MME

GCSE MATHEMATICS

AQA | Edexcel | OCR | WJEC

Iterative Methods

Please write clearly in block capitals

Forename:
Surname:

Materials

For this paper you must have:
- mathematical instruments

You ***can*** use a calculator.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- You may ask for graph paper, tracing paper and more answer paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

1 The table below shows values x_1, x_2, x_3, x_4 and x_5 for the equation $x_{n+1} = \sqrt{4 + x_n}$

x_n	x_{n+1}
x_1	2
x_2	2.449
x_3	2.539
x_4	2.557
x_5	2.561

1(a) Use the table above to estimate a solution to $x_{n+1} = \sqrt{4 + x_n}$, to 1 decimal place.

[1 mark]

Answer _____

1(b) Explain your answer.

[1 mark]

Answer _____

Turn over for next question

2

2 Using $x_{n+1} = \sqrt[3]{6 - 4x_n}$

With $x_0 = 0.5$

Find the value or x_1, x_2 and x_3.

Give your answers to 3 decimal places.

You must show all your working.

[3 marks]

Answer _____

3

3 Using $x_{n+1} = -3 - \frac{2}{x_n{}^2}$

With $x_0 = -3$

3(a) Find the values of x_1, x_2 and x_3

[3 marks]

Answer _____

3(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation

$$x^3 + 3x^2 + 2 = 0$$

[2 marks]

Answer _____

4(a) Circle the correct iteration formula which can be found by rearranging the equation $10 = x^2 + x$

[1 mark]

$x_{n+1}^2 = 10 - x_n$

$x_{n+1} = \sqrt{10 - x_n}$

$x_{n+1}^2 = \sqrt{10} - x_n$

$x_{n+1} = \sqrt[3]{10 - x_n}$

4(b) Circle the correct iteration formula which can be found by rearranging the equation $x^3 - 5x + 1 = 0$

[1 mark]

$x_{n+1} = \sqrt[3]{5x_n - 1}$

$x_{n+1} = \frac{x_n}{5} - \frac{1}{5}$

$x_{n+1} = \sqrt[3]{\frac{x_n}{5} - \frac{1}{5}}$

$x_{n+1} = 5 + \frac{x_n}{5}$

4(c) Circle the correct iteration formula which can be found by rearranging the equation $x^3 - 10x^2 - 10 = 20$

[1 mark]

$x_{n+1} = \sqrt[3]{10x_n^2 + 30}$

$x_{n+1} = 10 + \frac{x_n^2}{10}$

$x_{n+1} = \sqrt[3]{\frac{x_n^2 - 30}{10}}$

$x_{n+1} = \sqrt[3]{10 + \frac{x_n^2}{10}}$

Turn over for next question

3

5 $3x^3 + 6x = 4$ has a solution between 0 and 1

5(a) Find an appropriate iteration formula for the equation $3x^3 + 6x = 4$

[2 marks]

Answer _____

5(b) Starting with $x_0 = 0$, use your iteration formula from part (a) three times to find an estimate for the solution to $3x^3 + 6x = 4$.

[3 marks]

x_3= _____

Turn over for next question

5

6 A tank of water is slowly leaking.

One morning, the volume of water in the tank is V_A

The next morning, the volume of water in the tank is given by $V_{A+1} = 0.98V_A$

On Monday morning, there was 50 litres in the tank.

What will the volume of water be on Friday morning?

Give your answer to the nearest whole litre.

[3 marks]

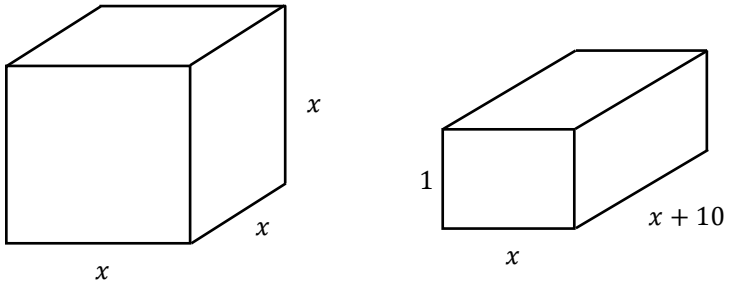
Answer _____

3

7 The diagram shows a cube and a cuboid.

All the measurements are in cm.

The volume of the cube is $20\ \text{cm}^3$ more than the volume of the cuboid.



Not drawn accurately

7(a) Show that $x^3 - x^2 - 10x = 20$

[2 marks]

Answer _____

7(b) Use an appropriate iteration formula to find x correct to 2 decimal places for

$$x^3 - x^2 - 10x = 20$$

You must show your working

[4 marks]

Answer _____

End of Questions

6

END