AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Integration (Answers)

Name:

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Total Marks: /44

C3 - Integration (Answers)
MEI, OCR, AQA, Edexcel

1. Calculate the following integrals. Remember to include a constant of integration where necessary:

   (a) $x^2 + c$. [1]

   (b) $-\cos x + c$. [1]

   (c) $\ln x + c$. [1]

   (d) 2. [1]

2. Calculate the following integrals by using integration by substitution:

   (a) $\frac{e^{x^2}}{2} + c$. [2]

   (b) $-\frac{1}{3}\cos(x^3) + c$. [2]

   (c) $\frac{1}{2}e^{(x+1)^2} + c$. [3]

   (d) $-\ln(\cos x) + c$. [3]

   (e) $-\frac{1}{2}\cos^2 x + c$. [3]

   (f) $\frac{\ln^2 x}{2} + c$. [3]

3. *Challange:* Using the fact that $\int_{-\infty}^{\infty} e^{-x^2}\ dx = \sqrt{\pi}$, evaluate the following integral:

$$\int_{-\infty}^{\infty} e^{(2-x)(2+x)}\ dx.$$

This question just requires some clever algebraic manipulation:

$$\begin{aligned}\int_{-\infty}^{\infty} e^{(2-x)(2+x)}\ dx &= \int_{-\infty}^{\infty} e^{4-x^2}\ dx \\ &= \int_{-\infty}^{\infty} e^4 e^{-x^2}\ dx \\ &= e^4 \int_{-\infty}^{\infty} e^{-x^2}\ dx \\ &= e^4\sqrt{\pi}.\end{aligned}$$

4. Calculate the following integrals by using integration by parts:

(a) $\sin x - x\cos x + c$. [3]

(b) $x\sin x + \cos x + c$. [3]

(c) $2x\sin x - (x^2 - 2)\cos x + c$. [3]

(d) $x\ln x - x + c$. [3]

(e) $\frac{1}{16}x^4(4\ln x - 1) + c$. [3]

Turn over

5. *Challange:* By using the technique of integration by parts, evaluate the following integral:

$$I = \int \sin(2x)\sin(x)\ dx.$$

[5]

This is a difficult question. We start by using integration by parts once using the following:

$$u = \sin(2x), \qquad v' = \sin x,$$
$$u' = 2\cos(2x), \qquad v = -\cos x.$$

This yields:

$$I = -\sin(2x)\cos x + 2\int \cos(2x)\cos x\ dx.$$

We now integrate by parts again to evaluate the integral on the right, this time using:

$$u = \cos(2x), \qquad v' = \cos x,$$
$$u' = -2\sin(2x), \qquad v = \sin x.$$

This yields

$$\begin{aligned} I &= -\sin(2x)\cos x + 2\left[\sin x\cos(2x) + 2\int \sin(x)\sin x\ dx\right] \\ &= -\sin(2x)\cos x + 2\sin x\cos(2x) + 4\int \sin(2x)\sin x\ dx \\ &= -\sin(2x)\cos x + 2\sin x\cos(2x) + 4I \end{aligned}$$

And so we have an equation to solve:

$$I = -\sin(2x)\cos x + 2\sin x\cos(2x) + 4I.$$

Rearranging to make I the subject we get:

$$I = \frac{1}{3}\left(\sin(2x)\cos x - 2\sin x\cos(2x)\right).$$

Full marks should be awarded for the answer above. However, we can make some simplifications to the answer using trigonometric identities:

$$\begin{aligned} I &= \frac{1}{3}\left(\sin(2x)\cos x - 2\sin x\cos(2x)\right) \\ &= \frac{1}{3}\left([\sin(2x)\cos x - \sin x\cos(2x)] - \sin x\cos(2x)\right) \\ &= \frac{1}{3}\left(\sin x - \sin x\cos(2x)\right) \\ &= \frac{1}{3}\sin x\left(1 - \cos(2x)\right) \\ &= \frac{1}{3}\sin x\left(1 - (\cos^2 x - \sin^2 x)\right) \\ &= \frac{1}{3}\sin x(2\sin^2 x) \\ &= \frac{2}{3}\sin^3 x. \end{aligned}$$

6. Consider the function $y = x \sin x$ sketched below:

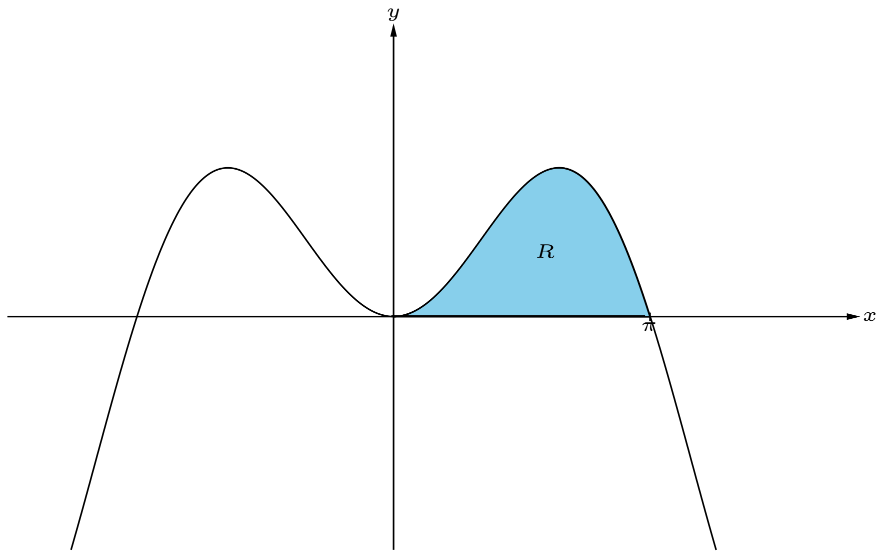

Figure 1: The graph of $y = x \sin x$.

(a) To calculate the area of the region R, we simply evaluate $\int_0^\pi x \sin x \, dx$. To do this we use integration by parts:

$$u = x, \qquad v' = \sin x,$$
$$u' = 1, \qquad v = -\cos x.$$

Thus,

$$\begin{aligned} \int_0^\pi x \sin x \, dx &= [-x \cos x]_0^\pi + \int_0^\pi \cos x \, dx \\ &= \pi + [\sin x]_0^\pi \\ &= \pi. \end{aligned}$$

And so the area of the region R is π!

[3]