hence subtracting our new equation 2 from equation 1 we get,

$$2A + 3C = 20$$
$$2A + 2C = 17$$

$$(2A - 2A) + (3C - 2C) = (20 - 17)$$
$$C = 3$$

Then, substituting this value back into the original equation 2, we get,

$$A + 3 = 8.5$$
$$A = 5.5$$

Therefore, the cost of a child ticket is £3, and the cost of an adult ticket is £5.50.

Question 5:

$$y = -x + 7$$
$$x^2 + y^2 = 25$$

$$x^2 + (-x+7)^2 = 25$$
$$2x^2 - 14x + 49 = 25$$
$$2x^2 - 14x + 24 = 0$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$

Hence $x = 3$ or $x = 4$
Substituting these two values back into the first equation,

$$y = -(3) + 7 = 4$$
$$y = -(4) + 7 = 3$$

Therefore, the two solutions are, $x = 4, y = 3$ and $x = 3, y = 4$

24 Proof

Question 1: We will try the first few even numbers (squaring them and adding 3) until we find an example that isnt prime. So, we get
$2^2 + 3 = 4 + 3 = 7,$ which is prime.
$4^2 + 3 = 16 + 3 = 19,$ which is prime.
$6^2 + 3 = 36 + 3 = 39,$ which is not prime.

Since 39 is divisible by 3, it must not be prime, so we have proved Luke's statement to be false.

Question 2: To show that the left and right hand sides of the equation are identical we expand the brackets on the left hand side, $5(3x-5)-2(2x+9) = 15x-25-4x-18 = 11x-43$. Hence we have shown the identity is true.

Question 3: To answer this question, we will need to expand and simplify the expression given to us, so we can hopefully write it in a way that shows it is clearly divisible by 2 (since that's the definition of even). So, expanding the first bracket, we get

$$(3n+1)^2 = 9n^2+3n+3n+1 = 9n^2+6n+1$$

Then, expanding the second bracket, we get

$$(n-1)^2 = n^2-n-n+1 = n^2-2n+1$$

Adding the expansions together, we get

$$(9n^2+6n+1)+(n^2-2n+1) = 10n^2+4n+2$$

Is this an even number? Well, if we take a factor of 2 out of the expression: $2(5n^2+2n+1)$, we see that since $5n^2+2n+1$ is a whole number because n is a whole number, the expression in question is equal to $2 \times$ (some whole number) and so must be even.

Question 4:
For any 3 consecutive odd numbers: $2n+1, 2n+3$, and $2n+5$, adding them together gives us, $(2n+1)+(2n+3)+(2n+5) = 6n+9$.
Since n is a whole number, $6n$ is even since Even × Even = Even and Even × Odd = Even. Then Even + Odd = Odd, so the result is odd.

Question 5: For any 3 consecutive even numbers: $2n, (2n+2)$ and $(2n+4)$, adding them together gives us, $2n+(2n+2)+(2n+4) = 6n+6 = 6(n+1)$, which is divisible by 6.

---

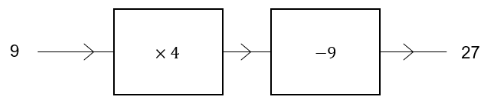
25 Function Machines

Question 1: a) Inputting 35, we first multiply by 3: $35 \times 3 = 105$. Then, add 15 to get, $105+15 = 120$
b) We must work backwards and do the opposite operations. So, first subtracting 15 from the given output, we get, $48-15 = 33$. Then, dividing by 3 we get, $33 \div 3 = 11$. Meaning that 11 is the input required.

Question 2: Inputting -5, we first multiply by -2: $-5 \times -2 = 10$. Then, adding 7 we get, $10+7 = 17$. Meaning that 17 is the output required.

Question 3: Inputting $3x$, we first multiply by $\frac{1}{2}$, $3x \times \frac{1}{2} = \frac{3}{2}x$
Then, dividing by 3 we get, $\frac{3}{2}x \div 3 = \frac{1}{2}x$, meaning that $\frac{1}{2}x$ is the output.

Question 4: An example of two operations are, multiplying by 4, so $9 \times 4 = 36$, subtracting 9, so $36-9 = 27$



Question 5: We have to see what we get if we input x into the function machine. First, multiplying x by 12, $12 \times x = 12x$. Then, subtracting 25 to get, $12x-25$. This is the output of inputting x but we know the output is equal to $2x$, so we are left with the equation, $12x-25 = 2x$. Rearrange to get,

$$10x-25 = 0$$
$$10x = 25$$
$$x = \frac{25}{10} = 2.5$$

---

26 Functions

Question 1: a) Substituting $x = 10$ into $f(x)$, we find, $f(10) = \frac{10}{3(10)-5} = \frac{10}{25} = \frac{2}{5} = 0.4$

b) Substituting $x = 2$ into $f(x)$, we find, $f(10) = \frac{10}{3(2)-5} = \frac{10}{1} = 10$

c) Substituting $x = -1$ into $f(x)$, we find, $f(-1) = \frac{10}{3(-1)-5} = \frac{10}{-8} = -\frac{5}{4}$

Question 2: a) Substituting $x = 4$ into $g(x)$, then $f(x)$,
$g(4) = (2 \times 4)-5 = 8-5 = 3$, $fg(4) = f(3) = \frac{15}{3} = 5$

b) For $gf(-30)$ we must first find $f(-30)$ and then substitute the result into $g(x)$, $f(-30) = \frac{15}{-30} = -\frac{1}{2}$

$$gf(-30) = g(-\frac{1}{2}) = 2(-\frac{1}{2})-5 = -1-5 = -6$$

c) To find an expression for $gf(x)$, substitute $f(x)$ in for every instance of x in $g(x)$,

$$gf(x) = 2(f(x))-5 = 2 \times (\frac{15}{x})-5 = \frac{30}{x}-5$$

Question 3: We need to write the function as $y = \frac{5}{x-4}$ and rearrange this equation to make x the subject. Then, we will swap every y with an x - and vice versa.

$$y(x-4) = 5$$
$$x-4 = \frac{5}{y}$$
$$x = \frac{5}{y}+4$$

Now, swap each x with a y and vice versa to get, $f^{-1}(x) = \frac{5}{x}+4$

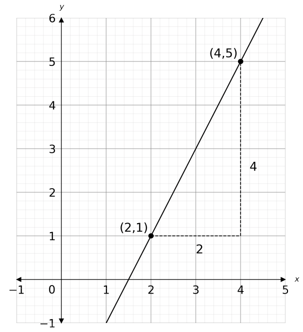
Question 4: We need to write the function as $y = \frac{4}{x}+3$ and rearrange this equation to make x the subject. Then, we will swap every y with an x - and vice versa.

$$y-3 = \frac{4}{x}$$
$$x(y-3) = 4$$
$$x = \frac{4}{y-3}$$

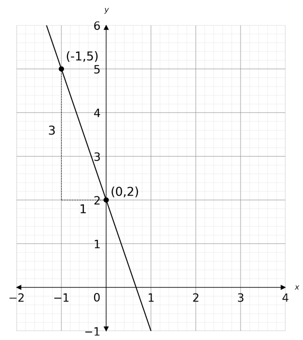
Now, simply swap each x with a y and vice versa to get, $y^{-1}(x) = \frac{4}{x-3}$.

1 Gradients of Straight Line Graphs

Question 1: Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{4}{2} = 2$

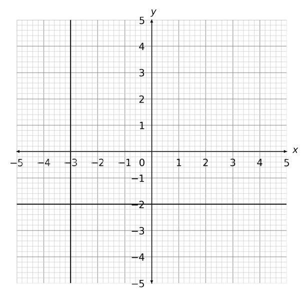


Question 2: Gradient $= -\frac{3}{1} = -3$



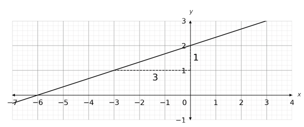
Question 3: Gradient $= \frac{-1-(-6)}{-8-2} = \frac{5}{-10} = -\frac{1}{2}$

Question 4: The lines $y = -2$ and $x = -3$ should be a straight line perpendicular to the axis at that point,



2 $y = mx + c$

Question 1: The y-intercept is 2, so $c = 2$
$m = \text{gradient} = \frac{1}{3}$
Therefore, the equation of the line is $y = \frac{1}{3}x + 2$

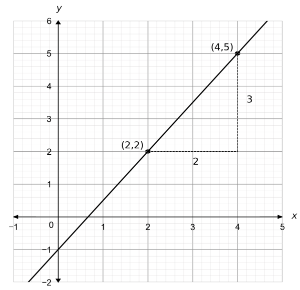


Question 2: $m = \text{gradient} = \frac{-6-34}{-3-2} = \frac{-40}{-5} = 8$.
Substitute one pair of coordinates into the equation $y = mx + c$ to find c, e.g. $(2, 34)$. So,

$$34 = 8 \times 2 + c$$
$$34 = 16 + c$$
$$c = 34 - 16$$
$$= 18$$

Therefore, the equation of the line is, $y = 8x + 18$.

Question 3: The y-intercept is -1, so $c = -1$.
$m = \text{gradient} = \frac{3}{2}$
Hence, $y = \frac{3}{2}x - 1$



3 Coordinates and Ratios

Question 1: $A = (-2, 2)$, $B = (-1, -2)$, $C = (3, 0)$.

Question 2: $A = (-2, -2)$, $B = (0, 3)$. By taking the average of the x and y coordinates of A and B separately, the midpoint is

$$\left(\frac{-2+0}{2}, \frac{-2+3}{2}\right) = \left(-1, \frac{1}{2}\right)$$

Question 3: Ratio $2:7$, total parts $= 9$, Thus $AC = \frac{2}{9}AB$
To find the distance AB, calculate the distance in x and y separately;

$$\text{Distance in } x = -16 - (-10) = -6$$
$$\text{Distance in } y = 1 - 37 = -36$$

The distance $AC = \frac{2}{9} \times (\text{distance}) AC$, so

$$\text{Distance in } x = \frac{2}{9} \times -6 = -\frac{4}{3}$$
$$\text{Distance in } y = \frac{2}{9} \times -36 = -8$$

Thus, location of C:

$$x = -10 + \left(-\frac{4}{3}\right) = -\frac{34}{3}$$
$$y = 37 + (-8) = 29$$

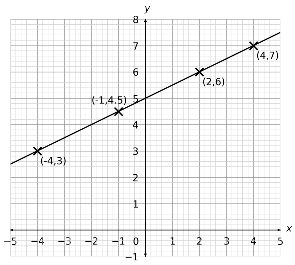
Hence, $C = \left(-\frac{34}{3}, 29\right)$

4 Drawing Straight Line Graphs

Question 1: Substitute the given values into the equation, e.g. when $x = -1$, $y = \frac{1}{2} \times (-1) + 5 = 4.5$, and so on.
The completed table looks like:

x	-4	-1	2	4
y	3	4.5	6	7

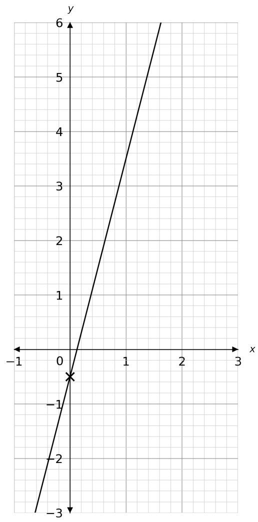
Plotting these points should give the following graph:



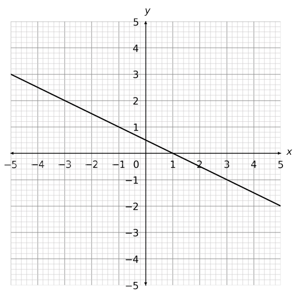
Question 2: Rearrange the equation to the form $y = mx + c$

$$2y = 8x - 1$$
$$y = 4x - \frac{1}{2}$$

So, the y-intercept is $-\frac{1}{2}$, and the gradient is 4 This is enough to draw the graph:



Question 3: Rearrange the equation to the form $y = mx + c$
$y = -0.5x + 0.5$
So, the y-intercept is $\frac{1}{2}$, and the gradient is $-\frac{1}{2}$. The result should look like the figure below



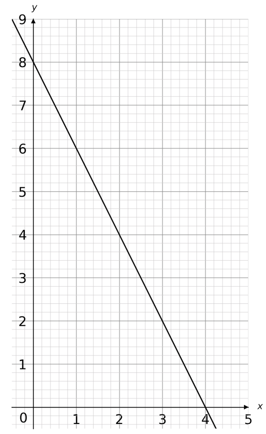
---

5 Parallel and Perpendicular Lines

Question 1:
a) Rearrange to get, $y = \frac{1}{2}x + \frac{1}{4}$
b) Rearrange to get, $y = -2x + \frac{5}{2}$
c) Rearrange to get, $y = \frac{1}{2}x + 45$

a) and c) have the same gradient $\left(\frac{1}{2}\right)$ so they are parallel.

Question 2: Rearrrange the equation to the form $y = mx + c$
$y = -2x - \frac{3}{5}$.
The new line must have the same gradient (-2) and pass through $(1, 6)$.

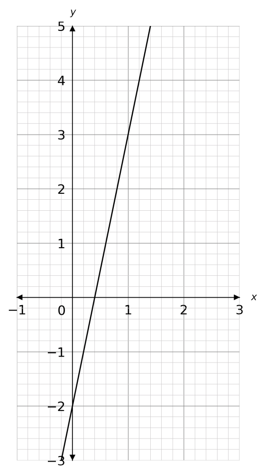


Question 3: For a line with gradient $\frac{3}{7}$, the gradient of the perpendicular line will be $-1 \div \frac{3}{7} = -\frac{7}{3}$ (i.e. the negative reciprocal).
Substitute the gradient and the point $(5, -4)$ into $y = mx + c$ in order to find c.

$$-4 = \left(-\frac{7}{3}\right) \times 5 + c = -\frac{35}{3} + c$$
$$c = -4 + \frac{35}{3} = -\frac{4}{1} + \frac{35}{3}$$
$$= -\frac{12}{3} + \frac{35}{3} = \frac{23}{3}$$

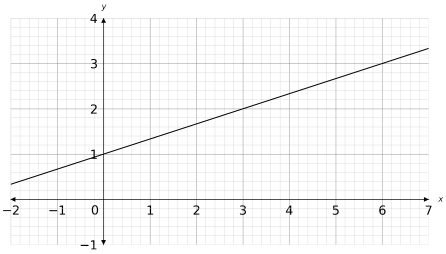
Therefore, the equation of the line is, $y = -\frac{7}{3}x + \frac{23}{3}$

Question 4: For a line with gradient 5, the gradient of a parallel line must also be 5. Substitute the gradient and the point $(1,3)$ into $y = mx + c$ in order to find c.
$3 = 1 \times 5 + c = 5 + c$, therefore $c = 3 - 5 = -2$.
So, $y = 5x - 2$.



Question 5: For a line with gradient -3, the gradient of the perpendicular line must be, $\left(-\frac{1}{-3}\right) = \frac{1}{3}$
Substitute the gradient and the point $(-9, -2)$ values into $y = mx + c$ in order to find c. Doing so, we get, $-2 = (-9) \times \frac{1}{3} + c = -3 + c$, therefore

$c = -2 + 3 = 1$. So, $y = \frac{1}{3}x + 1$.

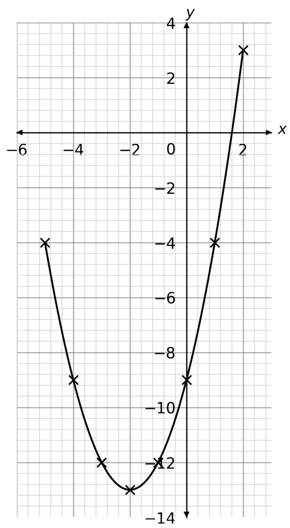


6 Harder Graphs

Question 1: Substitute in the values of x to get the missing values of y. The completed table should look like:

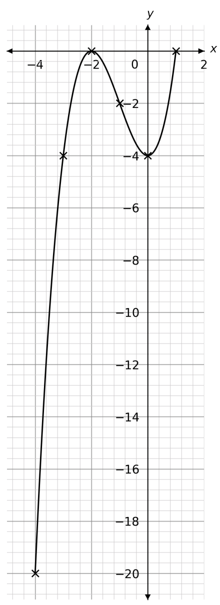
x	-5	-4	-3	-2	-1	0	1	2
y	-4	-9	-12	-13	-12	-9	-4	3

Plotting these coordinates on a pair of axes and joining them with a curve:



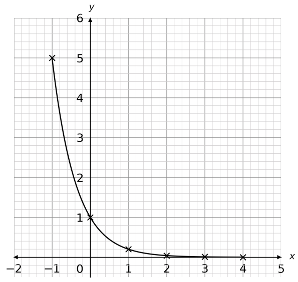
Question 2: The completed table and graph should look like:

x	-4	-3	-2	-1	0	1
y	-20	-4	0	-2	-4	0



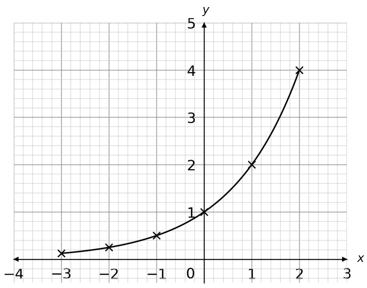
Question 3: The completed table and graph should look like:

x	-1	0	1	2	3	4
y	5	1	0.2	0.04	0.008	0.0016



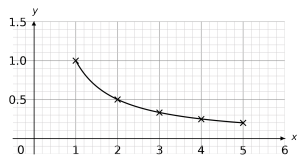
Question 4: The completed table and graph should look like:

x	-3	-2	-1	0	1	2
y	0.125	0.25	0.5	1	2	4



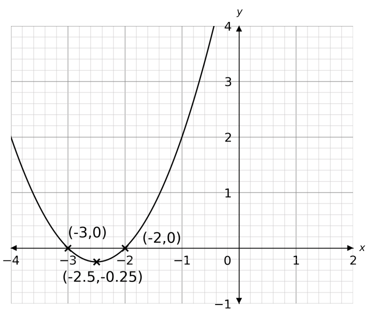
Question 5: The completed table and graph should look like:

x	1	2	3	4	5
y	1	0.5	$0.\dot{3}$	0.25	0.2



7 Turning Points of Quadratic Graphs

Question 1: $x^2 + 5x + 6 = (x+2)(x+3) = 0$
Thus the two roots are: $x = -2$ and $x = -3$. The turning point is halfway between the roots, which is $\frac{-2+(-3)}{2} = -2.5$.
To find the y coordinate, substitute the x value back into the equation, $y = (-2.5)^2 + 5(-2.5) + 6 = -0.25$.
The resulting sketch of the graph should look like



Question 2: Complete the square as follows,

$$x^2 - 3x + 11 = \left(x - \frac{3}{2}\right)^2 + 11 - \left(-\frac{3}{2}\right)^2$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{35}{4}$$

Thus, the coordinates of the turning point are $\left(\frac{3}{2}, \frac{35}{4}\right)$.

Question 3: Complete the square as follows,

$$-x^2 + 10x - 3 = -(x^2 - 10x + 3)$$
$$-(x^2 - 10x + 3) = -\left[(x-5)^2 + 3 - (-5)^2\right]$$
$$-\left[(x-5)^2 - 22\right] = -(x-5)^2 + 22$$

Therefore, the coordinates of the turning point are $(5, 22)$.

Question 4: Complete the square as follows,

$$y = 2(x^2 + 10x + 7)$$
$$y = 2\left[(x+5)^2 + 7 - 25\right]$$
$$= 2\left[(x+5)^2 - 18\right]$$
$$= 2(x+5)^2 - 36$$

Therefore, the coordinates of the turning point are $(-5, -36)$.

8 Circle Graphs and Tangents

Question 1: $x^2 + y^2 = 100$

Question 2: First, find the gradient of the line from the centre to $(12, 5)$.

$$\text{Gradient of radius } = \frac{\text{change in } y}{\text{change in } x} = \frac{5-0}{12-0} = \frac{5}{12}$$
$$\text{Gradient of tangent } = -1 \div \frac{5}{12} = -\frac{12}{5}$$

Substitute the gradient and the point $(-9, -2)$ into $y = mx + c$ in order to find c,

$$5 = -\frac{12}{5} \times 12 + c$$
$$5 = -\frac{144}{5} + c$$
$$c = 5 + \frac{144}{5} = \frac{169}{5}$$
$$y = -\frac{12}{5}x + \frac{169}{5}$$

Question 3: First find the gradient of the line from the centre to $(-8, -7)$.

$$\text{Gradient of radius } = \frac{\text{change in } y}{\text{change in } x} = \frac{-7-0}{-8-0} = \frac{7}{8}$$
$$\text{Gradient of tangent } = -\frac{8}{7}$$

Substitute the gradient and the point $(-8, -7)$ into $y = mx + c$ in order to find c,

$$-7 = -\frac{8}{7} \times -8 + c$$
$$-7 = \frac{64}{7} + c$$
$$c = -\frac{113}{7}$$
$$y = -\frac{8}{7}x - \frac{113}{7}$$

9 Solving Simultaneous Equations with Graphs

Question 1: Substitute in some values of x into the first equation, we get

$x = -2$ gives $y = -2 \times (-2) + 1 = 5$

$x = 0$ gives $y = -2 \times (0) + 1 = -1$

$x = 3$ gives $y = -2 \times (3) + 1 = -5$

So 3 coordinates on the line are $(-2, 5)$, $(0, -1)$, $(3, -5)$.

Then, doing the same for the second equation,

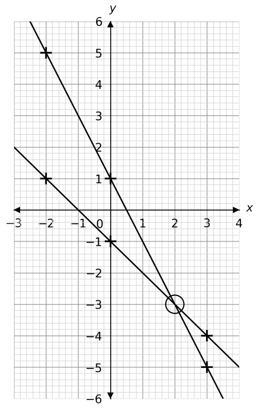
$x = -2$ gives $y = -(-2) - 1 = 1$

$x = 0$ gives $y = -(0) - 1 = -1$

$x = 3$ gives $y = -(3) - 1 = -4$

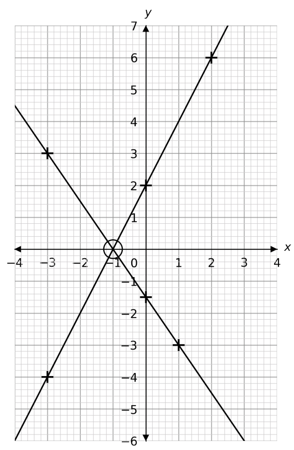
So, 3 coordinates on the line are $(-2, 1)$, $(0, -1)$, $(3, -4)$.

Plotting these points and drawing the lines gives the following graph.



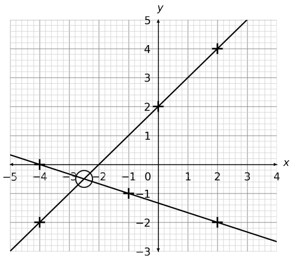
The two lines intersect at $(2, -3)$, therefore the solution is $x = 2$, $y = -3$

Question 2: Rearrange the second equation to be in a form we can use. We get: $y = -1.5x - 1.5$.
Plotting points for each equation gives the graph shown below.



The two lines intersect at $(-1, 0)$, therefore the solution is $x = -1$, $y = 0$

Question 3: Rearrange the second equation to the form $y = mx + c$, giving: $y = -\frac{1}{3}x - \frac{4}{3}$.
Plotting points for each equation gives the graph shown below.

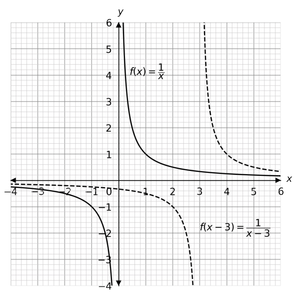


The two lines intersect at $(-2.5, -0.5)$, therefore the solution is, $x = -2.5,\ y = -0.5$

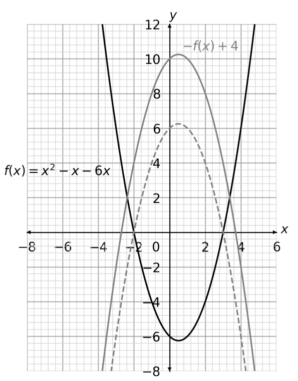
---

10 Graph Transformations

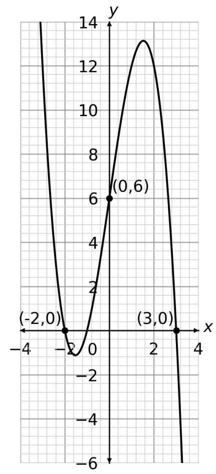
Question 1: $f(x-3) = \frac{1}{x-3}$ is a translation of positive 3 in the x-direction.



Question 2: $-f(x) + 4$ is both a reflection in the x-axis and a translation of 4 in the positive y-direction. Perform the reflection first and the translation second - the resulting graph should look as follows (the dotted line shows the intermediate step)



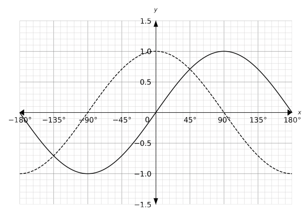
Question 3: $f(-x)$ means a reflection in the y-axis. The resulting graph should look like:



---

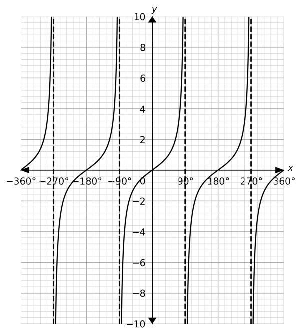
11 Sin, cos and tan Graphs

Question 1:

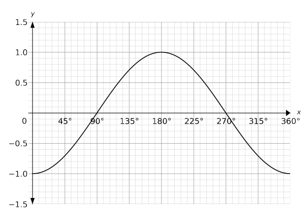


The solid black line represents the sin graph and the dotted line represents the cos graph.

Question 2: The tan graph has an asymptote at 90°, and every 180° before and after that.

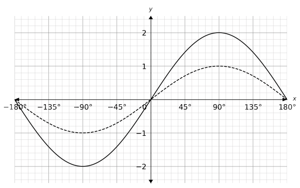


Question 3: This is a transformation of the form $y = -f(x)$, which corresponds to a reflection in the x axis. Hence, the graph should look like:



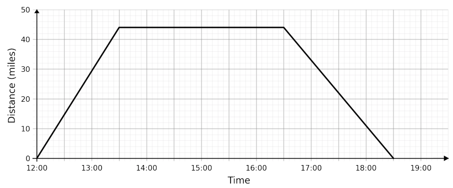
Question 4: Here we will plot $y = \sin(x)$ as a dotted line and $y =$

$2\sin(x)$ as a solid line. The resulting graph should look like,



12 Distance-Time Graphs

Question 1: The journey can be described as follows:
$12:00 - 13:30$, he travels from 0 miles away to 44 miles away;
$13:30 - 16:30$, he stays in one place;
$16:30 - 18:30$, he travels from 44 miles away to 0 miles away.
On a graph, this looks like:



Question 2: The fastest speed is given by the steepest gradient. Eliminating the middle period (i.e. the least steep section) and comparing the other two:

Period 1:

$$\text{Gradient } = \frac{\text{distance travelled}}{\text{time taken}} = \frac{600-0}{72-0} = 8.33 \text{ m/s}$$

Period 3:

$$\text{Gradient } = \frac{\text{distance travelled}}{\text{time taken}} = \frac{1,500-880}{282-180} = 6.08 \text{ m/s}$$

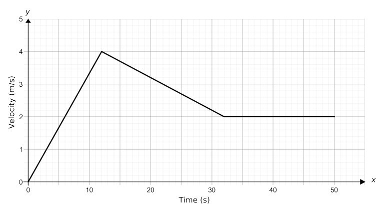
Therefore, the fastest speed travelled by Chris during the race was 8.33 m/s, to 3 sf.

Question 3: (a) Total Distance $= 48 + 10 = 58$ km (b) She stopped for 30 mins at the 32 km mark.

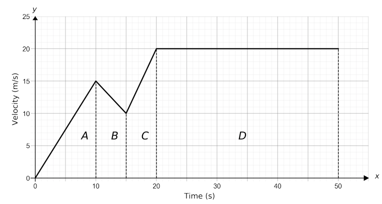
Question 4: The fastest average speed is given the steepest section of the graph. This is the final section which covered 48 km in one hour, thus, Maximum speed $= 48$ km/h

13 Velocity-Time Graphs

Question 1: The movement of the ball can be described as:
0 s to 12 s: the ball accelerates from 0m/s to 4m/s
12 s to 32 s: the ball decelerates from 4m/s to 2m/s ($0.1 \times 20 = 2$ m/s)
32 s to 50 s:the ball travels at constant speed.
The result should look like the graph below.



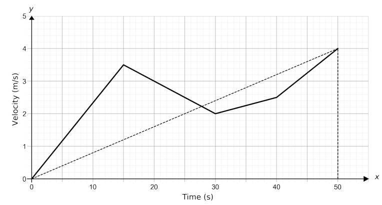
Question 2: The total distance travelled is given by the area under the graph. Dividing the area under the graph into 4 shapes gives:



$$\text{A} = \frac{1}{2} \times 10 \times 15 = 75 \text{ m}$$
$$\text{B} = \frac{1}{2} \times (10+15) \times 5 = 62.5 \text{ m}$$
$$\text{C} = \frac{1}{2} \times (10+20) \times 5 = 75 \text{ m}$$
$$\text{D} = 30 \times 20 = 600 \text{ m}$$

Therefore, the total distance travelled by the cyclist is, $75 + 62.5 + 75 + 600 = 812.5$ m

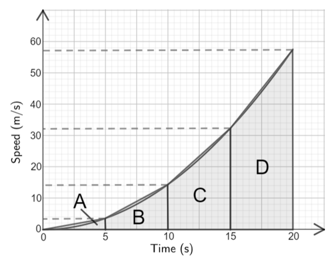
Question 3: Draw a line from the origin to the endpoint of the graph, as seen below. The average acceleration is given by the gradient of this line.



Hence the average acceleration is, gradient $= \frac{4-0}{50-0} = 0.08$ m/s^2

14 Area Under a Graph

Question 1: The distance is given by the area under the graph. Divide the graph into 4 equal strips of width 5 seconds.



Shape A is a triangle,

$$\text{Area of A } = \frac{1}{2} \times 5 \times 3.5 = 8.75$$

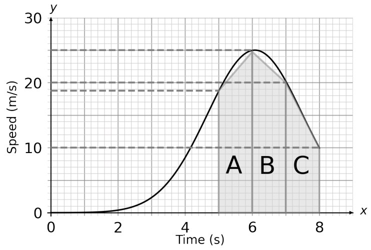
The other 3 shapes are trapeziums. Reading the remaining y-values from the graph, gives

$$\text{Area of B } = \frac{1}{2} \times (3.5+14) \times 5 = 43.75$$
$$\text{Area of C } = \frac{1}{2} \times (14+32) \times 5 = 115$$
$$\text{Area of D } = \frac{1}{2} \times (32+57) \times 5 = 222.5$$

The total distance travelled is, $8.75 + 43.75 + 115 + 222.5 = 390$ m.

Note: Any answer between 385 m and 395 m is acceptable in this case.

Question 2: Set up the graph as shown below.



$$\text{area of A} = \frac{1}{2} \times (19 + 25) \times 1 = 22$$

$$\text{area of B} = \frac{1}{2} \times (25 + 20) \times 1 = 22.5$$

$$\text{area of C} = \frac{1}{2} \times (20 + 10) \times 1 = 15$$

The total distance travelled is, $22 + 22.5 + 15 = 59.5$ m.
Note: Any answer between 59 m and 60 m is acceptable in this case.

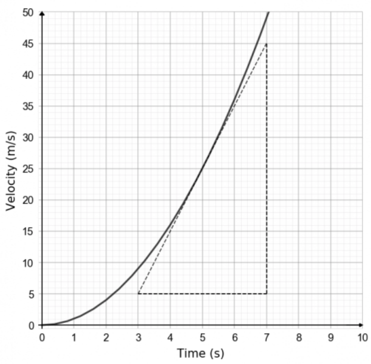
15 Real Life Graphs

Question 1: Diesel price per litre $\approx \frac{40}{35} = £1.14$.
Petrol price per litre $\approx \frac{35}{35} = £1.00$.
Hence the difference in price per litre, $£1.14 - £1.00 = £0.14$

Question 2: Cost per bike $\approx £38.00$
Total cost (4 bikes) $= £38.00 \times 4 = £152$

Question 3: $£8 = \$11.20$. Thus, $£800 = \$1120$

Question 4: The acceleration at 5 seconds after launch is the gradient of the line at that point. To find the gradient, draw a large triangle with the hypotenuse being the tangent to the line at $x = 5$.



$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{45 - 5}{7 - 3} = 10 \text{ m/s}^2$$

1 Ratios

Question 1: a) The sum of the ratio is $4+5=9$. Since the ratio share for blonde students is 4, this means that the fraction of blonde students is $\frac{4}{9}$

b) $\frac{4}{9}$ of the students have blonde hair, so the fraction of students with brown hair is $\frac{5}{9}$. If there are a total of 450 students in the school, the number of students with brown hair is:

$$\frac{5}{9} \times 450 = 250 \text{ students.}$$

Question 2: Total parts $= 2+5=7$
7 parts $= 35$, therefore 1 part $= 35 \div 7 = 5$
2 parts $= 2 \times 5 = 10$
5 parts $= 5 \times 5 = 25$
Hence the ratio is $10:25$

Question 3: The ratio is 2 parts blue to 13 parts white $(2:13)$. Lucy buys 16 blue tiles, which is 2 parts. so, 1 part $= 16 \div 2 = 8$
No. of white tiles $= 13 \times 8 = 104$ tiles
Cost of blue tiles $= £2.80 \times 16 = £44.80$
Cost of white tiles $= 104 \times £2.35 = £244.40$
Hence, the total cost is

$$£44.80 + £244.40 = £289.20$$

Question 4: Deducting the 20% spent on the magazine subscription gives 80% of the original amount:
$0.8 \times £200 = £160$.
Steve therefore has £160 pounds remaining which he spends on football stickers, sweets and fizzy drinks in the ratio of $5:2:1$.
Total parts $= 5+2+1=8$
Thus, the amount spent on football stickers is:

$$\frac{5}{8} \times £160 = £100$$

Question 5: a) The ratio of books read by Jon to books read by Kate is $2:1$. The ratio of books read by Alieke to books read by Jon is $4:1$. Scaling up the second ratio so Jon has 2 parts gives the following 3 way ratio:
Alieke : Jon : Kate $= 8:2:1$

b) The difference between the ratio share is 7 parts $(8-1=7)$.
The difference in the ratio share is 7 parts, and the difference in the number of books read is 63. Thus, 1 part $= 63 \div 7 = 9$ books
Total parts $= 8+2+1=11$
Hence the total number of books is

$$11 \times 9 \text{ books} = 99 \text{ books}$$

2 Direct and Inverse Proportion

Question 1: F is inversely proportional to the square of r (i.e. r^2), so $F \propto \frac{1}{r^2}$
Rewriting this equation using k to represent the constant of proportionality:

$$F = \frac{k}{r^2}$$

When $F = 50, r = 3$. Substituting these values into the equation and solving for k:

$$50 = \frac{k}{3^2} = \frac{k}{9}$$
$$k = 50 \times 9 = 450$$

Rewriting the original equation with a value for the constant k:

$$F = \frac{450}{r^2} = \frac{450}{12^2} = 3.125$$

Question 2: When $x = 4, y = 7.5$. Substituting these values into the equation and solving for k.

$$y = \frac{k}{x}$$
$$7.5 = \frac{k}{4}$$
$$k = 7.5 \times 4 = 30$$

So,

$$y = \frac{30}{x}$$

When $x = 60, y = \frac{30}{60} = 0.5$
When $y = 12,\ x = \frac{30}{12} = 2.5$
The completed table is therefore:

x	4	60	2.5
y	7.5	0.5	12

Question 3: a) M is directly proportional to h, which can be written as $M \propto h$, or $M = kh$.
When $M = £155.80, h = 9.5$ hours.
Substituting these values into the equation and solving for k:

$$155.80 = k \times 9.5$$
$$k = \frac{155.80}{9.5} = 16.4$$

Thus, $M = 16.4h$

b) When $M = £688.80$, $h = 688.80 \div 16.4 = 42$ hours.

Question 4: a) x is directly proportional to y, so $x \propto y$ or $x = ky$. Substituting in the known values for x and y and solving for k:

$$x = ky$$
$$2 = k \times 8$$
$$k = \frac{2}{8} = \frac{1}{4}$$

So,

$$x = \frac{1}{4}y$$

b) When $y = 32$, $x = 32 \div 4 = 8$

c) When x is 50, $y = 50 \times 4 = 200$

Question 5: a) The time taken (t) is inversely proportional to the square of the number of staff on duty (s), i.e.,

$t \propto \frac{1}{s^2}$ or $t = \frac{k}{s^2}$.
Substituting in the known values for t and s and soliving for k:

$$t = \frac{k}{s^2}$$
$$20 = \frac{k}{4^2}$$
$$20 = \frac{k}{16}$$
$$20 \times 16 = k$$
$$k = 320$$

So,

$$t = \frac{320}{s^2}$$

b) $k = 320$ and $s = 8$. Substituting these values into the formula gives,

$$t = \frac{k}{s^2}$$
$$t = \frac{320}{8^2}$$
$$t = \frac{320}{64} = 5 \text{ minutes}$$

Therefore, doubling the number of staff means the orders take 5 minutes to be received instead of 20 minutes, so the orders are received 4 times faster.

---

3 Percentages

Question 1: 10% of $180 = 180 \div 10 = 18$
Therefore, 30% of $180 = 3 \times 18 = 54$
1% of $180 = 180 \div 100 = 1.8$
Therefore, 3% of $180 = 3 \times 1.8 = 5.4$
33% of $180 = 30\% + 3\% = 54 + 5.4 = 59.4$

Question 2: $(99 \div 150) \times 100 = 66\%$

Question 3:

$$\begin{aligned}\text{Percentage change} &= \left(\frac{\text{change}}{\text{original}}\right) \times 100 \\ &= \left(\frac{£25{,}338 - £24{,}600}{£24{,}600}\right) \times 100 \\ &= \left(\frac{£738}{£24{,}600}\right) \times 100 \\ &= 3\%\end{aligned}$$

Question 4: A 10% price reduction means the new value is 90% of the original value. A further 10% price reduction is therefore 90% of the new value, i.e. 90% of 90:
$90 \times 0.9 = 81$
Thus, the new value is 81% of the original price, which is a reduction of $100 - 81 = 19\%$

---

4 Reverse Percentages

Question 1: The original price of the t-shirt is:
$£13.50 \div 0.75 = £18$

Question 2: The original price of the car is:
$£11{,}550 \div 1.05 = £11{,}000$

Question 3: The total mass of the bar is:
$9.90\text{g} \div 0.18 = 55\text{g}$

Question 4: The total capacity of the stadium is:
$19805 \div 0.85 = 23{,}300$

Question 5: The total population of the U.K. is:
$9{,}300{,}000 \div 0.14 = 66{,}428{,}571$

Question 6: The original price of the car is:
$£9{,}680 \div 0.44 = £22{,}000$

---

5 Compound Growth and Decay

Question 1: Using the compound growth formula:

$$\begin{aligned}\text{Amount after 4 years} &= \$1{,}400{,}000 \times \left(1 + \frac{2.4}{100}\right)^4 \\ &= \$1{,}400{,}000 \times 1.024^4 \\ &= \$1{,}539{,}316.28\end{aligned}$$

Question 2: Using the compound decay formula:

$$\begin{aligned}\text{No. of tigers in 5 years} &= 234 \times \left(1 - \frac{18}{100}\right)^5 \\ &= 234 \times 0.82^5 \\ &= 87 \text{ tigers (nearest whole number)}\end{aligned}$$

This is less than 100, therefore Riley is correct.

Question 3: Using the compound growth formula and solving for x:

$$\begin{aligned}£292{,}662.70 &= £268{,}000 \times \left(1 + \frac{x}{100}\right)^2 \\ \frac{£292{,}662.70}{£268{,}000} &= \left(1 + \frac{x}{100}\right)^2 \\ 1.0920 &= \left(1 + \frac{x}{100}\right)^2 \\ \sqrt{1.0920} &= 1 + \frac{x}{100} \\ 1.0450 &= 1 + \frac{x}{100} \\ 0.0450 &= \frac{x}{100} \\ x &= 4.5\%\end{aligned}$$

Question 4: Using the compound decay formula:

$$\begin{aligned}\text{Value after 3 years} &= £850{,}000 \times \left(1 - \frac{6}{100}\right)^3 \\ &= £850{,}000 \times 0.94^3 \\ &= £705{,}996.40\end{aligned}$$

Using this as the new value for N_0 in the compound decay formula:

$$\begin{aligned}\text{Value after a further 2 years} &= £705{,}996.40 \times \left(1 - \frac{4}{100}\right)^2 \\ &= £705{,}996.40 \times 0.96^2 \\ &= £650{,}646.28\end{aligned}$$

Which as a percentage of the original value is:

$$\left(\frac{£850,000 - £650,646.28}{£850,000}\right) \times 100 = 23\%$$

Question 5: Using the compound decay formula:

$$£15,187.50 = £36,000 \times \left(1 - \frac{x}{100}\right)^3$$

Solving for x gives,

$$\frac{15,187.50}{36,000} = \left(1 - \frac{x}{100}\right)^3$$

$$\sqrt[3]{\frac{15,187.50}{36,000}} = 1 - \frac{x}{100}$$

$$\frac{x}{100} = 1 - \sqrt[3]{\frac{15,187.50}{36,000}}$$

$$\frac{x}{100} = 0.25$$

$$x = 25\%$$

Hence using the compound decay formula with $n = 5$,

$$\text{Value after 5 years} = £36,000 \times \left(1 - \frac{25}{100}\right)^5$$

$$= £8543 \text{ (nearest pound)}$$

6 Conversions

Question 1: Deducting the 3% fee gives 97% remaining. So, money left $= \frac{97}{100} \times 500 = £485$
Hence the amount in dollars is, $485 \times 1.56 = \$756.60$

Question 2: 2.3 km = 2,300 m. So, the distance in feet is, $2,300 \div 0.3048 = 7,546$ feet (to the nearest foot).

Question 3: The volume of his fish tank in cm^3 is: $120 \text{ cm} \times 180 \text{ cm} \times 100 \text{ cm} = 2160000 \text{ cm}^3$, where $1 \text{ m}^3 = 1000000 \text{ cm}^3$. So the volume of his fish tank in m^3 is: $2160000 \div 1000000 = 2.16 \text{ m}^3$

Question 4: 13.1 miles $\times 1.61$ kilometres $= 21.091$ kilometres
The total time to complete the half marathon at a pace of 5.5 minutes per kilometre is therefore:

$$21.091 \text{ kilometres} \times 5.5 \text{ minutes per kilometre} = 116 \text{ mins}$$

$$\equiv 1 \text{ hour } 56 \text{ minutes}$$

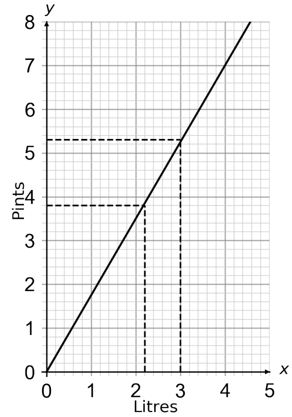
(Remember that 30 seconds is $\frac{1}{2}$ a minute = 0.5 minutes)

Question 5: Value in Jan $= £5,000 \times €1.16 = €5,800$
Converting back into pounds in Feb gives, $€5,800 \div €1.15 = £5,043$
Hence the profit is $£5,043 - £5,000 = £43$ to the nearest pound.

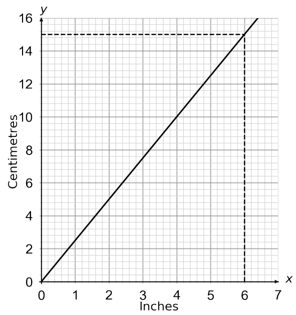
7 Conversion Graphs

Question 1: a) Locate the value 3 on the horizontal (x) axis and draw a line vertically upwards until it touches the black solid line of the graph. Then draw a line from this point horizontally to the left to find the corresponding value on the vertical (y) axis. The value falls between 5.2 and 5.4 pints, so the approximate answer is 3 litres = 5.3 pints (accept ±0.1 pints, see graph below)

b) Locate 3.8 on the vertical (y) axis and draw a line horizontally to the right until it touches the black solid line of the graph. Then draw a line from this point vertically down to find the corresponding value on the horizontal (x) axis. This value falls between 2 and 2.2 litres. Since it is closer to 2.2 than 2, the approximate answer is 3.8 pints = 2.15 litres (accept ±0.1 litres, see graph below)



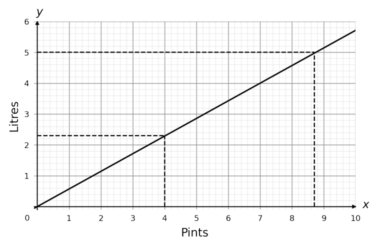
Question 2: a) 1 inch ≈ 2.5 cm. Plot the points $(1, 2.5), (2, 5), (3, 7.5)$ on the graph and draw a line through each point.



b) To convert 15cm to inches, locate 15cm on the vertical (y) axis and draw a horizontal line to the right until it touches the line of the graph. Then draw a line vertically down to find the corresponding value on the horizontal (x) axis. The value on the x-axis is 6, so 15 cm ≈ 6 inches

Question 3: a) Locate 4 on the horizontal (x) axis and draw a line up until it touches the line of the graph. Then go across to the corresponding value on the vertical (y) axis. This line touches between 2.2 litres and 2.4 litres, so 4 pints ≈ 2.3 litres

b) Locate 5 on the vertical (y) axis and draw a line across to the right until it touches the line of the graph. Then go down to the corresponding value on the horizontal x axis. This line touches between 8.6 pints and 8.8 pints, so 5 litres ≈ 8.7 pints



8 Best Buys

Question 1: Brand B contains 3 times as much as brand A ($200 \text{ ml} \times 3 = 600 \text{ ml}$)
600 ml of brand A costs $= £0.80 \times 3 = £2.40$. This is more than £2.20, so brand B is better value for money.

Question 2: 30% of $120 = 0.3 \times 120 = 36$ extra pencils. This means that for £4.20 you receive $120 + 36 = 156$ pencils. So, the price per pencil for Brand A is $£4.20 \div 156 = £0.0269$.
Brand B sells 200 pencils for £6.20, so the price per pencil is: $£6.20 \div 200 = £0.031$.
Therefore brand A is better value for money since the price per pencil is less.

Question 3:

$$\text{Supermarket A} : 2.40 \div 215 = £0.0111... \text{ per gram}$$

$$\text{Supermarket B} : 4.10 \div 403 = £0.0101... \text{ per gram}$$

$$\text{Supermarket C} : 3.40 \div 297 = £0.0114... \text{ per gram}$$

Therefore supermarket B offers the best value for money.

Question 4: a) 250g = 0.25 kg of Gorgonzola. So the cost of Gorgonzola is $0.25 \times £11.60 = £2.90$.

b) The price of Edam is: $£4.48 \div 400 = £0.0112$ per gram.
The price of Gorgonzola is: $£11.60 \div 1000 = £0.0116$ per gram.
Therefore the Edam cheese is better value as it costs less per gram.
Converting the prices from pounds per gram to prices in pence per gram:
Edam: $£0.0112 \times 100 = 1.12$ pence per gram
Gorgonzola: $£0.0116 \times 100 = 1.16$ pence per gram
The difference between one gram of Edam and one gram of Gorgonzola is therefore: $1.16 - 1.12 = 0.04$ pence per gram

Question 5:

$$\text{Swift Cabs total cost} = £3.00 + £3.80 + (£1.60 \times 10) = £22.80$$

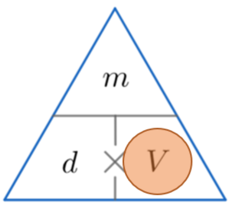
$$\text{Zoom Taxis total cost} = £3.80 + (11 \times £1.60) = £21.40$$

$$\text{Relaxi Cabs total cost} = 11 \times £2.10 = £23.10$$

The best value company is Zoom Taxis, the next best value is Swift Cabs, and the worst value is Relaxi Cabs.

9 Density Mass Volume

Question 1: 2 kg = 2000 g

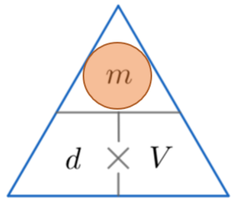


Therefore, the volume of the olive oil can be calculated as follows:

$$\text{Volume} = 2000 \text{ g} \div 0.925 \text{ g/cm}^3 = 2162 \text{ cm}^3$$

Question 2: The cube has a side length of 7 m, so the volume of the cube is: $7 \times 7 \times 7 = 343 \text{ m}^3$. Thus,

$$\text{Mass} = 343 \times 10,800 = 3,704,400 \text{ kg}$$



Question 3: Mass = 2460 kg and Volume = 1.2 m^3. Substituting these values into the formula: Density $= 2460 \text{ kg} \div 1.2 \text{ m}^3 = 2050 \text{ kg/m}^3$

Question 4: a) Total Volume = Volume of A + Volume of B
Rearranging the density formula to make volume the subject gives, volume = mass ÷ density. So,

$$\text{Volume of A} = 1200 \text{ g} \div 5 \text{ g/cm}^3 = 240 \text{ cm}^3$$

$$\text{Volume of B} = 600 \text{ g} \div 3 \text{ g/cm}^3 = 200 \text{ cm}^3$$

So, total volume $= 240 \text{ cm}^3 + 200 \text{ cm}^3 = 440 \text{ cm}^3$

b) Density = mass ÷ volume
where total mass $= 1200 \text{ g} + 600 \text{ g} = 1800 \text{ g}$
Therefore, density $= 1800 \text{ g} \div 440 \text{ cm}^3 = 4.09 \text{ g/cm}^3$

Question 5: If the ratio of metal A to metal B is $3:7$, that means that $\frac{3}{10}$ of the mass of metal C comes from metal A and the remaining $\frac{7}{10}$ is metal B. So,

The mass of metal A: $2500 \text{ g} \times \frac{3}{10} = 750g$
The mass of metal B: $2500 \text{ g} \times \frac{7}{10} = 1750g$

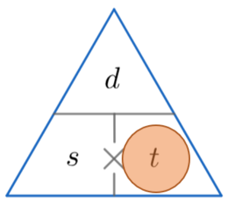
Since density = mass ÷ volume, then volume = mass ÷ density

The volume of metal A: $750 \text{ g} \div 3.2 \text{ g/cm}^3 = 234.375 \text{ cm}^3$
The volume of metal A: $1750 \text{ g} \div 5.5 \text{ g/cm}^3 = 318.18 \text{ cm}^3$

If metal A has a volume of 234.375 cm^3 and metal B has a volume of 318.18 cm^3, then their combined volume is the volume of metal C.
Volume of metal $C = 234.375 + 318.18 = 552.5568 \text{ cm}^3$.
Hence, the density of metal $C = 2500 \text{ g} \div 552.5568 \text{ cm}^3 = 4.5 \text{ g/cm}^3$

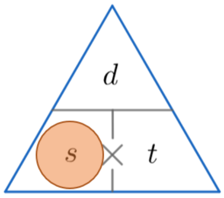
10 Speed Distance Time

Question 1: Time $= \frac{d}{s} = \frac{100}{8.5} = 11.76$ s (2 dp)



Question 2: 30 minutes = 0.5 hours, so Gustavo's speed can be calculated as follows: speed $= \frac{d}{t} = \frac{36}{0.5} = 72$ mph. Gustavo is exceeding the

speed limit.



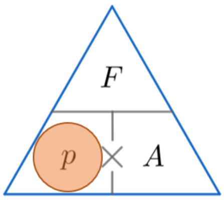
Question 3: Dividing the journey into two parts, A and B
Distance in Part A = 3 hours × 55 mph = 165 miles.
90 minutes = 1.5 hours. Thus,
Distance in Part B = 1.5 hours × 48 mph = 72 miles.
Therefore the total distance travelled is 165 + 72 = 237 miles

Question 4: 210 million = 210,000,000. Converting the time from minutes and seconds to seconds, 11 minutes = 11 × 60 seconds = 660 seconds. 11 minutes and 40 seconds = 660 seconds + 40 seconds = 700 seconds.
Hence the speed of light can be calculated as follows:
Speed of light = 210,000,000 km ÷ 700 seconds = 300,000 km/s

Question 5: Converting 35 years to seconds: 35 years = $35 \times 365 \times 24 \times 60 \times 60 = 1.104 \times 10^9$ seconds. Hence, distance = $17 \times (1.104 \times 10^9) = 1.88 \times 10^{10}$ km

11 Pressure Force Area

Question 1: Since the square has a side length of 3 m, the area is:
$3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2$

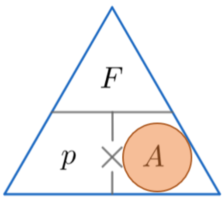


Substituting the values for the area and the force into the pressure equation as follows:

$$\text{pressure} = \frac{F}{A} = \frac{185.6}{9} = 20.6222... = 20.6 \text{ N/m}^2 \text{ (3 sf)}$$

Question 2:

$$A = \frac{F}{p} = \frac{740}{2312.5} = 0.32 \text{ m}^2$$



Question 3:

$$\text{Force} = 16 \text{ m}^2 \times 2480 \text{ N/m}^2 = 39,680 \text{ N}$$

Question 4: The area of the circular face of the cylinder in contact with the ground is: $4872 \text{ N} \div 812 \text{ N/m}^2 = 6 \text{ m}^2$

The formula for the area of a circle is $A = \pi r^2$. Rearranging to make the radius, r, the subject gives

$$\sqrt{\frac{a}{\pi}} = r$$

Substituting in the values gives,

$$\sqrt{\frac{6 \text{ m}^2}{\pi}} = 1.38 \text{ m}$$

Hence, diameter = 2 × 1.38 = 2.76 m

Question 5: The base of the pyramid has an area of $8 \times 8 = 64 \text{ m}^2$
So,

$$\text{Pressure exerted by the pyramid} = 440 \text{ N} \div 64 \text{ m}^2 = 6.875 \text{ N/m}^2$$

The cube exerts the same pressure as the square-based pyramid, so the pressure exerted by the cube is also 6.875 N/m^2.
Hence, the area of the cube in contact with the ground can be calculated as follows:

$$\text{Area} = 110 \text{ N} \div 6.875 \text{ N/m}^2 = 16 \text{ m}^2$$

Thus, the side length of the cube can be calculated by taking the square root of the area:

$$\text{Side length of cube} = \sqrt{16} = 4 \text{ m}$$

1 Geometry Basics

Question 1:
$\angle CDB = 180° - 103° = 77°$ (angles on a straight line sum to 180°).

Question 2:
$x = 360° - 100° - 105° - 50° = 105°$ (angles around a point sum to 360°).

Question 3: Base angles in an isosceles triangle are equal and angles in a triangle add up to 180°.
$y = 180° - 61° - 61° = 58°$.

Question 4: Base angles in an isosceles triangle are equal and angles in a triangle add up to 180°,

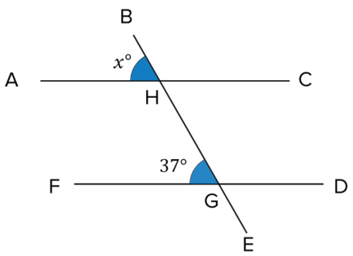
$$x + x + 55° = 180°$$
$$2x = 180° - 55° = 125°$$
$$x = 62.5°$$

.

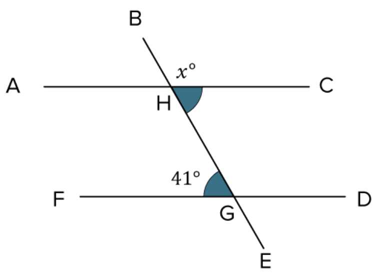
Question 5: $x = 180° - 115° = 65°$ (angles on a straight line sum to 180°).
$y = 180° - 25° - 65°$ (angles in a triangle sum to 180°), so $y = 90°$.

2 Corresponding and Alternate Angles

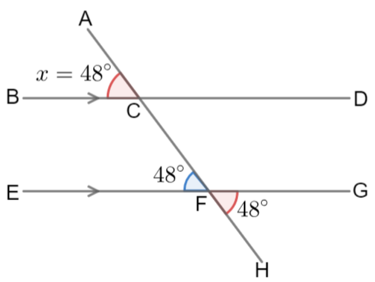
Question 1: $\angle$AHB = $\angle$FGH (corresponding angles), so $\angle x = 37°$.



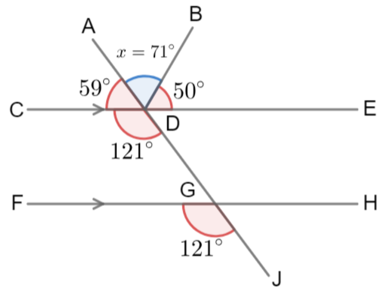
Question 2: $\angle$FGH = $\angle$GHC (alternate angles), so $\angle$GHC = 41°.
$x = 180° - 41° = 139°$ (angles on a straight line sum to 180°).



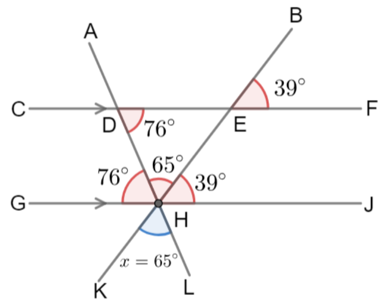
Question 3: $\angle$HFG = $\angle$EFC (vertically opposite angles), so $\angle$EFC = 48°.
$\angle$EFC = $\angle$BCA ($\angle x$) (corresponding angles), so: $\angle$BCA = $x = 48°$.



Question 4: $\angle$FGJ = CDG (corresponding angles), so $\angle$CDG = 121°.
$\angle$CDA = 180 − 121 = 59° (angles on a straight line sum to 180°).
$x = 180 - 59 - 50 = 71°$ (angles on a straight line sum to 180°).



Question 5: $\angle$BEF = $\angle$EHJ (corresponding angles), so $\angle$EHJ = 39°.
$\angle$EDH = $\angle$DHG (alternate angles), so angle DHG = 76°.
$\angle$DHE = 180 − 76 − 39 = 65° (angles on a straight line sum to 180°).
$\angle$DHE = $\angle x$ (vertically opposite angles), so $x = 65°$.



3 2D Shapes

Question 1: Irregular pentagon.

Question 2: Trapezium.

Question 3: $x° = 180° - 56° = 124°$ (adjacent angles in a parallelogram sum to 180°).

4 Interior and Exterior Angles

Question 1: Sum of interior angles = $180 \times (5-2) = 540°$. Hence each interior angle is, $x° = 540° \div 5 = 108°$.

Question 2: Sum of interior angles = $180 \times (8-2) = 1080°$. Hence each interior angle is, $x° = 1080° \div 8 = 135°$.

Question 3: Sum of interior angles = $180 \times (5-2) = 540°$.
Hence,

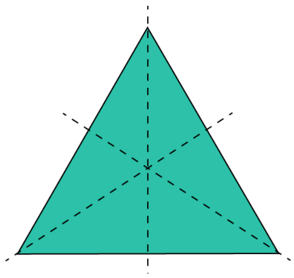
$$33 + 140 + 2x + x + (x + 75) = 540$$
$$4x + 248 = 540$$
$$4x = 292$$
$$x = 292 \div 4 = 73°$$

Question 4: Sum of interior angles = $180 \times (4-2) = 360°$.
$\angle$CDB = $180 - (y + 48) = 132 - y$ (angles on a straight line sum to 180°).
$\angle$CAB = 180 − 68 = 112 (angles on a straight line sum to 360°).
So, $112 + 90 + 2y + (132 - y) = 360°$ (sum of interior angles).

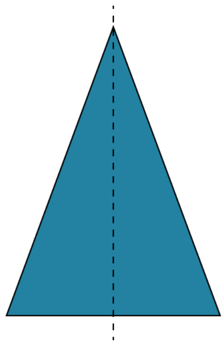
$$y + 334 = 360$$
$$y = 360 - 334 = 26°$$

5 Symmetry

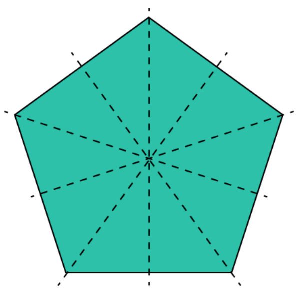
Question 1: An equilateral triangle has three lines of symmetry.



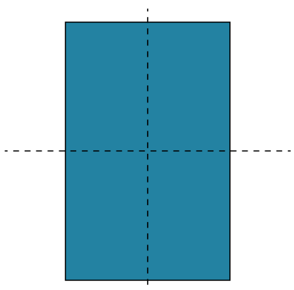
Question 2: An isosceles triangle only has one line of symmetry.



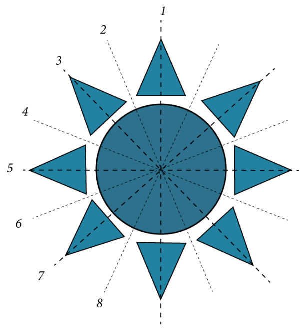
Question 3: A regular pentagon has 5 lines of symmetry.



Question 4: There are a number of shapes you could make with two lines of symmetry, the most straightforward being a rectangle.



Question 5: The shape has 8 lines of symmetry .



6 Areas of Shapes

Question 1:

$$\text{Area } = \frac{1}{2} \times b \times h = \frac{1}{2} \times 11.5 \times 12 = 69 \text{ cm}^2$$

Question 2: Form a right-angled triangle with hypotenuse = 5 cm and base = 3 cm (8 cm − 5 cm = 3 cm).
Thus, Perpendicular height = $\sqrt{5^2 - 3^2} = \sqrt{16} = 4$ cm.
Hence, Area = $\frac{1}{2}(a+b)h = \frac{1}{2}(5+8) \times 4 = 26 \text{ cm}^2$.

Question 3: Area = base × height = 8 × 15 = 120 cm^2.

Question 4: Area = $\frac{1}{2}ab\sin(C)$

$$\frac{1}{2} \times 2.15 \times x \times \sin(26) = 1.47$$
$$1.075\sin(26) \times x = 1.47$$
$$x = \frac{1.47}{1.075\sin(26)}$$
$$= 3.12\text{m (2 dp)}$$

7 Circles

Question 1: a) Circumference = $\pi d = \pi \times 8.4 = \frac{42}{5}\pi$ mm
b) Area = πr^2, $r = 8.4 \div 2 = 4.2$ mm.
So, area $= \pi \times 4.2^2 = 55.417... = 55.4 \text{ mm}^2$ (3sf).

Question 2: Area = $\pi r^2 = \pi \times 5^2 = 25\pi \text{ cm}^2$

Question 3: Area $= \pi r^2 = 200 \text{ cm}^2$, and $r = x$.

$$200 = \pi x^2$$
$$\frac{200}{\pi} = x^2$$
$$x = \sqrt{\frac{200}{\pi}}$$
$$= 7.97... = 8.0 \text{ cm (1 dp)}$$

Question 4: Circumference $= \pi d = 120\text{mm}$, $d = x$

$$120 = \pi \times x$$
$$\frac{120}{\pi} = x$$
$$x = 38.2 \text{ mm (3 sf)}$$

8 Perimeter

Question 1: Area $= x^2 = 64$, so $x = 8$.
Perimeter $= 8+8+8+8 = 32$ m.

Question 2: Length of one side $= 21 \div 6 = 3.5$ cm.

Question 3: Length of diameter $= 2r = 2 \times 5 = 10$ cm.
Length of curved edge $= \frac{1}{2}\pi d = \frac{1}{2} \times \pi \times 10 = 5\pi$ cm.
Total perimeter $= 10 + 5\pi = 25.7$ cm (1 dp).

Question 4: Missing lengths: $120 - 55 = 65$ cm , $195 - 70 = 125$ cm.
Total Perimeter $= 120 + 70 + 65 + 125 + 55 + 195 = 630$ cm.

Question 5: $AB = BC = x + 5$

$$\begin{aligned}(x+5)+(x+5)+3x &= 45\text{cm}\\ 5x+10 &= 45\\ 5x &= 35\\ x &= 35 \div 5 = 7\text{cm}\end{aligned}$$

9 The 8 Circle Theorems

Question 1: $x = 98° \div 2 = 49°$ (angle at centre $= 2\times$ angle at circumference).

Question 2: $\angle$CAB $= 90°$ (Diameter subtends a right angle).
$x = 180° - 90° - 32° = 58°$ (Angles in a triangle sum to 180°).

Question 3: $\angle$BAD $= 90°$ (Diameter subtends a right angle).
$\angle$BAE $= 90 + 31 = 121°$.
$\angle$CDE $= 180 - 121 = 59°$ (Opposite angles in a cyclic quadrilateral sum to 180°).
Thus, $\angle$EDA $= 59 - 18 = 41°$.

Question 4: AB = BD (Tangents to a circle from the same point are equal), so $\angle$BAD $= \angle$ADB. Let $\angle$BAD $= x$,

$$\begin{aligned}x + x + 42 &= 180°\\ 2x &= 180 - 42\\ &= 138°\\ x &= 69°\end{aligned}$$

Thus, $\angle$AED $= 69°$ (alternate segment theorem).

Question 5: $\angle$ABC $= 180° - 71° - 23° = 86°$ (angles in a triangle sum to 180°).
Hence, $x° = \angle$ABC $= 86°$ (Alternate segment theorem).

10 Circle Sector, Segments and Arcs

Question 1: Area $= \pi r^2$, $r = 5.24 \div 2 = 2.62$ cm.
Thus, Area $= 2.62^2 \times \pi = 21.6$ cm (3 sf).

Question 2: Area of sector $= \frac{\text{angle}}{360} \times \pi r^2$.
Thus, Area $= \frac{72°}{360°} \times \pi(5)^2 = \frac{72°}{360°} \times 25\pi = 5\pi$ m^2.

Question 3:

$$\begin{aligned}\text{Area of sector} = 26.15 &= \frac{x°}{360°} \times \pi \times 15^2\\ 26.15 &= \frac{x°}{360°} \times 225\pi\\ \frac{26.15}{225\pi} &= \frac{x°}{360°}\\ x &= \frac{26.15}{225\pi} \times 360°\\ x &= 13.3° \text{ (1 dp)}\end{aligned}$$

Question 4: Arc length $= \frac{\text{angle}}{360} \times 2\pi r = \frac{165}{360} \times 2\pi \times 14 = 40.3$mm.
Thus, total perimeter $= 14 + 14 + 40.3 = 68.3$mm.

Question 5:

$$\begin{aligned}\text{Area of sector} = 160 &= \frac{x°}{360°} \times \pi \times 9^2\\ 160 &= \frac{x°}{360°} \times 81\pi\\ \frac{160}{81\pi} &= \frac{x°}{360°}\\ x &= \frac{160}{81\pi} \times 360°\\ x &= 226° \text{ (to 3 sf)}\end{aligned}$$

11 Congruent Shapes

Question 1: B and F are congruent, E and G are congruent.

Question 2: P and Q are congruent, M and K are congruent.

Question 3: A and H are congruent, D and G are congruent.

Question 4: B and C are congruent (SSS).

Question 5: The two triangles can be shown to be congruent by the SAS rule, i.e. [AB $-\angle$CBA $-$ CB] = [XZ $-\angle$ZXY $-$ XY].

12 Similar Shapes

Question 1:
a) Scale factor, $SF = 5 \div 2 = 2.5$.
So $x = 2.5 \times 3 = 7.5$ cm.
b) Area scale factor, $SF_A = 2.5^2 = 6.25$.
Area of Q $= 6 \times 6.25 = 37.5$ cm^2.

Question 2:
a) $SF = 42 \div 14 = 3$.
b) AC $= 51 \div 3 = 17$ cm.

Question 3:
a) $SF = 6 \div 3 = 2$.
b) BE $= 4.4 \times 2 = 8.8$ cm.

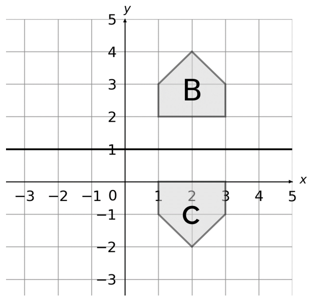
Question 4:
$SF = 3x \div x = 3$, so area scale factor, $SF_A = 3^2 = 9$.
Thus, SA of larger sphere : SA of smaller sphere $= 9:1$.

Question 5: $AC = 20 + 5 = 25$ cm. Let unknown length AF $= x$,

$$\frac{x}{5} = \frac{25}{x}$$
$$x^2 = 125$$
$$x = \sqrt{125}$$
$$SF = \frac{25}{\sqrt{125}}$$
$$= \frac{25}{5\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

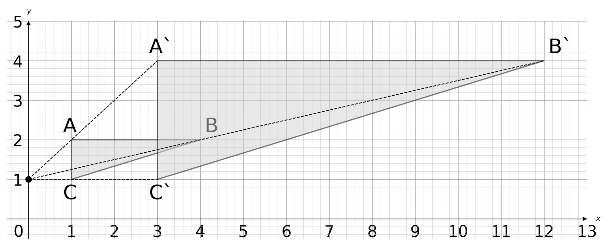
13 Transformations

Question 1:

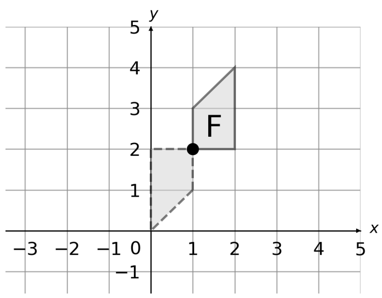


Question 2: Translation, by the vector $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

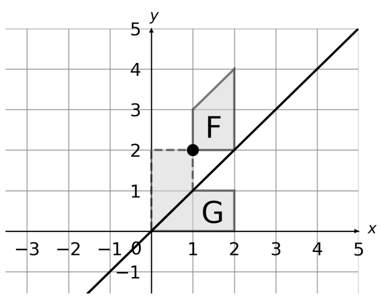
Question 3:



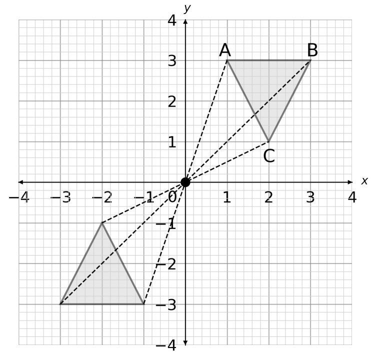
Question 4: First, perform the rotation,



Then reflect in the line $y = x$

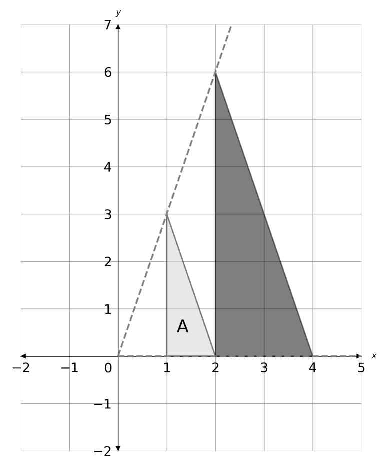


Question 5: A scale factor of -1 produces an image of the same size rotated 180° about the centre of enlargement.



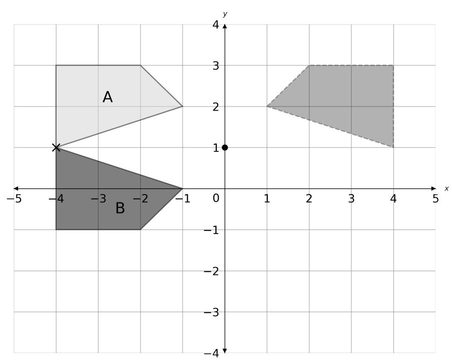
14 Invariant Points

Question 1: a) See enlarged shape below.



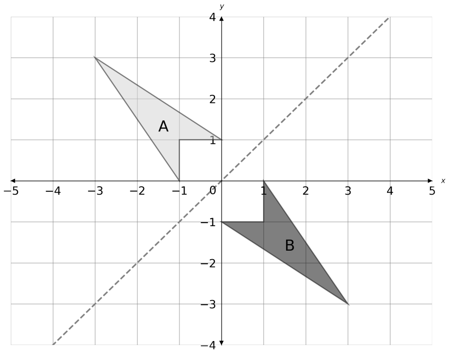
b) The number of invariant points is zero.

Question 2: A reflection in the line $x = 0$ is a reflection in the y-axis. Rotating the resulting shape 180° about (0, 1) we get the shape labelled B below.

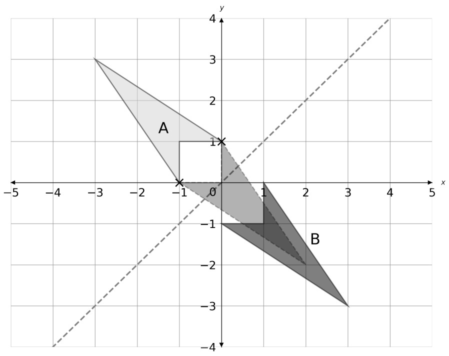


b) The coordinates of the invariant point are $(-4, 1)$.

Question 3: a) Reflection in $y = x$ gives the following shape:

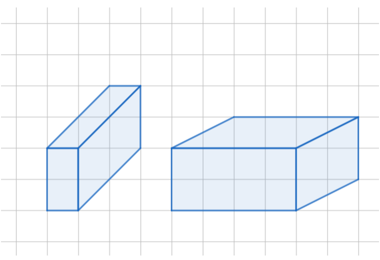


b) Translation by the vector $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ gives 2 invariant points.



15 3D Shapes

Question 1: Two examples of cuboids are shown below.



Question 2:
a) Square-based pyramid
b) Cone
c) Cylinder

Question 3: Total faces $= 2$ (ends) $+ 3$ (sides) $= 5$.

Question 4: 6 faces, 12 edges, 8 vertices.

Question 5: 1 face, 0 edges, 0 vertices.

16 Volume of 3D shapes

Question 1: Volume $= 3 \times 12 \times 16 = 576$ cm^3

Question 2: Volume $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 5^2 \times 12 = 100$ m^3

Question 3:

$$\text{Area of cross section} = \frac{1}{2} \times (45 + 60) \times 20 = 1{,}050 \text{ cm}^2$$
$$\text{Volume of prism} = 1{,}050 \times 80 = 84{,}000 \text{ cm}^3$$

Question 4:

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$
$$54 = \frac{1}{3} \times 18 \times (x + 5)$$
$$54 = 6(x + 5)$$
$$x + 5 = 9$$
$$x = 4 \text{ cm}$$

Question 5:

$$\text{Volume of cylinder} = \pi \times r^2 h = \pi \times (2.3)^2 \times 5.6 \approx 93.07\text{m}^3$$
$$\text{Volume of hemisphere} = \frac{1}{2} \times (\frac{4}{3}\pi \times (2.3)^3) \approx 25.48\text{m}^3$$
$$\text{Total volume} = 93.0665... + 25.4825... = 119 \text{ m}^3 \text{ (3 sf)}$$

17 Surface Area

Question 1: 3 pairs of faces:
front/back area $= 2 \times (4 \times 2.5) = 20$ mm^2.
top/bottom area $= 2 \times (2.5 \times 6) = 30$ mm^2.
left/right area $= 2 \times (4 \times 6) = 48$ mm^2.
total area $= 20 + 30 + 48 = 98$ mm^2.

Question 2: Let $l =$ slant height

$$\text{Surface area} = \pi r l + \pi r^2$$
$$3l\pi + 3^2\pi = 120$$
$$3l\pi + 9\pi = 120$$
$$l = \frac{120 - 9\pi}{3\pi}$$
$$= 9.7 \text{ cm (1 dp)}$$

Question 3:
Surface area of sphere $= 4\pi r^2 = 4\pi(8.5)^2 = 907.9$ m^2.
No. of tins required $= 907.9 \div 10 = 90.8$, i.e. 91 tins are required.
Total cost $= 91 \times £9.60 = £873.60$.

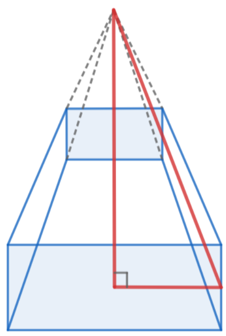
Question 4:
Area of the two triangular faces $= 2 \times (\frac{1}{2} \times 6 \times 8) = 48$ cm^2.
Area of the rectangular base $= 6 \times 11 = 66$ cm^2.
Slanted height of the prism $=$ AB $= \sqrt{8^2 + 3^2} = \sqrt{73}$ cm.
Area of the two sides $= 2 \times 11 \times \sqrt{73} = 22\sqrt{73}$.
Total surface area $= 48 + 66 + 22\sqrt{73} = 301.97$ cm^2.

Question 5:
Area of square base $= 12 \times 12 = 144$ cm^2.
Length of midpoint of DC to E $= \sqrt{10^2 + 6^2} = \sqrt{136}$ mm.
Hence, area of 4 triangle faces $= 4 \times (\frac{1}{2} \times 12 \times \sqrt{136}) = 279.89$ cm^2.
Total surface area $= 279.89 + 144 = 423.89$ cm^2.

18 Frustums

Question 1:
Height of original cone = 50 cm.
Height of missing portion = 50 − 30 = 20 cm.
Scale factor = 50 ÷ 20 = 2.5.
Thus, radius of missing portion = 10 ÷ 2.5 = 4 cm.
Volume of original cone $= \frac{1}{3}\pi \times 10^2 \times 50 = \frac{5000}{3}\pi$.
Volume of missing portion $= \frac{1}{3}\pi \times 4^2 \times 20 = \frac{320}{3}\pi$.
Volume of frustum $= \frac{5000}{3}\pi - \frac{320}{3}\pi = 4{,}900\text{cm}^3$ (3 sf).

Question 2: Area of trapezium (side face) $= \frac{1}{2}(a+b)h$ where a and b are the two parallel sides (4 m and 8 m) in this case. To find h, construct a right-angled triangle between the apex, the centre of the base, and the centre of one of the base's sides.



The base of this triangle is half the base of the whole pyramid (4 m), and the height is 7.5 m.
Thus, $c^2 = 4^2 + 7.5^2$ therefore $c = \sqrt{72.25} = 8.5$m.
The whole pyramid and the missing portion are similar shapes. Scale factor = 8 ÷ 4 = 2.
Perpendicular height of trapezium, h (side face) = 8.5 ÷ 2 = 4.25m.
Area of trapezium $= \frac{1}{2}(4+8) \times 4.25 = 25.5 \text{ m}^2$.
Total area $= (4 \times 25.5) + 4^2 + 8^2 = 182 \text{ m}^2$.

Question 3:
The whole cone and the missing portion are similar shapes. Scale factor $= 3 \div 7 = \frac{3}{7}$.
Height of the smaller cone $= 10 \times \frac{3}{7} = \frac{30}{7}$ cm.
Height of frustum $= 10 - \frac{30}{7} \approx 5.71$ cm (2 dp).

Question 4:
The whole cone and the missing portion are similar shapes. Scale factor is = 12 ÷ 3 = 4
Slanted height of the smaller cone is 15 ÷ 4 = 3.75 cm.
Slanted height of frustum, $x = 15 - 3.75 = 11.25$cm.

Question 5:
Volume of whole pyramid $= \frac{1}{3} \times$ base area × height.

Area of base $= \frac{1}{2}ab\sin(C)$, where $a = b = 11$ cm and $C = 60°$, so,

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}ab\sin(C) \\ &= \frac{1}{2}(11)^2\sin(60) \\ &= \frac{1}{2} \times 121 \times \frac{\sqrt{3}}{2} \\ &= \frac{121\sqrt{3}}{4} \end{aligned}$$

So,

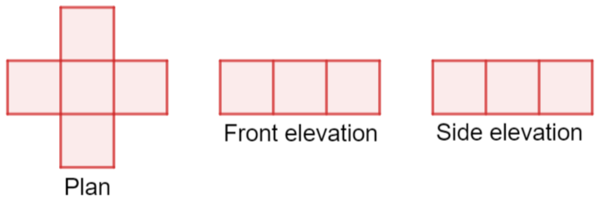
$$\text{Volume of whole pyramid} = \frac{1}{3} \times \frac{121\sqrt{3}}{4} \times 20 = 349.3 \text{ cm}^3$$

$$\text{Volume of top pyramid} = \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 7^2 \times 14 = 99\text{cm}^3$$

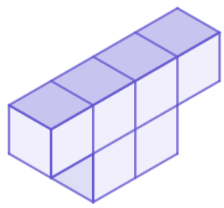
$$\text{Volume of frustum} = 349.3 - 99 = 250.3\text{cm}^3$$

19 Projections, Plans and Elevations

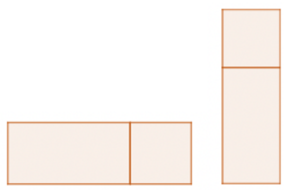
Question 1: All 3 projections are shown below.



Question 2: See 3D diagram below.

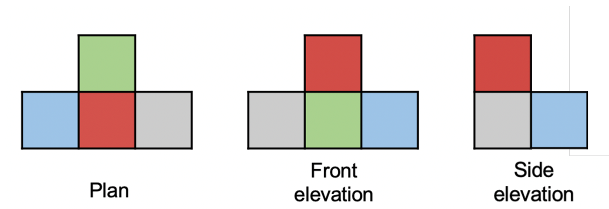


Question 3: The plan (left) and side (right) elevations are shown below.



Question 4: The plan (left) and side (right) elevations are shown below.

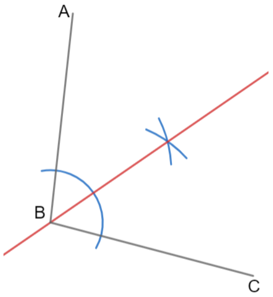
Question 5: All 3 projections are as seen below.



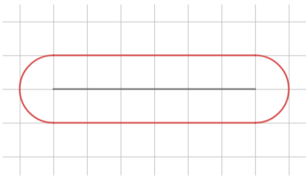
20 Loci and Construction

Question 1: The bisector of an angle is a line segment which divides the angle into two equal parts.

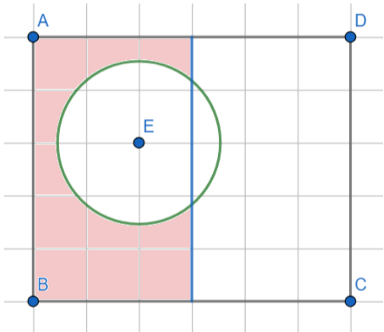
Question 2: The correct construction is a bisection of an angle, as shown below.



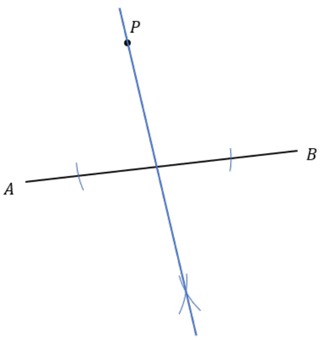
Question 3:



Question 4: Considering only the area 3m away from the house, draw a line parallel to CD and 3 cm away from it. The locus of points which are 1.5m away from the tree at E will be a circle of radius 1.5cm. Shade the area outside the two excluded regions, as shown below.

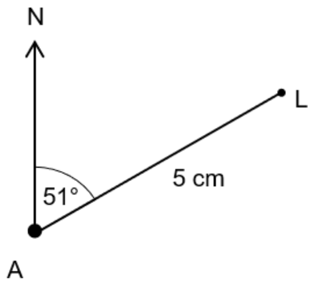


Question 5: Construction of a line perpendicular to AB passing through point P as shown:

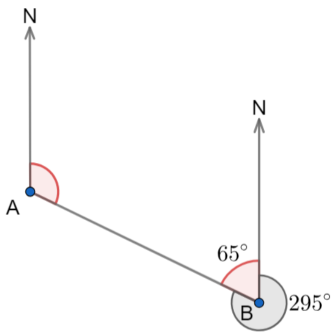


21 Bearings

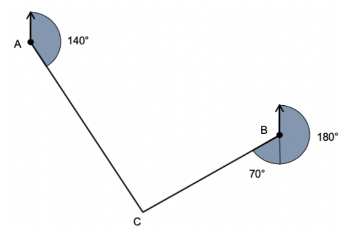
Question 1: Let the lighthouse be L and the boat be B. L from B is given by an angle of 051° and a distance of 5 cm. The final diagram should look like,



Question 2: $360° - 295° = 65°$ (angles around a point sum to 360°).
Two North lines are parallel, so $180 - 65° = 115$ (co-interior/allied angles sum to 180°).
Hence, Bearing of B from A $= 180° - 65° = 115°$.



Question 3: C is the point of intersection lines drawn along both bearings.



Question 4: By use of a protractor or otherwise, the angle is measured to be 60 degrees, so the bearing is, 060°

Question 5: Two North lines are parallel, so $180° - 60° = 120°$ (co-interior/allied angles sum to 180°).
So, $360° - 120° = 240°$ (angles around a point sum to 360°).
Hence, Bearing of A from B = 240°.

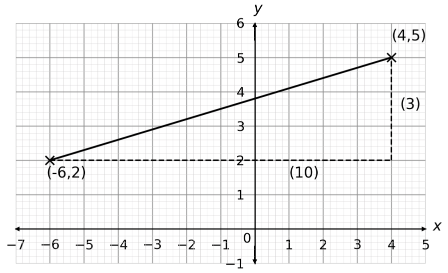
1 Pythagoras

Question 1: The missing side is the hypotenuse. Substituting the other 2 sides into the equation $a^2 + b^2 = c^2$ gives:

$$c^2 = 8^2 + 14^2$$
$$c^2 = 64 + 196 = 260$$
$$c = \sqrt{260} = 16.1245...$$

So $BC = c = 16.1$ cm (1 dp)

Question 2: Construct a right-angled triangle by plotting the given points:



The distance between the points is given by the hypotenuse of the right-angled triangle. Substituting the known sides into the equation $a^2 + b^2 = c^2$ gives:

$$c^2 = 10^2 + 3^2$$
$$c^2 = 100 + 9 = 109$$
$$c = \sqrt{109} = 10.4403... = 10.4 \text{ cm (3 sf)}$$

Question 3: The missing side is the hypotenuse. Substituting the known sides into the equation $a^2 + b^2 = c^2$ gives:

$$c^2 = 5.9^2 + 6.7^2$$
$$c^2 = 34.81 + 44.89 = 79.7$$
$$c = \sqrt{79.7} = 8.927... = 8.9 \text{ cm (1 dp)}$$

Question 4: Substituting the known sides into $c^2 = a^2 + b^2$ gives: $5.1^2 = LN^2 + 3.1^2$
Solving for LN,

$$5.1^2 = LN^2 + 3.1^2$$
$$26.01 = LN^2 + 9.61$$
$$LN^2 = 16.4$$
$$LN = \sqrt{16.4} = 4.0496... = 4.0 \text{ cm (1 dp)}$$

Question 5: Substituting the known sides into $c^2 = a^2 + b^2$ (and letting the height of the wall be a) gives:

$$2.9^2 = a^2 + 1.3^2$$
$$a^2 = 8.41 - 1.69 = 6.72$$
$$a = \sqrt{6.72} = 2.592 = 2.6 \text{ cm (1 dp)}$$

2 Trigonometry

Question 1: 'CAH': $\cos(43°) = \frac{35}{p}$.
$p = \frac{35}{\cos(43°)} = 47.85646... = 47.9$ m (3 sf).

Question 2: 'SOH': $\sin(q) = \frac{13}{15}$
$q = \sin^{-1}\left(\frac{13}{15}\right) = 60.0735.. = 60.1°$(1 dp)

Question 3: According to 'SOH CAH TOA', the sin of w must be equal to the opposite side divided by the hypotenuse. We can use Pythagoras to find the hypotenuse. If the hypotenuse is c, then a and b are both 2, so the equation $a^2 + b^2 = c^2$ becomes: $c^2 = 2^2 + 2^2 = 4 + 4 = 8$, so $c = \sqrt{8} = 2\sqrt{2}$.

'SOH': $\sin(w) = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

Question 4: 'SOH': $\sin(30°) = \frac{CB}{12}$
$CB = 12\sin(30°) = 6.0$ cm (1 dp).

Question 5: 'TOA': $\tan(x) = \frac{4}{7}$.
$x = \tan^{-1}\left(\frac{4}{7}\right) = 29.7448813...$, $x = 29.7°$(1 dp).

3 Trigonometry Common Values

Question 1: Each term is one of a small list of common trigonometry values that students are required to remember. Here,

$$\sin(60°) = \frac{\sqrt{3}}{2}, \cos(30°) = \frac{\sqrt{3}}{2}$$

Thus the calculation is, $\sin(60°) + \cos(30°) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

Question 2: $\tan(45°) = 1$, $\sin(30°) = \frac{1}{2}$ and $\tan(60°) = \sqrt{3}$.
Thus the calculation is,

$$\frac{\tan(45°)}{\sin(30°)} \times 10\tan(60°) = \frac{1}{\frac{1}{2}} \times 10(\sqrt{3}) = 2 \times 10\sqrt{3} = 20\sqrt{3}$$

Question 3: $\tan(45°) = 1$, $\cos(30°) = \frac{\sqrt{3}}{2}$ and $\cos(60°) = \frac{1}{2}$.
Thus the calculation is,

$$\frac{\tan(45°) + \cos(30°)}{\tan(45°)} \times \cos(60°) = \frac{1 + \frac{\sqrt{3}}{2}}{1} \times \frac{1}{2} = \frac{2 + \sqrt{3}}{2} \times \frac{1}{2} = \frac{2 + \sqrt{3}}{4}$$

Question 4: We are given the hypotenuse and want to find the opposite side length, hence, $\sin(30°) = \frac{x}{16}$. Using $\sin(30°) = \frac{1}{2}$,

$$x = 16 \times \sin(30°) = 16 \times \frac{1}{2} = 8 \text{ cm}$$

Question 5: We are given the adjacent length and want the find the opposite side length, hence, $\tan(30°) = \frac{x}{12}$. using $\tan(30°) = \frac{1}{\sqrt{3}}$,

$$x = 12 \times \tan(30°) = 12 \times \frac{1}{\sqrt{3}} = 12 \times \frac{\sqrt{3}}{3} = 4\sqrt{3}$$

4 Sine Rule

Question 1: Let the missing angle = A, such that
$A = 180° - 40° - 94° = 46°$ (Angles in a triangle sum to 180°)

Substituting the values into the sine rule and solving for x gives:

$$\frac{x}{\sin(46°)} = \frac{10.5}{\sin(94°)}$$
$$x = \frac{10.5}{\sin(94°)} \times \sin(46°) = 7.5715...$$
$$x = 7.57 \text{ (3 sf)}$$

Question 2: Applying the sine rule and solving for x:

$$\frac{x}{\sin(30°)} = \frac{5}{\sin(80°)}$$
$$x = \frac{5}{\sin(80°)} \times \sin(30°) = 2.5385... = 2.54 \text{ cm (3 sf)}$$

Question 3: Applying the sine rule and solving for x:

$$\frac{\sin(x°)}{12} = \frac{\sin(15°)}{7}$$
$$\sin(x) = \frac{12 \times \sin(15°)}{7} = 0.4436897916$$
$$x = \sin^{-1}(0.4436897916) = 26.33954244°$$

To find the obtuse angle, subtract the acute angle from 180°: $180° - 26.33954244° = 153.6604576 = 154°$ (3 sf).

Question 4: Applying the sine rule and solving for x:

$$\frac{\sin(x°)}{6.5} = \frac{\sin(52°)}{12}$$
$$\sin(x) = 6.5 \times \frac{\sin(52°)}{12} = 0.4268391582$$
$$x = \sin^{-1}(ANS) = 25.26713177 = 25.3° \text{ (3 sf)}$$

Question 5: Applying the sine rule and solving for x:

$$\frac{x}{\sin(35°)} = \frac{6}{\sin(68°)}$$
$$x = \frac{6}{\sin(68°)} \times \sin(35°)$$
$$= 3.711732685... = 3.71 \text{ cm (3 sf)}$$

5 Cosine Rule

Question 1: Let $a = x$, therefore $A = 19°$, since it is the angle opposite. Let $b = 86$ cm and $c = 65$ cm. Substituting these values into the cosine rule formula and solving for x gives

$$x^2 = 86^2 + 65^2 - (2 \times 86 \times 65 \times \cos(19°))$$
$$= 7,396 + 4,225 - 11,180\cos(19°)$$
$$x = \sqrt{7,396 + 4,225 - 11,180\cos(19°)} = 32 \text{ cm (2 sf)}$$

Question 2: Rearrange the cosine rule to make $\cos A$ the subject: $\cos A = \frac{b^2+c^2-a^2}{2bc}$. Substituting the values into the formula gives,

$$\cos x = \frac{5^2 + 7^2 - 6^2}{2 \times 5 \times 7} = \frac{25 + 49 - 36}{70}$$
$$x = \cos^{-1}\left(\frac{25 + 49 - 36}{70}\right) = 57.1°(3s.f.)$$

Question 3: Substituting values into the cosine rule formula, gives

$$x^2 = 6^2 + 4.5^2 - (2 \times 6 \times 4.5 \times \cos(40°))$$
$$x^2 = 36 + 20.25 - 54\cos(40°)$$
$$x = \sqrt{56.25 - 54\cos(40°)} = 3.9 \text{ cm (2 sf)}.$$

Question 4: $\cos A = \frac{b^2+c^2-a^2}{2bc}$ So,

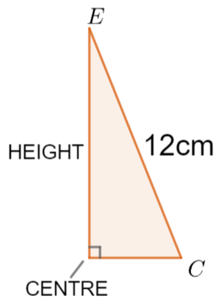
$$\cos x = \frac{7^2 + 10^2 - 12^2}{2 \times 7 \times 10} = \frac{49 + 100 - 144}{140}$$
$$x = \cos^{-1}\left(\frac{5}{140}\right) = 88.0° \text{ (1 dp)}$$

Question 5: Using the cosine rule formula,

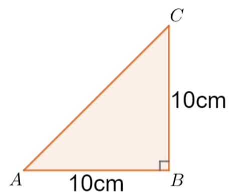
$$x^2 = 5.2^2 + 3.5^2 - (2 \times 5.2 \times 3.5 \times \cos(28°))$$
$$x^2 = 27.04 + 12.25 - 36.4\cos(28°)$$
$$x = \sqrt{39.29 - 36.4\cos(28°)} = 2.7 \text{ cm (2 sf)}$$

6 3D Pythagoras and Trigonometry

Question 1: A line drawn from the apex at E down to the centre of the base, represents the perpendicular height, since the apex is directly above the centre. Consider the triangle formed by this line, the line which goes from the centre to C, and the line EC.



The distance from the centre to C is half the distance from A to C. The length AC can be calculated from the width of the square-based triangle; consider the triangle ABC.



Therefore, the length AC,

$$AC^2 = AB^2 + BC^2 = 10^2 + 10^2$$
$$AC = \sqrt{100 + 100} = 10\sqrt{2}$$

Therefore, the distance from the centre of the base to C is $5\sqrt{2}$. Thus, the perpendicular height of the pyramid is,

$$12^2 = (\text{HEIGHT})^2 + (5\sqrt{2})^2$$
$$(\text{HEIGHT})^2 = 12^2 - (5\sqrt{2})^2 = 144 - 50 = 94$$
$$\text{HEIGHT} = \sqrt{94} \text{ cm}$$

Question 2:

$$BY^2 = 6^2 + 6^2$$
$$BY = \sqrt{36 + 36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ cm}$$

$$AY^2 = 6\sqrt{2}^2 + 9^2$$
$$AY = \sqrt{72+81}$$
$$= \sqrt{153}$$
$$= 3\sqrt{17}\text{ cm}$$

Question 3:

$$CE^2 = 9^2 + 6^2 + 12^2$$
$$CE = \sqrt{81+36+144}$$
$$= \sqrt{261}$$
$$= 3\sqrt{29}\text{ cm}$$

Question 4:

$$DB^2 = 14^2 + 14^2$$
$$DB = \sqrt{196+196}$$
$$DB = 14\sqrt{2}$$

Hence the length from D to the centre of the square, O, is half this value, $DO = 7\sqrt{2}$.

$$\tan(EDB) = \frac{O}{A} = \frac{11}{7\sqrt{2}}$$
$$\text{Angle } EDB = \tan^{-1}\left(\frac{11}{7\sqrt{2}}\right) = 48.0°$$

7 Column Vectors

Question 1:

$$2\mathbf{a} = 2 \times \begin{pmatrix}3\\8\end{pmatrix} = \begin{pmatrix}6\\16\end{pmatrix}$$
$$2\mathbf{a}+\mathbf{b} = \begin{pmatrix}6\\16\end{pmatrix} + \begin{pmatrix}-7\\2\end{pmatrix} = \begin{pmatrix}-1\\18\end{pmatrix}$$

Question 2: $3\mathbf{a} - 2\mathbf{b} = 3 \times \begin{pmatrix}2\\7\end{pmatrix} - 2 \times \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}6\\21\end{pmatrix} - \begin{pmatrix}-10\\6\end{pmatrix} = \begin{pmatrix}16\\15\end{pmatrix}$

Question 3:

$\mathbf{a} + 2\mathbf{b} - \mathbf{c} = \begin{pmatrix}6\\2\end{pmatrix} + 2 \times \begin{pmatrix}5\\-3\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix} + \begin{pmatrix}10\\-6\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}14\\-5\end{pmatrix}$

8 Vectors

Question 1: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 3\mathbf{a} + 2\mathbf{b}$,
$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$,
Hence, $\overrightarrow{AM} = \frac{1}{2}(3\mathbf{a}+2\mathbf{b}) = \frac{3}{2}\mathbf{a} + \mathbf{b}$.

Question 2: $\overrightarrow{EF} = \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CF}$,
$\overrightarrow{ED} = \frac{1}{2}\mathbf{b}$,
$\overrightarrow{DC} = \overrightarrow{AB}$, hence $\overrightarrow{DC} = 2\mathbf{a}$,
$\overrightarrow{CF} = \frac{3}{5} \times 2\mathbf{a} = \frac{6}{5}\mathbf{a}$.
Hence, $\overrightarrow{EF} = \frac{1}{2}\mathbf{b} + 2\mathbf{a} + \frac{6}{5}\mathbf{a} = 3.2\mathbf{a} + \frac{1}{2}\mathbf{b}$

Question 3: $\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = -\mathbf{a} + 2\mathbf{b}$,
$\overrightarrow{BD} = \frac{3}{5}\overrightarrow{BC} = -\frac{3}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$,

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$$
$$= \mathbf{a} + \left(-\frac{3}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}\right)$$
$$= \frac{2}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$$
$$\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AD}$$
$$= \frac{1}{3}\left(\frac{2}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}\right)$$
$$= \frac{2}{15}\mathbf{a} + \frac{2}{5}\mathbf{b}$$

Question 4: $\overrightarrow{AC} = \overrightarrow{AB} - \overrightarrow{OB} + \overrightarrow{OC}$
$\overrightarrow{AC} = \mathbf{b} - \mathbf{a} + \mathbf{b} = 2\mathbf{b} - \mathbf{a}$

Question 5: $\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$
$\overrightarrow{EA} = -\overrightarrow{AE} = -(3\mathbf{a} - 2\mathbf{b}) = -3\mathbf{a} + 2\mathbf{b}$
$\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$
$\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right)$
$\overrightarrow{DC} = 2\mathbf{a} + 4\mathbf{b}$
$\overrightarrow{AD} = 2\overrightarrow{AE} = 2(3\mathbf{a} - 2\mathbf{b}) = 6\mathbf{a} - 4\mathbf{b}$
Now,
$\overrightarrow{AB} = \frac{1}{2}(6\mathbf{a} - 4\mathbf{b} + 2\mathbf{a} + 4\mathbf{b}) = \frac{1}{2}(8\mathbf{a}) = 4\mathbf{a}$
Finally,
$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB} = -3\mathbf{a} + 2\mathbf{b} + 4\mathbf{a} = \mathbf{a} + 2\mathbf{b}$.

If $\overrightarrow{EB}$ and $\overrightarrow{DC}$ are parallel, then one must be a multiple of the other. Multiplying $\overrightarrow{EB}$ by 2 gives, $2 \times \overrightarrow{EB} = 2(\mathbf{a} + 2\mathbf{b}) = 2\mathbf{a} + 4\mathbf{b} = \overrightarrow{DC}$.
Since $\overrightarrow{DC} = 2\overrightarrow{EB}$, the two lines must be parallel.

1 Probability Basics & Listing Outcomes

Question 1: $P(A) + P(B) = 55\% + 40\% = 95\%$
$P(C) = 100\% - 95\% = 5\%$

Question 2: a) Probability of Jimmy not watching a romantic comedy:
$1 - 0.56 = 0.44$

Since the probability of Jimmy watching a sci-fi movie or a horror film is equal, the probability of Jimmy watching a sci-fi movie must be half of this amount: $0.44 \div 2 = 0.22$

b) $0.22 + 0.56 = 0.78$

Question 3: BY was already given in the question, so the full list of other possible outcomes is:
BO, BW, NO, NY, NW, PO, PY, PW

Question 4: a)

$$1 = 0.25 + 5x + 4x$$
$$0.75 = 9x$$
$$x = \frac{1}{12}$$

Blue: $5x = \frac{5}{12}$
b) Green: $4x = \frac{4}{12} = \frac{1}{3}$

Question 5: a)
There are 25 odd numbers in total between 1 and 50:
1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

There are 10 multiples of 5 between 1 and 50:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50

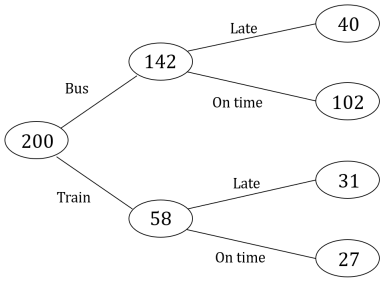
However, some of these multiples of 5 also feature on the odd number list, so cannot be counted twice. So, ignoring the odd multiples of 5, there are only 5 multiples of 5 remaining.
25 odd numbers + 5 (even) multiples of 5 = 30 numbers in total.
Hence, P(Multiple of 5 or odd) = $\frac{30}{50} = \frac{3}{5}$

b) The factors of 48 are as follows: 1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8. This means that 10 numbers out of the 50 in the hat are factors of 48.
Hence, P(factor of 48) = $\frac{10}{50} = \frac{1}{5}$

2 Frequency Trees

Question 1: 200 people in total.
58 travelled by train.
$200 - 58 = 142$ travelled by bus .
$142 - 40 = 102$ by bus and on time.

$71 - 40 = 31$ by train and late.
$58 - 31 = 27$ by train and on time.



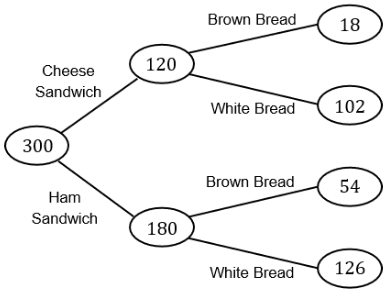
Question 2: a) 15% of the pupils chose a cheese sandwich on brown bread and that this figure represents a total of 18 pupils:

$$(18 \div 15) \times 100 = 120 \text{ pupils who select cheese}$$

120 pupils chose a cheese sandwich, 18 of them had brown bread, then $120 - 18 = 102$ cheese sandwich on white bread.
The cheese sandwich to ham sandwich ratio is $2:3$. So $\frac{2}{5}$ of these pupils had a cheese sandwich and $\frac{3}{5}$ of these pupils had a ham sandwich. $\frac{3}{5} = 180$ pupils chose ham.

120 cheese + 180 ham = 300 pupils total.
Of the people that chose a ham sandwich, $\frac{3}{10}$ opted for brown bread.
$\frac{3}{10} \times 180 = 54$ ham sandwich on brown bread.

$180 - 54 = 126$ ham sandwich on white bread.



b) There were a total of 300 pupils and 54 pupils chose a ham sandwich on brown bread: $\frac{54}{300}$ or $\frac{9}{50}$

Question 3: Votes were shared between the 3 parties in the ratio of $7:6:3$. So the Conservative Party received $\frac{7}{16}$ of the votes, the Labour party $\frac{6}{16}$ and the Green Party $\frac{3}{16}$. If 24,750 voted for the Green party and this represented $\frac{3}{16}$ of the total number of votes received, then the total number of votes is

$$(24,750 \div 3) \times 16 = 132,000 \text{ votes}$$

The total number of votes received by the Conservative party was:

$$\frac{7}{16} \times 132,000 = 57,750 \text{ votes}$$

The number of votes received by the Labour party was:

$$\frac{6}{16} \times 132,000 = 49,500 \text{ votes}$$

The votes cast by men and by women for the Labour party were in a ratio of $3:2$. This means that $\frac{3}{5}$ of the votes were cast by men and $\frac{2}{5}$ by women. The total number of votes cast by men was:

$$\frac{3}{5} \times 49,500 = 29,700 \text{ votes}$$

The total number of votes cast by women was:

$$\frac{2}{5} \times 49,500 = 19,800 \text{ votes}$$

The number of female votes received by the Green party was 65% of the number of female votes received by the Labour party.

$0.65 \times 19,800 = 12,870$ votes

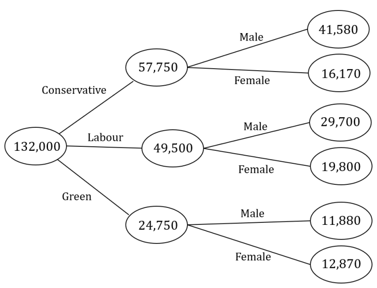
Green party male votes:

$24,750 - 12,870 = 11,880$ male votes

The number of male votes for the Conservative party was 40% more than the male votes received by the Labour party. The number of male votes received by the Labour party was 29,700, so there are $29,700 \times 1.4 = 41,580$ male Conservative votes.

The Conservative party received 57,750 votes in total:

$57,750 - 41,580 = 16,170$ female Conservative votes



---

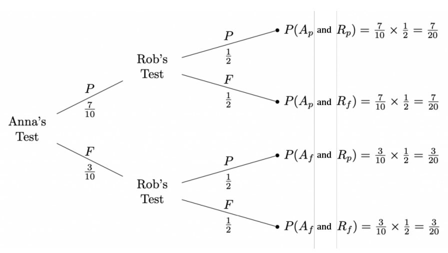
3 Probability and Tree Diagrams

Question 1: (a) Let "Anna passing" be event A_p and "Rob" passing be event R_p. The probability of both passing is: $P(A_p \text{ and } R_p) = 0.35$

$0.7 \times P(R_p) = 0.35$

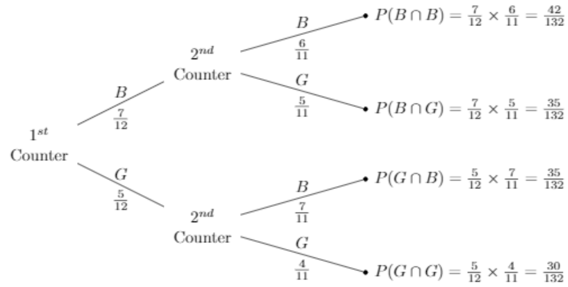
$P(R_p) = 0.35 \div 0.7 = 0.5$

(b) The probability of both Anna and Rob failing their driving test can be found using a tree diagram:

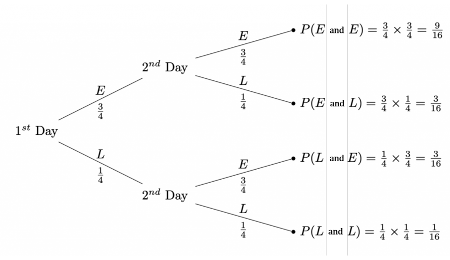


Hence the probability of them both failing is $\frac{3}{20} = 0.15$.

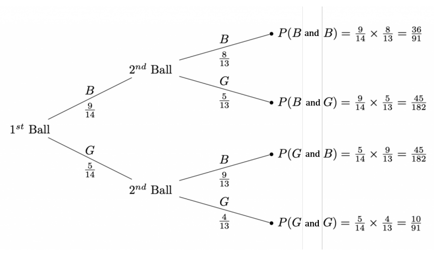
Question 2: Draw a tree diagram without replacement. Adding together the probabilities of the result being blue then blue or green then green: $\frac{7}{22} + \frac{5}{33} = \frac{31}{66}$



Question 3: Bottom line: Probability of being late of both days is $\frac{1}{16}$



Question 4: Draw a tree diagram without replacement of footballs.

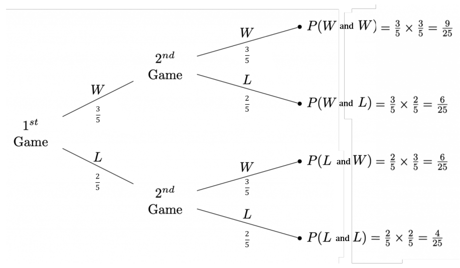


Adding together the probabilities of the result being two different colours:

$$\frac{45}{182} + \frac{45}{182} = \frac{90}{182} = \frac{45}{91}$$

Hence as this is just below a half it is more likely that the coach picks two balls that are of the same colour.

Question 5: (a) The resultant tree diagram should look something like:



(b)

$$\frac{9}{25} + \frac{6}{25} + \frac{6}{25} = \frac{21}{25} \text{ or }, 1 - \frac{4}{25} = \frac{21}{25}$$

4 Relative Frequency

Question 1: a) 12 students out of the total of 52 used public transport: $\frac{12}{52} = 0.231$ to 3 decimal places.

b) 16 students out of the total of 52 walked to school: $\frac{16}{52} = 0.308$ to 3 decimal places

c) 9 of the 52 students cycled, so the number who didn't cycle is: $52 - 9 = 43$.
$\frac{43}{52} = 0.827$ to 3 decimal places

Question 2: In Bev's experiment, the relative frequency of rolling a 6 is: $\frac{63}{400} = 0.1575$. The probability of rolling a 6 is: $\frac{1}{6} = 0.166666....$ The two results are fairly similar, i.e. the die does not appear to be biased, so Bev's statement is false.

Question 3: a) $\frac{32}{32+48} = 0.4 = 40\%$.
Hannah's statement is correct.

b) The total number of cars spotted was $32 + 41 + 48 + 111 = 232$ cars. The total number of silver cars spotted was $32 + 41 = 73$ silver cars.
$\frac{73}{232} = 0.315$ to 3 decimal places.

c) $\frac{73}{232} \times 5{,}000 = 1{,}573$ silver cars

Question 4: a) Bob throws the die 100 times and throws a 7 on 15 occasions: $15 \div 100 = 0.15$

b) Susan throws the die 500 times and throws a 7 on 60 occasions: $60 \div 500 = 0.12$

c) Susans results are more accurate because the more times you conduct an experiment, the more accurate the estimate will be. The expected probability of rolling a 7 is $\frac{1}{8} = 0.125$, which is closer to Susan's result.

Question 5: a) 16 students $= \frac{1}{10} = 0.1$.
Total students $= 16 \times 10 = 160$.

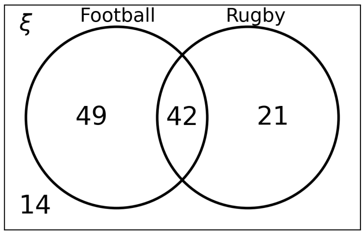
b) $\frac{4}{10} \times 160$ students $= 64$ students chose Pizza Cottage.

c) $\frac{1}{2} \times 160$ students $= 80$ students chose Dazza's Fish and Chips.

5 Venn Diagrams

Question 1: a) 14 outside, 42 in the intersection.
$7:3$ means that $\frac{7}{10}$ of the 70 students play football only and $\frac{3}{10}$ of the 70 students play rugby only.

The number of students who play football only is: $\frac{7}{10} \times 70 = 49$ students
The number of students who play rugby only is: $\frac{3}{10} \times 70 = 21$ students



b) 21 students play rugby only out of the total 126 students: $\frac{21}{126} = \frac{1}{6}$

Question 2: a) The number of students in year 10 is

$$40 + 52 + 14 + 36 = 142 \text{ students}$$

b) There are 142 students in total and, of these, 14 study both history and geography: $\frac{14}{142} = \frac{7}{71}$

c) The total that do not study geography is $52 + 36 = 88$ students.
36 out of the 88 who do not study geography do not study history: $\frac{36}{88} = \frac{18}{44} = \frac{9}{22}$

Question 3: a) All **80** students like at least **1** of the animals. So, the numbers in the Venn diagram should add up to 80.

15 students like all **3** animals. So, insert '15' in the intersection of all 3 circles.

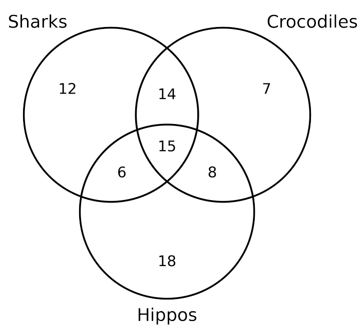
14 students like sharks and crocodiles but do not like hippos. So, insert 14 in the section where sharks and crocodiles intersect.

23 students like crocodiles and hippos. 15 students like sharks, crocodiles and hippos, so there are $23 - 15 = 8$ students who like crocodiles and hippos only. So, insert 8 in the intersection of crocodiles and hippos.

21 students like sharks and hippos. 15 students like sharks, crocodiles and hippos. So, $21 - 15 = 6$ students like sharks and hippos, but not crocodiles. Therefore insert 6 in the intersection of sharks and hippos.

44 students like crocodiles. 14 students like crocodiles and sharks, 15 students like sharks, crocodiles and hippos, and 8 like crocodiles and hippos, so $14+15+8 = 37$ students like crocodiles. Therefore, there are $44 - 37 = 7$ students who like crocodiles only. Therefore insert 7 in the part of the crocodile circle that does not overlap with any other section.

12 students only like sharks. So insert 12 in the sharks only section. Finally, $80 - 12 - 14 - 7 - 6 - 15 - 8 = 18$ students that only like hippos.



b) There are 80 students in total, of which 18 only like hippos: $\frac{18}{80}$ or $\frac{9}{40}$

c) There are 14 students that like crocodiles and sharks and 8 students that like crocodiles and hippos, so there are 22 students that like crocodiles and one other animal. This means that 22 out of the 80 only like crocodiles and one other type of animal: $\frac{22}{80} = \frac{11}{40}$

---

6 Set Notation

Question 1: a) A is the subset consisting of even numbers: $A = \{104, 110, 112, 114\}$

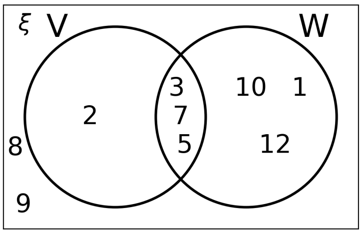
$A \cup B = \{103, 104, 110, 112, 114\}$

b) A' is the group of odd (not even) numbers from the universal set ξ: $A = \{103, 105, 109\}$

Question 2: $V = \{2, 3, 5, 7\}$

$V \cap W = \{3, 5, 7\}$. So, 3, 5 and 7 go in the intersection. The only remaining number from V, the number 2, needs to be placed inside the V circle, but outside the W circle.

1, 10 and 12 from $V \cup W$ need to be placed inside W, but outside V. Finally, 8 and 9 need to placed outside the circles, but still inside the rectangle.



Question 3: Write the variable followed by a colon before the inequality, and then put everything inside curly brackets. The results are as follows:

a) $\{x : x \geq 12\}$

b) $\{z : z < -2\}$

c) $\{a : a > 0\}$

d) $\{x : 13 < x\}$

Question 4:

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{2, 3, 4, 6, 8, 12\}$

$C = \{4, 9, 16\}$

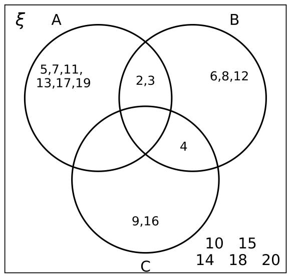
A, B and C share no numbers.

A and B share 2 and 3.

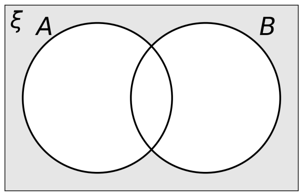
A and C share no numbers.

B and C share 4.

Then, place the numbers that have not yet been mentioned from each set in their respective circles, but not in any intersections.



Question 5: $(A' \cap B')$ means anything not in A and not in B, which is everything outside of the circles, so the Venn diagram should be shaded similar to:



---

7 Types of Data

Question 1:
a) Continuous.
b) Continuous.
c) Categorical.
d) Discrete.

Question 2:
a) Primary.
b) Continuous.

Question 3: Tahani is wrong because although a shoe size is based on foot length, the length of a person's foot can be of any value, whereas shoe sizes have limited values (5, 5 and a half, 6, 6 and a half etc.).

Question 4:
a) Primary.

b) By collecting the data himself, he can ensure that the numbers are all accurately recorded. A second advantage is that he can make efforts to make sure his sample is representative (he can ask people of different genders, races, ages, etc.).

Question 5: a) Since the data that Steve collects from the first half of the class is worded data, this is categorical data.

b) Since the data that Steve collects from the second of the class is numerical, this is quantitative data. Since the data can only take certain values (numbers between 1 and 10), the data is discrete quantative data.

c) The first disadvantage of collecting data in this way is that it is harder to analyse. The second disadvantage is that there are only 6 options for the worded responses, whereas there are eleven options for numbered responses between 0 and 10.

8 Mean, Median, Mode and Range

Question 1:
Since the number 350 occurs 3 times, it is the most common value, so: Mode $= 350$.
Range $= 590 - 280 = 310$.

Question 2: Order the set of values:
$154, 163, 164, 168, 170, 179, 185, 188$.

There are 8 values, so: $\frac{8+1}{2} = 4.5$ So the median is half-way between the 4th value and the 5th value. The 4th value is 168 and the 5th value is 170, so the median is 169.

Question 3: a) The sum of the reaction times is $0.25 + 0.34 + 0.39 + 0.38 + 0.39 + 1.67 + 0.28 + 0.3 + 0.42 + 0.46 = 4.88$.
Then, Mean $= \frac{4.88}{10} = 0.488$

b) 1.67 is the outlier as it is vastly higher than all the other values. If this outlier were removed, then the mean would be lower.

Question 4: Total length: $7 \times 1.35\text{m} = 9.45\text{m}$

When the extra plank of wood is added, the mean length of a plank of wood increases to 1.4m. This means there are now 8 planks of wood, with a combined length of: $8 \times 1.40\text{m} = 11.2\text{m}$. Therefore, the length of this extra plank of wood is: $11.2\text{m} - 9.45\text{m} = 1.75\text{m}$.

Question 5: The combined weight of all 8 members is:
$63 + 60 + 57 + 66 + 62 + 65 + 69 + 58 = 500\text{kg}$.
The combined weight of the team is: $1.02 \times 500 = 510\text{kg}$.
The mean weight following this weight gain is: $510\text{kg} \div 8 = 63.75\text{kg}$.

9 Frequency Tables

Question 1: a) The highest frequency is the 1 bathroom category, so the mode is 1.
There are $30 + 21 + 5 + 7 + 3 = 66$ values in total, so $n = 66$, and $\frac{66+1}{2} = 33.5$. This means that the median is halfway between the 33rd and the 34th value.

The first 30 values are in the 1 bathroom category, and the following 23 values are in the 2 bathroom category. Therefore values 33 and 34 are in the 2 bathroom category so the median is 2 bathrooms.

b) It is not possible to calculate the mean due to the fact that there is a category of 5 bathrooms or more. It is not clear exactly how many bathrooms people in this category have.

Question 2: a) The total of frequency column is 240. $240 - 15 - 76 - 32 - 9 = 108$ divers. Therefore, $x + y = 108$ divers.
The ratio of x to y is $7:5$. This means that x is $\frac{7}{12}$ of the total and y is $\frac{5}{12}$ of the total.

$x = \frac{7}{12} \times 108$ divers $= 63$ divers
$y = \frac{5}{12} \times 108$ divers $= 45$ divers
b) The modal number of shark encounters is 2 shark encounters.

c) There are 240 divers, so $n = 240$, and $\frac{240+1}{2} = 120.5$. This means that the median is halfway between the 120th and the 121st value.

The first 9 values are in the 0 shark encounters category, and the following 32 values are in the 1 shark encounter category, so the first 41 values fall in the 0 or the 1 shark encounter categories. The following 76 values fall into the 2 shark encounters category, so the first 117 values fall in the 0 or 1 or 2 shark encounter categories. The following 63 values fall in the 3 shark encounters category, so values 120 and 121 must be in this category. Hence, the median is simply 3 shark encounters.

d) Multiply the number of shark encounters by the frequency:

9×0 shark encounters $= 0$ shark encounters
32×1 shark encounters $= 32$ shark encounters
76×2 shark encounters $= 152$ shark encounters
63×3 shark encounters $= 189$ shark encounters
45×4 shark encounters $= 180$ shark encounters
15×5 shark encounters $= 75$ shark encounters

Total number of shark encounters $= 0 + 32 + 152 + 189 + 180 + 75 = 628$
The mean number of shark encounters is: 628 shark encounters $\div 240$ divers $= 3$ shark encounters (to the nearest whole number)

Question 3:
a)

Name	Frequency
Abigail	4
Dawn	6
Elizabeth	4
Gemma	8
Leanne	3
Sophie	5
Tanya	2

b) Total: $4 + 6 + 4 + 8 + 3 + 5 + 2 = 32$ students

If 8 out of the 32 students voted for Gemma, then $32 - 8 = 24$ did not.

So, $\frac{24}{32} = \frac{3}{4}$ of students did not vote for her.

10 Grouped Frequency Tables

Question 1: Rewriting the list of heights for each group,

The $0 < h \leq 20$ group: $7, 9, 15, 19, 19$
The $20 < h \leq 30$ group: $21, 22, 25, 25, 27, 28, 30$
The $30 < h \leq 40$ group: $31, 32, 32, 33, 35, 37, 38, 39$
The $40 < h \leq 70$ group: $46, 51, 55, 61$

Height, h (cm)	Frequency
$0 < h \leq 20$	5
$20 < h \leq 30$	7
$30 < h \leq 40$	8
$40 < h \leq 70$	4

Question 2: a) The bottom two groups in the table amount to the total number of people who took over 2 minutes, which is: $19 + 19 = 38$ people

b) 90 seconds $= 90 \div 60 = 1.5$ minutes.
The first two groups in the table represent the people who completed the puzzle in under 90 seconds, so: $8 + 22 = 30$ people. 30 people out of 100 completed the puzzle in under 90 seconds: $\frac{30}{100} = \frac{3}{10}$

Question 3: a)

Time in shop (minutes)	Frequency
0 – 5	3
6 – 10	5
11 – 15	4
16 – 20	6
21 +	2

b) The number of people who spent more than 10 minutes in the bike shop is $4 + 6 + 2 = 12$ customers. If their average spend was £12.50 each, then the total spent is: $£12.50 \times 12 = £150$.

c) There is a total of $4 + 6 = 10$ customers who spent more than 10 minutes but less than 21 in the shop.

In total there were $3+5+4+6+2 = 20$ customers in total. 10 out of the 20 customers spent more than 10, but less than 21 minutes: $\frac{10}{20} = \frac{1}{2}$.

d) The total time spent in the shop by all the customers: $16 + 23 + 4 + 9+4+18+45+20+8+6++3+14+12+17+12+19+9+16+10+15 =$ 280 minutes.
There was a total of 20 customers in the shop, so the mean amount of time spent in the shop was:

$$\frac{280 \text{ minutes}}{20 \text{ customers}} = 14 \text{ minutes}$$

11 Estimating the Mean

Question 1: Find the midpoints of the first column:

Journey time, t (mins)	Frequency	Midpoint
$0 < t \leq 10$	2	5
$10 < t \leq 20$	45	15
$20 < t \leq 30$	25	25
$30 < t \leq 40$	3	35

Then, multiply the frequency of each group by its midpoint to estimate the total journey time per group.

2×5 minutes $= 10$ minutes
45×15 minutes $= 675$ minutes
25×25 minutes $= 625$ minutes
3×35 minutes $= 105$ minutes

Estimated total journey time (all groups):
10 minutes + 675 minutes + 625 minutes + 105 minutes = 1415 minutes.

Total number of journeys: $2 + 45 + 25 + 3 = 75$ journeys.
Then the estimated mean can be calculated as follows:

$$\frac{1415 \text{ minutes}}{75 \text{ journeys}} = 18.9 \text{ minutes (1 dp)}$$

Question 2: Find the midpoints of the first column:

Distance, d (cm)	Frequency	Midpoint
$0 < d \leq 50$	4	25
$50 < d \leq 100$	18	75
$100 < d \leq 150$	56	125
$150 < d \leq 200$	32	175
$200 < d \leq 250$	8	225

Then, multiply the frequency of each group by its midpoint.

$4 \times 25\text{cm} = 100\text{cm}$
$18 \times 75\text{cm} = 1,350\text{cm}$
$56 \times 125\text{cm} = 7,000\text{cm}$
$32 \times 175\text{cm} = 5,600\text{cm}$
$8 \times 225\text{cm} = 1,800\text{cm}$

Estimated total jump length: $50\text{cm} + 1,350\text{cm} + 7,000\text{cm} + 5,600\text{cm} + 1,800\text{cm} = 15,850\text{cm}$

Total number of jumps: $4 + 18 + 56 + 32 + 8 = 118$ jumps.
Then the estimated mean is:

$$\frac{15,850\text{cm}}{118 \text{ jumps}} = 134.3 \text{ cm (1 dp)}$$

Question 3: a) Find the midpoints of the first column:

Suzanna		
Number of hits, h	Frequency	Midpoint
$0 < h \leq 10$	3	5
$10 < h \leq 20$	25	15
$20 < h \leq 30$	28	25
$30 < h \leq 40$	19	35
$40 < h \leq 50$	8	45
$50 < h \leq 60$	2	55

Then, multiply the frequency of each group by its midpoint.

3×5 hits $= 15$ hits
25×15 hits $= 375$ hits
28×25 hits $= 700$ hits
19×35 hits $= 665$ hits
8×45 hits $= 360$ hits
2×55 hits $= 110$ hits

Total number of times the bullseye was hit:
15 hits + 375 hits + 700 hits + 665 hits + 360 hits + 110 hits = 2,225 hits

Total number of participants: 3 + 25 + 28 + 19 + 8 + 2 = 85 participants. Then, the estimated mean can be calculated as follows:

$$\frac{2,225 \text{ bullseyes hit}}{85 \text{ participants}} = 26.2 \text{ hits (1 dp)}$$

b) By organising the data in batches of 20 hits, rather than batches of 10 hits, each row in Flolellas table will combine 2 of Suzannas rows, so the table will be half the size. It will look as follows:

Number of hits, h	Frequency
$0 < h \leq 20$	28
$20 < h \leq 40$	47
$40 < h \leq 60$	10

To calculate the estimated mean, work out the new midpoints, as follows:

Number of hits, h	Frequency	Midpoint
$0 < h \leq 20$	28	10
$20 < h \leq 40$	47	30
$40 < h \leq 60$	10	50

Using Floellas table, the estimated mean can be calculated as follows:

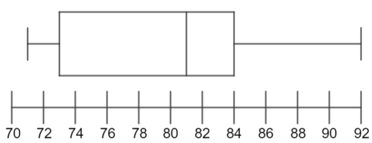
$$\frac{(28 \times 10) + (47 \times 30) + (10 \times 50)}{85} = 25.8 \text{ hits (1 dp)}$$

When the answer is rounded to the nearest whole number, the estimated mean is the same.

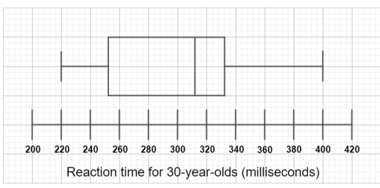
12 Box Plots

Question 1: Range $= 15.8 - 10 = 5.8$ seconds.
Interquartile range $= 12.4 - 10.5 = 1.9$ seconds.

Question 2: Smallest value = Largest value − Range $= 92 - 21 = 71$. The completed box plot should be similar to the one below:



Question 3: The smallest value is 220 and the largest value is 400, so we will have to work out the remaining values.
The median is the $\frac{7+1}{2}$ = 4th term, which is 312. The lower quartile is the $\frac{7+1}{4}$ = 2nd term, which is 252. The upper quartile is the $\frac{3(7+1)}{4}$ = 6th term, which is 332. So,



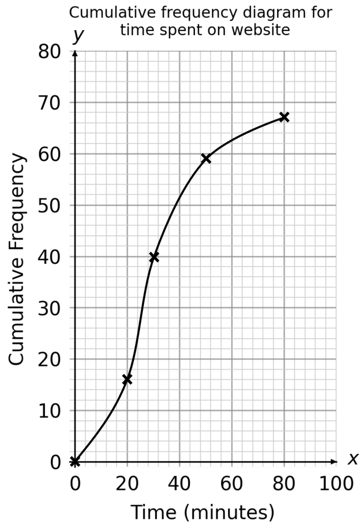
Comparing the two box plots, the second one has a higher median, meaning that the 30-year-olds were on average slower at reacting than the 20-year-olds. Additionally, the interquartile range is greater for the 30-year-olds than it is for the 20-year-olds, which means that the reaction times for 30-year-olds are more spread out than those for 20-year-olds.

13 Cumulative Frequency

Question 1: a) Add up the frequencies as we move downwards in the table and write the result in a cumulative frequency column:

Time, t (minutes)	Frequency	Cumulative Frequency
$0 < t \leq 20$	16	16
$20 < h \leq 30$	24	40
$30 < h \leq 50$	19	59
$50 < h \leq 80$	8	67

b) For the cumulative frequency diagram, plot the time in minutes along the x-axis and the cumulative frequency totals on the y-axis. Plot each of the cumulative frequency figures with the corresponding class interval maximums. Join each point with a smooth curve which passes through the origin.

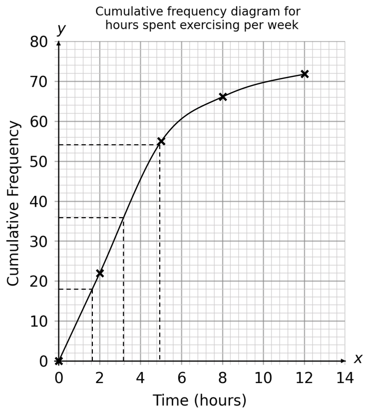


Question 2: The minimum number of hours was approximately 0 and the maximum was 12. The graph displays exercising data from 72 people since this is where the graph ends. The following values are required to draw a box plot:

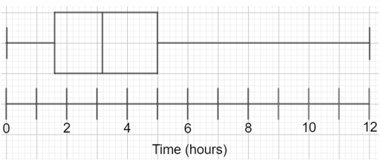
a) the minimum value b) the maximum value c) the median value d) the lower quartile e) the upper quartile

The median value is the middle value. Since there are 72 values in total, then the median value is the 36th value (since 36 is half of 72). Reading from the graph, the median is 3.2. Similarly, the lower quartile is the $72 \div 4 = 18$th term, which is 1.6 hours. The upper quartile is the 54th term which is 5 hours.

These can be seen by the lines on the graph below.



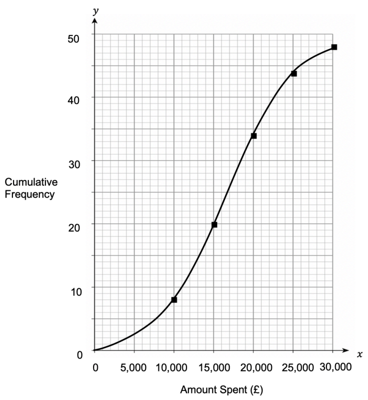
Using these values to construct the box plot gives the following:



Question 3: a) Calculate the cumulative frequency values and your cumulative frequency table should look like:

Price, p	Frequency	Cumulative Frequency
$0 < p \leq 10{,}000$	8	8
$10{,}000 < h \leq 15{,}000$	12	20
$15{,}000 < h \leq 20{,}000$	14	34
$20{,}000 < h \leq 25{,}000$	10	44
$25{,}000 < h \leq 30{,}000$	4	48

Plot the cumulative frequency total against the highest value in the band.

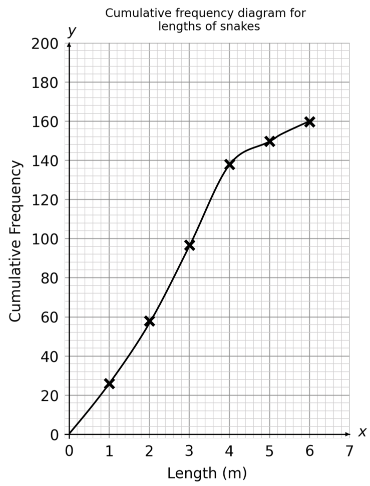


b) The median is the 24th value, which corresponds to approximately £16,000.
c) Locate £17,500 on the x-axis and see what cumulative frequency total this corresponds to, which is 26. This means that 26 cars have a value that is up to £17,500. Therefore the remaining cars must have a value which is greater than £17,500.
The number of cars which have a value of more than £17,500 is simply $48 - 26 = 22$ cars.

Question 4: a) To draw the cumulative frequency graph, work out the cumulative frequency totals:

Shake length, l	Frequency	Cumulative Frequency
$0 < l \leq 1$	23	23
$1 < l \leq 2$	35	58
$2 < l \leq 3$	39	97
$3 < l \leq 4$	41	138
$4 < l \leq 5$	12	150
$5 < l \leq 6$	10	160

Plot the cumulative frequency totals on the vertical axis against snake length on the horizontal axis and joining the points together with a smooth line:



b) The lower quartile is the 40th value, which corresponds to 1.5m.
The upper quartile is the 120th value, which corresponds to 3.5m.
Therefore the interquartile ranges is $3.5 - 1.5 = 2$ metres.

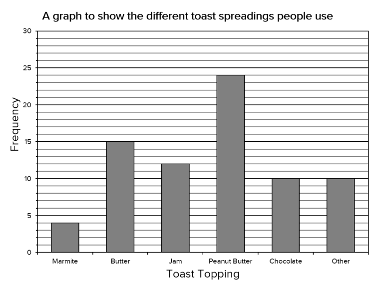
c) The median is the 80th value, which is 2.6.
At the other snake sanctuary, the median snake length is 1.78 metres.
The difference in median snake length is: 2.6 m $-$ 1.78 m = 0.82 m.

Using the percentage difference formula:

$$\frac{\text{difference}}{\text{original value}} \times 100 = \frac{0.82 \text{ m}}{2.6 \text{ m}} \times 100 = 31.5\%$$

14 Bar Graphs

Question 1: The completed bar graph should look like the one shown below. There must be gaps between the bars and everything (including the axes and the individual bars) should be clearly labelled.



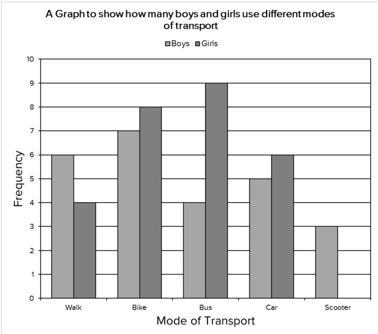
Question 2: a) According to the scale on the y-axis, 1 small line accounts for 1,000 book sales. So, the number of audiobooks sold is 3,000, the number of hardbacks sold is 5,000, and the number of paperbacks sold is 12,000.

Therefore the percentage of sales that were audiobooks is:

$$\frac{3,000}{3,000+5,000+12,000} \times 100 = 15\%$$

b) The ratio is: $5,000 : 12,000 = 5 : 12$

Question 3:



Question 4: a) The number of children who have between 0 and 5 servings per week is a total of 16. Therefore, the number of adults and children combined who have between 0 and 5 servings is $4+16 = 20$.

b) The number of children who eat over 20 servings of fruit and vegetables is 4. The number of children who eat between 6 and 10 servings of fruit and vegetables is 25. Therefore, the difference between these two categories is $25 - 4 = 21$.

c) The mode is the most frequently occurring value. The $16-20$ bar is the highest with 22. Therefore the mode is $16-20$ servings.

d) Adults eat more portions of fruit and vegetables per week than children.

Question 5: a) 12 girls.

b) The number of boys who played video games for $11-15$ hours was 8 and the number of boys who played $6-10$ hours was 2. Therefore the number of boys who played for less than 16 hours was $8+2 = 10$ boys.

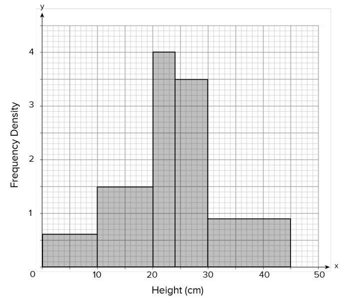
c) The number of girls who played for $6-10$ hours was 12 and the number who played for $0-5$ hours was 8. Therefore the number of girls who played video games for 10 hours or less was $12+8 = 20$ girls.
In total there are $8+12+20+14+8+10+6+2 = 80$ girls. Thus 20 out of the 80 girls play video games for 10 hours or less, which is 25%.

15 Histograms

Question 1: Calculate the frequency density (frequency ÷ class width) for each class and create a new column for these values.

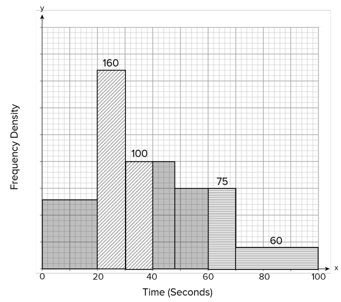
Height, h (cm)	Frequency	Frequency Density
$0 < h \le 10$	6	0.6
$10 < h \le 20$	15	1.5
$20 < h \le 24$	16	4
$24 < h \le 30$	21	3.5
$30 < h \le 50$	18	0.9

Construct the histogram with height on the the x-axis and frequency density on the y-axis.



Question 2: The number of values in each class = area of each bar. 54 people can hold their breath for at least a minute, so this means that the area of the bars from 60 seconds upwards represents 54 people. People who can hold their breath for 1 minute or more is represented by the whole of the last bar ($70-100$ seconds) and the right-hand part of the second-to-last bar ($60-70$ seconds). To work out the area in these two bars, we simply need to count the small squares: $(5 \times 15) + (15 \times 4) = 75 + 60 = 135$ If 135 small squares represents 54 people, we can work out how many people one small square represents:
1 person $= \frac{135}{54} = 2.5$ small squares.

Now, we need to work out how many small squares there are between 20 and 40 seconds: $(5 \times 37) + (5 \times 20) = 160 + 100 = 285$.
This is illustrated on the graph.



Therefore, the number of people who can hold their breath for between 20 and 40 seconds is: $\frac{285}{2.5} = 114$ people.

Question 3: a) Using Frequency = frequency density × bandwidth.
The number of riders (the frequency) who rode between 0 and 20 kilometres can be calculated as follows: $4 \times 20 = 80$ riders.
The number of riders (the frequency) who rode between 20 and 30 kilometres can be calculated as follows: $10 \times 10 = 100$ riders.
Therefore the number of riders who rode between 0 and 30 kilometres is: $80 + 100 = 180$ riders

b) We already know from the previous question that 80 riders rode between 0 and 20 kilometres and that a further 100 riders rode between 20 and 30 kilometres.
In the $30-57$ kilometres category, we have a band width of 27 kilometres and a frequency density of 2, so the number of riders can be calculated as follows: $27 \times 2 = 54$ riders.

In the $57-70$ kilometres category: $13 \times 9 = 117$ riders.
In the $70-90$ kilometres category: $20 \times 6 = 120$ riders.

Tabulate our data, with a midpoint column and a midpoint × frequency column.

The tabulated data should look like the below:

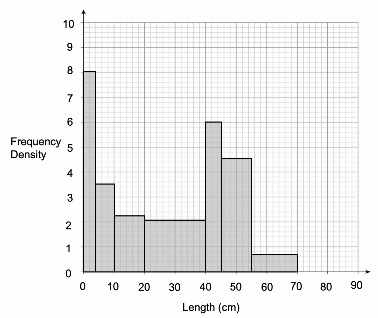
Distance (km)	Frequency	Midpoint	$M \times F$
$0 < h \leq 20$	80	10	800
$20 < h \leq 30$	100	25	2500
$30 < h \leq 57$	54	43.5	2349
$57 < h \leq 70$	117	63.5	7429.5
$70 < h \leq 90$	120	80	9600

The total of the frequency column is the total number of riders. The total of the midpoint multiplied by frequency column is the total distance travelled by all of the riders.
Therefore the estimated mean is:
Estimated mean $= 24,564$ kilometres $\div$ 492 riders $\approx$ 52 kilometres (to the nearest km).

Question 4: Calculate the frequency density for each category:

$0-4$ cm length category: Frequency density $= 32 \div 4 = 8$.
$10-20$ cm length category: Frequency density $= 22 \div 10 = 2.2$.
$20-40$ cm length category: Frequency density $= 42 \div 20 = 2.1$.
4045 cm length category: Frequency density $= 30 \div 5 = 6$.
5570 cm length category: Frequency density $= 9 \div 15 = 0.6$.

Using these values to plot the histogram:



Using Frequency = frequency density $\times$ bandwidth,
$4-10$ cm length category: frequency $= 3.5 \times 6 = 21$
$45-55$ cm length category: frequency $= 4.6 \times 10 = 46$

The completed table should look as follows:

length, l (cm)	Frequency
$0 < l \leq 4$	32
$4 < l \leq 10$	21
$10 < l \leq 20$	22
$20 < l \leq 40$	42
$40 < l \leq 45$	30
$45 < l \leq 55$	46
$55 < l \leq 70$	9

Question 5: a) The area of the $35-40$ pounds bar is: 2.5×30 small squares $= 75$ small squares.
Therefore 15 bags of flour is represented by 75 small squares. If 15 bags $= 75$ small squares, then 1 bag $= 5$ small squares.

Between 80 and 95 pounds there are 75 small squares, and between 95 and 100 pounds, there are a further 125 small squares, so the total is 200 small squares. Since 5 small squares represents a single bag of flour, then 200 squares represents $200 \div 5 = 40$ bags of flour.

b) The answer to part a) can only be an estimate because the data is grouped.

c) There are 185 bags of flour in total. The median bag of flour is given by $\frac{n+1}{2} = \frac{186}{2} = 93$, i.e. the 93$^{\text{rd}}$ bag is the median weight. Tabulating the data from each weight class is a good way to find which weight class the 93rd bag belongs to:

Weight class (pounds)	No. of squares	No. of bags	Cumulative frequency
$30-40$	150	30	30
$40-55$	225	45	75
$55-65$	175	35	110
$65-70$	125	25	135
$70-95$	125	25	160
$95-100$	125	25	185

Thus, the 93$^{\text{rd}}$ bag is in the $55-65$ pound weight class.

There are $110-75 = 35$ bags in the $55-65$ pound weight class, so the 93$^{\text{rd}}$ bag will be the 18$^{\text{th}}$ bag in this category. This can be written as $\frac{18}{35}$, i.e. it will fall $\frac{18}{35}$ of the way between $55-65$ pounds. Since this is a weight category of 10 pounds, $\frac{18}{35} \times 10 = 5.14$ pounds. Since the category starts at 55 pounds, then the weight of the median bag (the 93$^{\text{rd}}$) bag is $55+5.14 = 60.14$ pounds

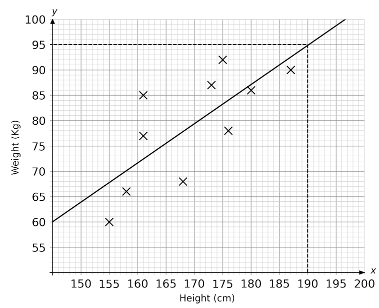
16 Scatter Graphs

Question 1: a) As the x variable increases, the y variable also increases. This indicates that there is a positive correlation. Since all the points are close together in a straight line, this graph has strong positive correlation.

b) There is no clear pattern here, so this graph has no correlation.

c) As the x variable increases, the y variable decreases, so there is a negative correlation. Since all the points are reasonably close to the line of best fit, this graph has moderate negative correlation.

Question 2: a) The results of plotting the ten points on a graph should look like:

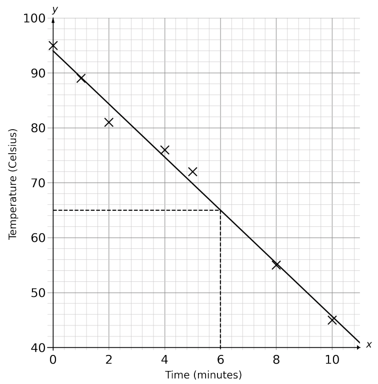


b) The line of best fit will cut through what you believe to be the middle of all the points,

To predict the weight of someone with a height of 190cm, locate 190 on the horizontal x-axis and draw a vertical line up to your line of best fit. Then draw across from this point to the corresponding value on the y-axis. The prediction, according to this line of best fit, is 95kg.

(Your line of best fit may be slightly different, in which case any answers between 93kg and 97kg are acceptable.)

Question 3: a)



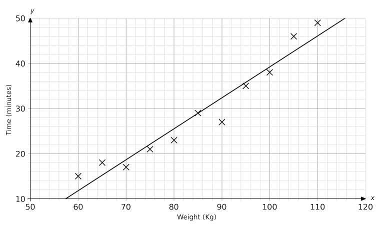
b) Draw a line of best fit that cuts through the middle of as many of the dots as possible. As the x variable increases, the y variable decreases, so there is a negative correlation. Since all the points are very close to the line of best fit, this graph has strong negative correlation.

c) Since the y variable decreases as the x variable increases, the temperature of the cup of tea is reducing over time.

d) The estimated temperature is 66°.

e) It would be inappropriate to find an estimate for the temperature after 45 minutes as 45 minutes is beyond the range of the data and tea would not get colder than room temperature.

Question 4: a)

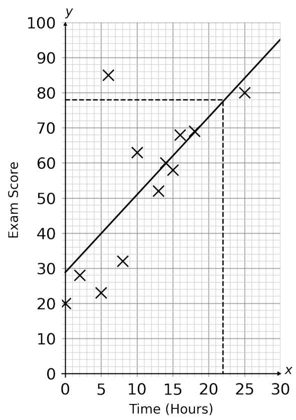


b) As the x variable increases, the y variable increases, so there is a positive correlation. Since all the points are very close to the line of best fit, this graph has strong positive correlation.

c) Since the y variable increases as the x variable increases, this tells us that the time taken to run 5 kilometres is greater for a heavier runner.

d) It would be inappropriate to find an estimate for the time taken for a runner of 40 kilograms since 40 kilograms is beyond the range of the data.

Question 5: a)



b) Draw in a line of best fit. Since the line of best fit goes up and, generally, the points are close to this line, there is a positive correlation.

c) Since the y values (the exam scores) increase as the x values (time spent revising) increase, more time spent revising is likely to give a better exam score.

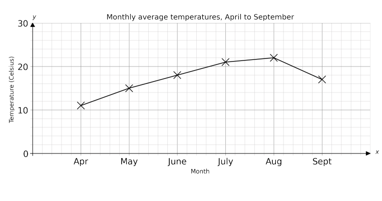
d) An outlier is any point which is a long way from the line of best fit. This is the point that corresponds to the student who scored 85 with only 6 hours of revision.

e) 22 hours of revision corresponds to an exam score of approximately 78 marks (accept 77-79 marks).

f) It would be inappropriate to find an estimate for an exam score for a student doing 85 hours of revision as this is beyond the range of the data.

17 Line graphs

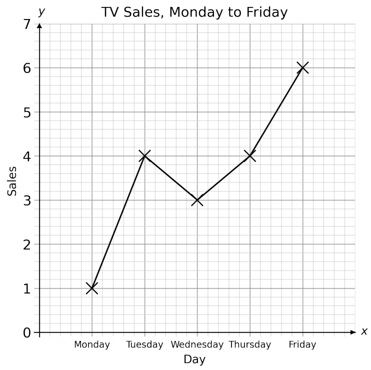
Question 1: The line graph should have the months on the x-axis and the temperature on the y-axis. It should also have the axes clearly labelled and an appropriate title at the top.



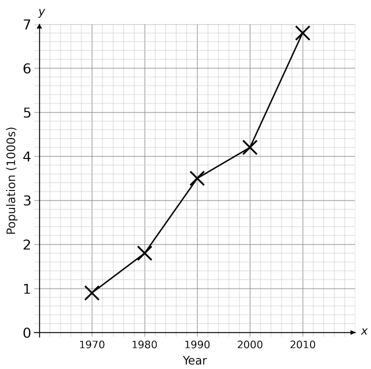
Question 2: The first mistake Roger made is that he did not label one of his axes.

The second mistake he made is that he plotted the 2014 point at 600 when it should be at 700.

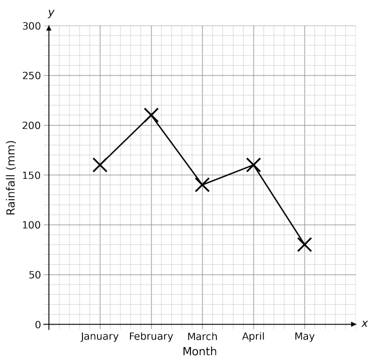
Question 3:



Question 4:



Question 5:



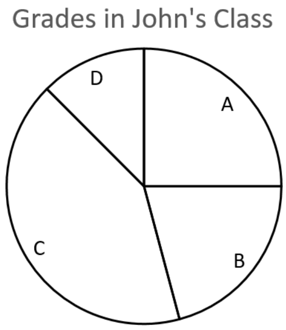
18 Pie Charts

Question 1: $\frac{90°}{360°} \times 32 = 8$ students

Question 2: $\frac{60°}{360°} \times 510$ cars $= 85$ yellow cars.

Question 3: John recorded the grades of 24 pieces of homework $(6+5+10+3=24)$.
The angle for the grade A slice must be: $\frac{6}{24} \times 360° = 90°$.
The angle for the grade B slice must be: $\frac{5}{24} \times 360° = 75°$.
The angle for the grade C slice must be: $\frac{10}{24} \times 360° = 150°$.
The angle for the grade D slice must be: $\frac{3}{24} \times 360° = 45°$.

Drawing the circle with a compass, and measuring the angles with a protractor,



Question 4: a) $40° = 600$ cars.
$1° = 600 \div 40 = 15$ cars per $1°$.
$85° = 85 \times 15 = 1275$ Renault cars.

b) $1°$equates to 15 cars, so, $15 \times 360 = 5,400$ cars.

Question 5: Oliver has 12 hours of leisure time in total and this is represented by a $60°$ slice of the pie chart. Hence, Oliver plays golf for: $\frac{60°}{360°} \times 12$ hours $= 2$ hours.

If Lewis spends 2 hours of the available 9 playing golf, this will be represented by $\frac{2}{9} \times 360° = 80°$.